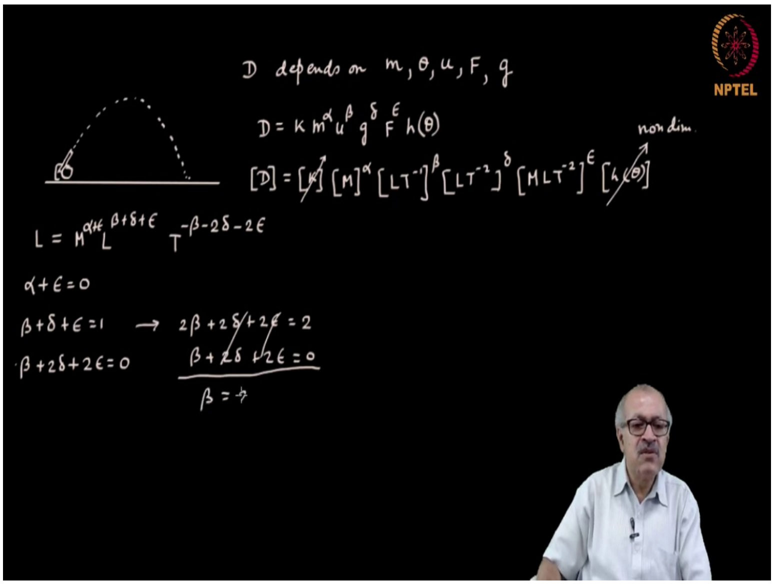
Research Methodology
Prof. Soumitro Banerjee
Department of Physical Sciences
Indian Institute of Science Education and Research, Kolkata

Lecture - 54
Theoretical Research: Modeling Using Dimensional Analysis Part 02

In this case we had a situation where the power equation was exactly solvable, so that you could obtain an expression almost exactly. The only unknown term was the h of theta. But in all problems the power equations may not be solvable because the number of equations might be smaller than the number of unknowns. When that happens, we normally try to express it in terms of one of the powers, and express it in terms of just one unknown and then proceed. Let me illustrate that with an example.

(Refer Slide Time: 01:18)



Suppose, we consider the same problem where we do not know about the Newton's laws. And we are trying to figure out how far will a cannon's throw would go, but this time we consider the air friction. So, when we try to do that, let us see how we can proceed using dimensional analysis.

In the earlier part, we assumed that the air friction is negligible, and after having done the derivation we might perform the test and might find that it does have some effect. Therefore, that cannot be negligible and, so we have to derive the expression involving the air friction. How do we go about it?

Again we have to guess what does D depend on without any prior knowledge. It might depend on the mass, it might depend on the angle, it might depend on the initial velocity and it might depend on the force of air friction, let us call it F, and of course, it will depend on g.

So, these are the quantities we assume that it depends on. So we again write the equation as D is some a constant k times m to the power alpha, u to the power beta, g to the power delta. Now, we have to introduce this. Let us call F to the power some other Greek letter, let us say epsilon. And then theta will again be a non-dimensional quantity and therefore, it can take any transcendental functional form.

$$D = k\, m^{\alpha}\, u^{\beta}\, g^{\delta}\, F^{\epsilon}\, h(\theta)$$

So, this is the basic equation we start with, and then we cast it in the form of the dimensional equation. So, the dimension of D should be equal to the dimension of the RHS. K we can ignore if we assume that K is non-dimensional.

The dimension of m is M to the power alpha, dimension of u is L T inverse to the power beta, dimension of g is L T minus 2 to the power delta. Then F is force, is equal to mass into acceleration, so M times L T minus 2 to the power epsilon. h does not have a dimension, so we cannot we can ignore that.

$$\begin{aligned} [D] &= [k][m]^{\alpha}[u]^{\beta}[g]^{\delta}[F]^{\epsilon}[h(\theta)] \\ L &= M^{\alpha}\left(LT^{-1}\right)^{\beta}\left(LT^{-2}\right)^{\delta}\left(MLT^{-2}\right)^{\epsilon} \\ &= M^{\alpha+\epsilon}\, L^{\beta+\delta+\epsilon}\, T^{-\beta-2\delta-2\epsilon} \end{aligned}$$

We can write h of theta, but then we have to ignore that. Not ignored; we assume that this is non-dimensional.

We need to work out this M to the power alpha, L to the power beta, L to the power delta and L to the power epsilon. L to the power beta plus delta plus epsilon. T to the power minus beta, T to the power minus 2 delta, T to the power minus 2 epsilon. So, T to the power minus beta minus 2 delta minus 2 epsilon.

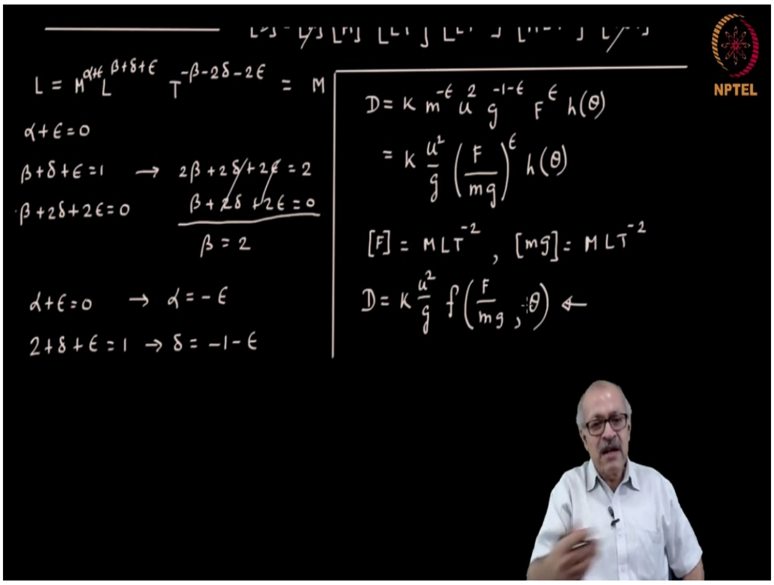
So, now we need to do a dimensional analyis, write down the dimensional equations. There is no M on the left hand side. So, alpha plus epsilon should be equal to 0. There is an L with power 1. So, beta plus delta plus epsilon should be equal to 1. There is no T on the left hand side. So, we can write it plus minus beta plus 2 delta plus 2 epsilon is equal to 0.

$$\alpha + \epsilon = 0, \quad \beta + \delta + \epsilon = 1, \quad -\beta - 2\delta - 2\epsilon = 0$$

Now, I can see that from these two equations we can extract the value of beta. All that we have to do is to multiply this equation by 2 and subtract from here. So, if we do that, this becomes 2 beta plus 2 delta plus 2 epsilon is equal to 2 and this one is beta plus 2 delta plus 2 epsilon is equal to 0. And if you subtract this cancels off this is beta is equal to 2.

So, we get beta equal to 2 without any difficulty. But we cannot get the rest because there are how many unknowns: alpha, epsilon, delta three unknowns and we do not have as many equations.

(Refer Slide Time: 08:44)



And so, what remains in the equations after having obtained beta, is alpha plus epsilon is equal to 0 and beta is there. So, this equation becomes 2 plus delta plus epsilon is equal to 1 and this one also gives the same equation. So, we have alpha plus epsilon equal to 0. This we now express everything in terms of one of these unknown quantities. Suppose that unknown quantity is epsilon.

In that case, we have to express alpha as minus epsilon. From here we get delta is equal to minus 1 minus epsilon. We have obtained this: beta is known, alpha is known, delta is known. So, everything is known. Let us now substitute into this equation. We can write D is equal to K times m to the power alpha and alpha was minus epsilon. So, m to the power minus epsilon. u to the power beta and beta is 2. g to the power delta, so g to the power minus 1 minus epsilon. F to the power epsilon and h of theta.

$$
\begin{aligned}
D &= k\, m^{-\epsilon}\, u^2\, g^{-1-\epsilon}\, F^{\epsilon} h(\theta) \\
&= k\, \frac{u^2}{g} \left(\frac{F}{mg}\right)^{\epsilon} h(\theta)
\end{aligned}
$$

So, now, we have been able to express it in terms of just one unknown power. And if we rearrange it a bit, put all the epsilon terms in one place and non-epsilon terms separately. So, this becomes u square g to the power minus 1, u square by g. Then all the terms with epsilon put together becomes F to the power epsilon by m to the power minus epsilon. So, m in the denominator, g in the denominator to the power epsilon, times h of theta.

Now, you notice that this is the extent that dimension analysis can bring us. But we can go a little further. Because F has a dimension of M L T minus 2, and the dimension of mg should be M, L T minus 2. Therefore, they have the same dimension. Therefore, F by mg is non-dimensional.

And if F by mg is non-dimensional, there is no reason to expect that it will take only a power. A non-dimensional quantity can take any transcendental functional form without any difficulty. Therefore, we have actually derived from the dimensional analysis, positively is this part. But, we now know notice that this quantity and theta both individually are non-dimensional quantities. And therefore, there can be some kind of a functional form involving F by m g and theta.

This has to go together, because it together is non-dimensional. And therefore, it will be a transcendental function of two variables. One variable is a composite variable that totally is non-dimensional and theta is non-dimensional. So, this is how we write the equation.

$$D = k \frac{u^2}{g} h\left(\frac{F}{mg}, \theta\right)$$

This is as far as the dimensional analysis can bring us. But, you can immediately see that this points to a way of conducting directed experiments to obtain that functional form.

In that case we will do something so that we will use the same velocity and maybe keep this also constant and vary only this, thereby get different distances and draw a graph that will give us the dependence on theta. And similarly, you keep theta constant, u constant and throw with different masses and you will then get different values of this. From these two, you will get a functional dependence on both these quantities.

This is a way of obtaining a mathematical model without any use of a theoretical background. We have not used theory anywhere in this method. The resulting models are normally called the phenomenological models. A phenomenological model is something that does not contradict existing theory, but is not derived from existing theory. It is derived from something else, some other consideration, not from the first principles.

What we did in the last class was to derive from the first principles. But a phenomenological model is derived not from the first principles, but it should not contradict what is already known.

The models like this, that are derived by the method that I just shown, are typical examples of phenomenological models. Remember, the word phenomenology has different connotations in different fields. For example, in particle physics phenomenology would mean deriving experimentally testable predictions from existing theory. Supposing there are a few competing theories in place for a particular experimental situation. What will be the prediction of this one, what will be the prediction of that one, obtaining that is said to be doing phenomenology in particle physics. But a phenomenological model is something else. Also in philosophy, phenomenology concerns how we think and all that. We are not talking about that. A phenomenological model is a model that is not derived using existing theory, but that does not contradict the existing theory. So, these are the ways of obtaining some phenomenological models.

And once a phenomenological model is derived, theorists will get busy in trying to figure out why this particular functional form is true. This is how science progresses.