Research Methodology
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Lecture - 44
Statistical Methods in Hypothesis Testing: Z-Test and T-Test, Part 02

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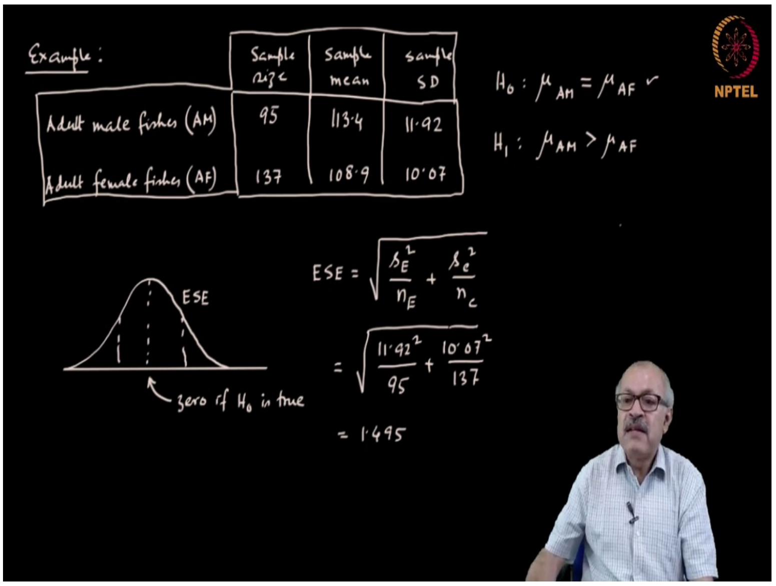

Let us do an example. Suppose there is a field biologist, who is trying to study the character of a specific species of fish, and she is interested in knowing: "Are the males bigger than the females?". Now, you know that there are some species in which the females are bigger than the males, in some species they do not have any difference, in some species, males are bigger than the females.

So, in this particular one, she has to get an idea of whether she can make the statement that the males are bigger than the females. For that, if she were able to collect all the fish and measure them, then she would get a correct idea. But she cannot do that and she has to take samples.

Now, suppose she has taken the following samples: adult male fishes of the particular species, let us call it adult male AM. We are also trying to figure out the adult females AF. Now, we will need to know the sample size. We will need to know the sample mean. She has collected samples and getting the means is not a big problem. So, she has collected and obtained the means and also sample standard deviations.

These are the data she has collected. The numbers were 95, 113.4 grams and 11.92 was the standard deviation. In this case, she got a larger number, and here 108.9 and 10.07. Now, we are trying to figure out this character. Again, we have to clearly state what is the null hypothesis.

Null hypothesis, again, will be that the mean of the adult male is equal to mean of the adult female. We should not start with assuming the alternative hypothesis. The alternative hypothesis will be that the mean of the adult male will be greater than mean of the adult female. We have to start with this assumption, we have to test whether we have enough justification to reject the null hypothesis. So, how do you do that?

In this case, we know that if we did this experiment again and again, that means, we collect different samples every time and we obtain the sample means, the difference of the means will be distributed in a normal distribution because the number is larger than 25. The standard deviation will be zero if H0 is true. We assume the H0 is true, therefore, the mean is zero.

The standard deviation, in this case, will be the estimated standard error, which we can calculate as s E square by the number E plus s C square by the number C is equal to square root of s E is 11.92 square by 95 plus this is 10.07 square by 137 and this, when calculated, yields 1.495.

$$\text{ESE} = \sqrt{\frac{s_{\text{AM}}^2}{n_{\text{AM}}} + \frac{s_{\text{AF}}^2}{n_{\text{AF}}}} = \sqrt{\frac{11.92^2}{95} + \frac{10.07^2}{137}} = 1.495$$

Therefore, the test statistic is this one minus this one, so, x adult male bar minus x adult female bar. This is the test statistic, divided by the ESE, that will be the z. Now, this x AM is 13.4; 113.4 minus 108.9 divided by the estimated standard error was 1.495. That yields 3.01. Now, 3.01 which is bigger than 1.96. If it is bigger than 1.96, then it is very unlikely to get a value of z like this if the null hypothesis is true.

$$z = \frac{\bar{x}_{\text{AM}} - \bar{x}_{\text{AM}}}{\text{ESE}} = \frac{113.4 - 108.9}{1.495} \approx 3.01$$

Therefore, we can state with more than 95 percent confidence that the null is false. If the null is false, then we basically assume that the alternative hypothesis, which says that the adult male is bigger than the adult female, that is true. So, this is how the we actually do the test.

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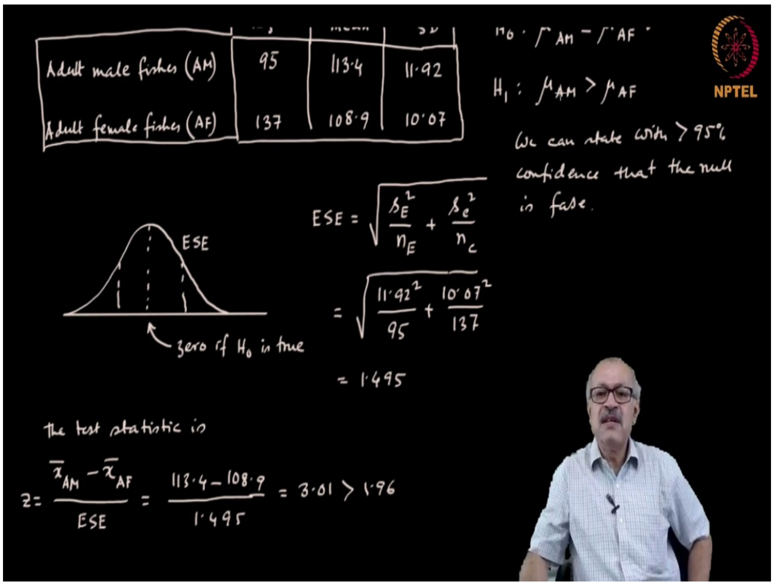

Now, the above method, where we use the z table to take decisions regarding the acceptability of the null hypothesis, that is applicable to situations where we are able to take at least 25 data points for each group. Normally, this would be so and that is the usual method used. But there can be situations, somewhat rare, but there still possibility exists, that there would be situations where we are unable to take 25 samples.

You were unable to take 25 samples because the disease is rather rare. So many people afflicted with the disease are not available in an area. There are various possibilities. For example, you are testing a hypothesis for which collecting the sample is very expensive, or labor intensive. There has to be proper justification as to why you fail to collect more than 25 samples.

But suppose you can justify that because there is a valid justification existing. In that case, we would not be able to use the z table because the distribution of the differences of the sample means will not follow a normal distribution.

Similar situations arose when we were studying measurement, where we were unable to take 25 measurements. There we saw that in that situation, the t-statistics becomes useful. Here also, we will use the same avenue; the t-statistics will become useful.

In that case, if the number of samples is smaller, for example, 10, 11 or something like that, then the number of samples also becomes a something that we use in order to infer something in the test from the statistics. While in case of the z-statistics, if the number of samples is more than 25, then we assume that the distribution is normal and therefore, we do not need to use the number of samples. But in case of the t-statistics, we do need to use the number because, for each degree of freedom, the distribution will be different.

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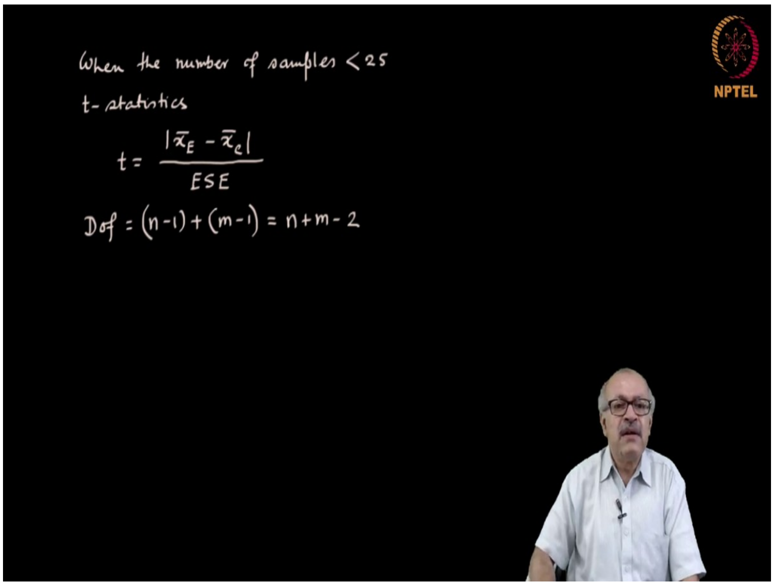

So, in this case, the methodology, the approach ─ everything remains the same. Only we have to use the t-table instead of the z-table. So, when the number of samples is less than 25, then we have to use the t-table, the t-statistics, that is what you have to use. In that case, the way we obtained the z, we obtain the value of t and the method is the same.

So, we use x E bar minus x C bar. Any one could be bigger than the other. So, this could be a positive value or the negative value. In order to get a positive value, we put a mod.

So, we get a number here and then, that has to be divided by the estimated standard error. This is how we obtained the z value; this is how we obtain the t value also.

But now, we have to consult the t-table, and the t-table demands that we specify the degree of freedom. Now, in this case, the experimental group had n number of samples, the control group had say m number of samples. So the degree of freedom in the experimental group is n minus 1 and the degree of freedom in the control group is m minus 1. You have to add these two: so, that will be n plus m minus 2. That will be the degree of freedom in this case.

On that basis, we consult the t-table, and we make a decision. We have already seen that in the t-table, things are given in terms of the degree of significance, which is 1 minus the degree of confidence.

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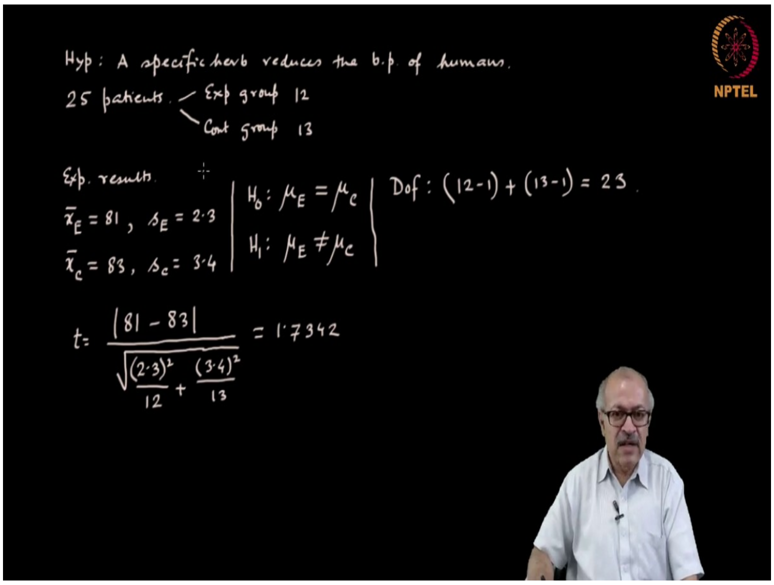

We will work out an example and on that basis, we will see how this is to be applied. Let us go to an example. Suppose somebody has proposed that a herb relieves a certain kind of disease, for example, blood pressure in humans: a particular herb reduces the blood pressure in humans. So, the hypothesis is: a specific herb reduces the BP of humans and therefore, we are trying to test whether that herb will be useful in case of hypertensive patients.

Suppose you have been able to collect total 25 patients, this will have to be divided into the experimental group and the control group. So, 25 patients, by itself, does not suffice. Suppose we divide into the experimental group and control group. The experimental group has 12 patients and the control group has 13 patients, and we are doing the test accordingly.

So, what do we do? We will apply the herb on the people in the experimental group for some period of time, and we will give a placebo (which is similar to it herb, but is not the herb itself) to the people in the control group. After some time, as hypothesized, we will measure the blood pressure. After having measured the blood pressure of all of them, we take the mean, we take the standard deviation and suppose the experimental results came as follows.

x E, this is the mean of the experimental group's the diastolic blood pressure came out to be 81 and the standard deviation of the exponental group came out to be 2.3. The control group's average came out to be 83 and the SD came out to be 3.4. It is clear that the mean value for the experimental group is lower than the mean value of the control group.

So, it appears that the herb was really effective in reducing the blood pressure. But let us check whether this particular conclusion stands on statistical scrutiny. So, let us proceed. We have to first state the null hypothesis and the experimental hypothesis based on the way the test is conducted.

The null hypothesis, H naught, will say that the mean of the experimental population should be equal to the mean of the control population. These are different: this is for the sample, this is for the population.

This is all the population of people who are hypertensive, who have high blood pressure, all this population, but having been applied with this herb. And this is not having been applied with herb. The null hypothesis will say that they will lead to the same behavior, while the sample showed a difference. So, we have to check whether the difference is due to some random fluctuations.

The alternative hypothesis will then be that mu E is not equal to mu C. As always, we have to root ourselves on the null hypothesis and check if we have sufficient ground to reject the null hypothesis.

We then calculate the t-value, which can be calculated from this. This is 81 minus 83, we have to take a mod of that, and then, in the denominator, is the estimated standard error. The estimated standard error would be the square root of this number 2.3 whole square by the number in that group which is 12 plus this will be 3.4 whole square by 13. This turns out to be 1.7342. So, this is the t-value.

$$t = \frac{|81 - 83|}{\sqrt{\frac{2.3^2}{12} + \frac{3.4^2}{13}}} = 1.7342.$$

And now, we have to consult the table, but before we do that, we have to figure out what is the degree of freedom. 12 is the number in the experimental group. So, 12 minus 1 plus 13 minus 1 is equal to 12 plus 13, 25 minus 2, 23. That would be the degree of freedom and with that, we have to consult the table. So, let us consult the table.

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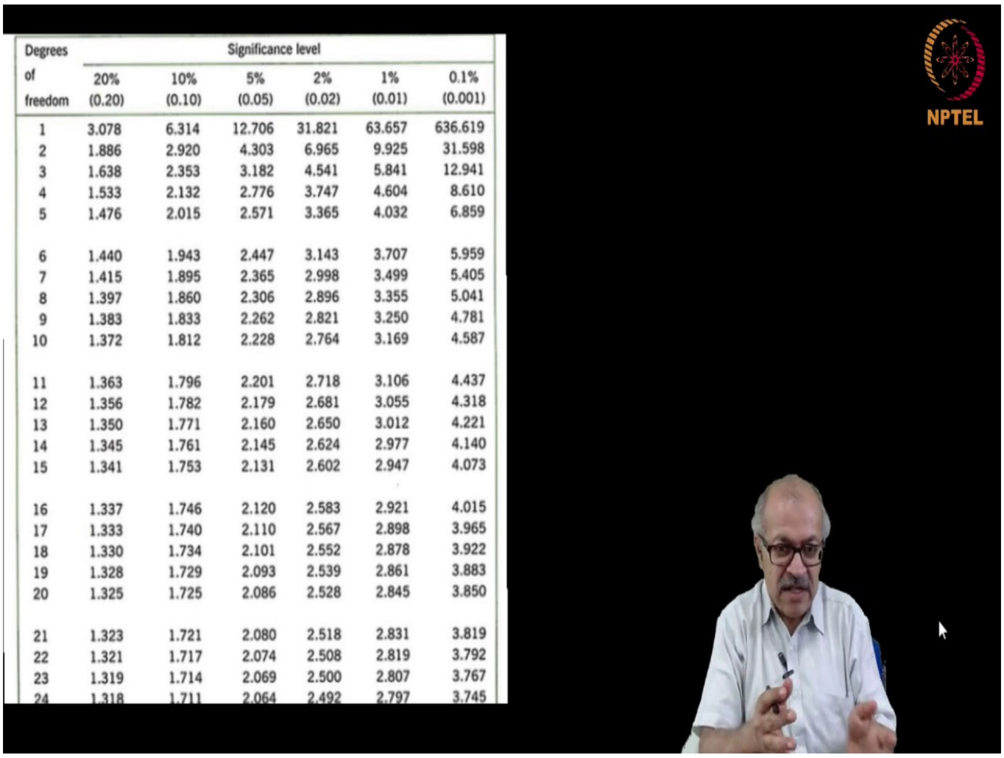

Degrees of freedom	Significance level 20% (0.20)	10% (0.10)	5% (0.05)	2% (0.02)	1% (0.01)	0.1% (0.001)
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745

Now, here is the t-table. As you can see, these are tabulated in terms of the degrees of freedom and the significance level. The significance level is 1 minus the confidence level. So, 0.0 5 significance level is equivalent to 95 percent confidence level. In this case, we found that the degree of freedom is 23. So, we come down this row, this column and come here, this is 23.

And along this, this is the column for 95 percent confidence or 0.05 significance level. We come down this and in the intersection, we find this particular number 2.069. So, this is the value that we have to take now. Accordingly, we go ahead with this value 2.069.

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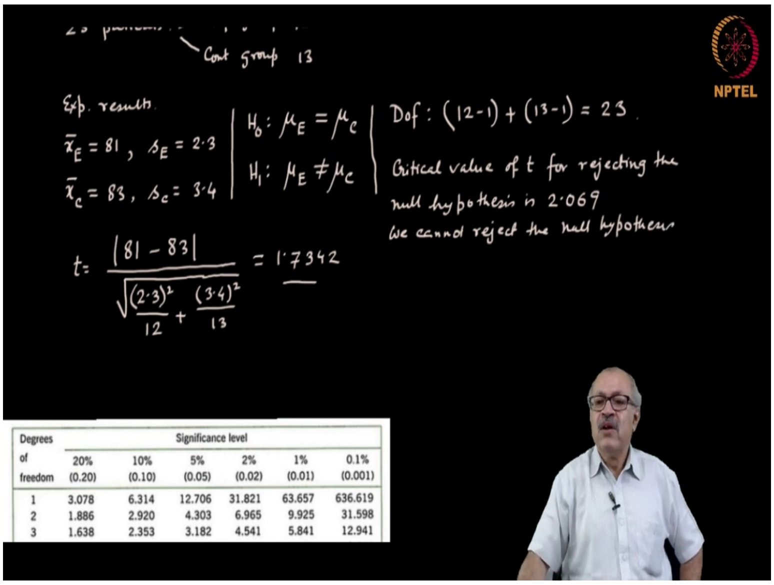

So, we were here. The value that we got, the critical value of t for rejecting the null hypothesis, is in this case 2.069. And we have got the t-value of 1.7342, which is below that. This means that the difference that we see is possible to get even if the null hypothesis is true, because of the variability of the data, because of the variability as signified by the standard deviation.

So, because of this, because of the random fluctuation of the data, if you repeatedly conduct the test by taking different collections of 25 people, dividing them into 12 and 13 groups, then you will get these numbers different and they will have a distribution.

The mean difference will have a distribution and for that distribution, it is quite likely to have this number, the difference, as one of the possibilities in the distribution, which means that this is within a statistical variability of the data if the null hypothesis is true. So, we say that we cannot reject the null hypothesis. Notice that in the t-table, we have the degree of freedom also embedded in the table. This is important for use of the t-table.

Now, in books you will find that different techniques are applicable to different situations, for example, if the two standard deviations are the same, then what will happen? If the number of samples in the two groups are the same, then what will happen?

There are all those details given in statistics textbooks. We will not go into those details. This is sort of a outline of the t-method. For our purpose, we will assume that we will use the t-table only sparingly, only in situations where we cannot use the z-statistics, only then. So, we will not get into the details of the t statistics. We will rather root ourselves on the z-statistics, but this is sort of an outline of how it is applied. For details, please refer to standard statistics textbooks. We will end this class here.