Research Methodology
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Lecture - 34
Error Bars and Confidence Interval, Part 02

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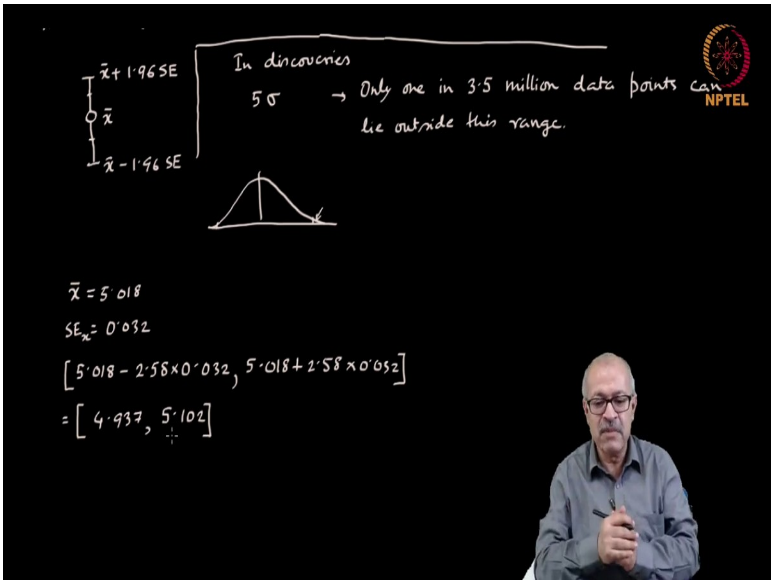

Whenever you make any statement of a measurement, you have to put the error bar and depending on the field in which you are reporting. There is some kind of a standard. Some fields demand 95 percent confidence, some fields demand 68.3 percent confidence. Depending on that, you can state your error bar.

But, a statement of a measurement without an error bar is in itself unacceptable. Because, it violates the basic principle of measurement that it has to be repeatable. If somebody else does this experiment somewhere else, he or she might be getting a different value of x bar.

But, his or her value will remain somewhere in this range and if that experiment is repeated again and again, it will be found that around 95 percent of the times the value that you get are remaining within this range. Therefore, the other experiments done by other people are actually conforming to the earlier measurement.

The measurement is repeatable in that sense. Notice a couple of things. Suppose I have made a measurement. Now, the reviewer says that this error bar is too large. See again the meaning of the error bar: that I am 95 percent confident that the actual value out there will lie within this range.

But if this range is too large, then the reviewer or the editor of the journal may think that it is too vague. It has to be more accurately measured, which means that they might demand that it has to made half, a shorter range. How do you do that? Simple. Because this range depends on the standard error and the standard error depends on the number of measurement samples, that n, square root of n. Therefore, if the error bar has to be halved, the number of samples has to be 4 times.

So, if you take 4 times the number of samples, then you can halve the error bar. This is point number 1. Point number 2 is that, you are after all taking samples from a population out there. If there is some kind of a bird whom you are sampling and you are trying to find out its average body weight, then there is a population out there, but the size of the error bar will not depend on the size of the population so long as the population size is much larger than the sample size. We have done a sampling and the population size is much larger. There is 1 million birds out there, I have caught some 15 and I have measured it. Then, it will not depend on the population size. The demand is that the population has to be much larger than the sample size. But, as I have said, it will depend on the sample size. So, if you want to make the error bar smaller, you have to take a larger number of samples.

There may be some cases where you might not be able to take 25 samples. What to do that in that situation? I will come to that later. For example, in the problem that I just calculated, suppose we had calculated x bar is equal to 5.018 and we had calculated SE of x as 0.032.

Then what should be with the error bar for a 99 percent confidence? It will be the range 5.018 minus (if it is 99 percent confidence) then it is 2.58 times the standard error 0.032, 5.018 plus 2.58 times 0.032. If you calculate that, it comes out to be 4.937 to 5.102.

So, with a confidence level of 99 percent you can state that the actual value will lie in a range something like this. So, this is a very concrete statement. Having made the measurement you are making a very concrete statement that I am 99 percent confident that this is the range in which it will lie. So, this is what comes out of a measurement. This is the scientific way of making and stating a measurement.

There are many situations, where you might not be able to collect 25 or more samples. Situations like that occur, for example in field biology or geology, where getting each data point involves going to the field, collecting samples, coming back with the samples, measuring them: expensive proposition.

In other fields like physics also, getting new data points might involve lot of expense and setting up new apparatus which might not be always possible. I would say that is not desirable. Wherever possible you should obtain 25 data points, because then you are confident about the result. But, suppose it is not possible. Then what do you do?

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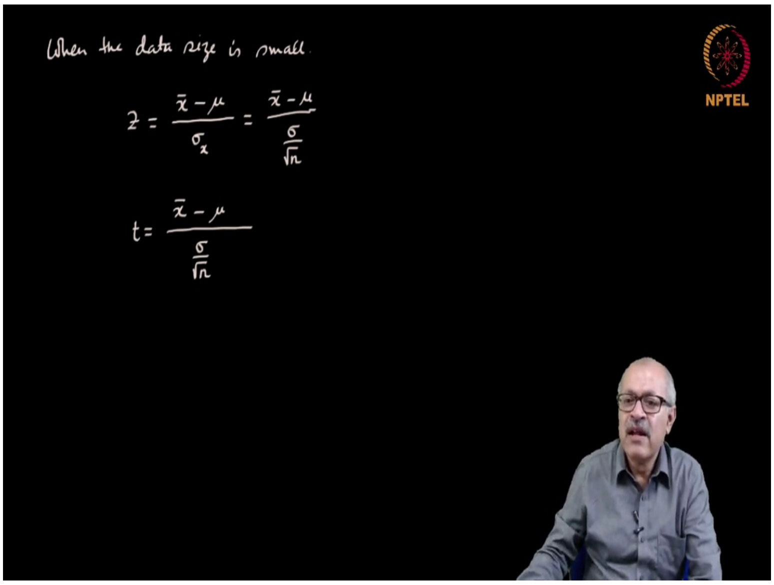

This is the situation when the data size is small. I have already said that in that situation, the central limit theorem will not be exactly applicable. The central limit theorem says that if the data size is the sample is 25 or more, then the distribution of the sample means would approximate a normal distribution. If the data size is small it will not approximate a normal distribution.

But, it will approximate some distribution. What distribution it approximates will depend on the data size. On that basis, we can still extract some meaningful results, even if the data size is small. But remember, this is not a shortcut. Wherever possible, you should actually take more than 25 data points and do it the way I earlier said. But in the cases where you cannot do that, there is still a way.

The way is that, in that case it will not go into a normal distribution. But it will be some distribution. That some distribution is called a t distribution. That distribution has been measured and we now have the t tables from which similar results can be obtained. Let me illustrate.

First, when we were doing the z measurement, what was the definition of z? It was basically the average value that you have measured minus the population mean divided by the sigma of that x. This is nothing but x measured minus mu, the mean value of x measured, by sigma by square root of n.

Now, if the n is small, then we do not call it a z value, and do not refer to the z table because then it will be erroneous. We call it the t value, the measured value of t. But the t, its definition is still the same. The measured mean minus the population mean divided by sigma by square root of n.

Again in this case, we do substitute sigma by the measured s, the standard deviation. We understand that it will incur some error. We understand that, but there is no other way. So, we do still use this logic in order to continue, in order to extract some kind of a meaningful result. Let me illustrate this with an example.

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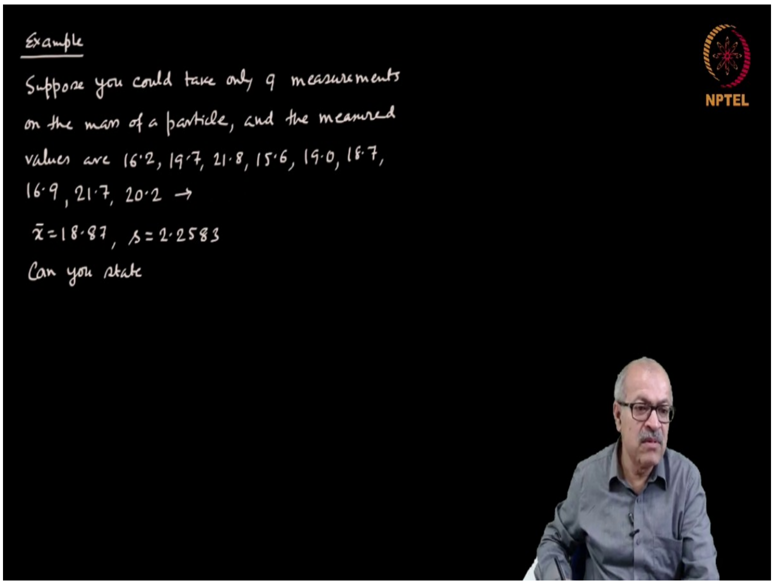

Suppose, you could take only 9 measurements on the mass of a particle. Let me write the measured values. The measured values are 16.2 (whatever unit), 19.7, 21.8, 15.6, 19.0, 18.7, 16.9, 21.7 and 20.2.

So, from this you can easily obtain x bar. x bar is 18.87 and s is 2.2583. Now the question is, on the basis of this result that you have got, can you state that the actual mass of the particle is (see the average that is calculated) below 21?

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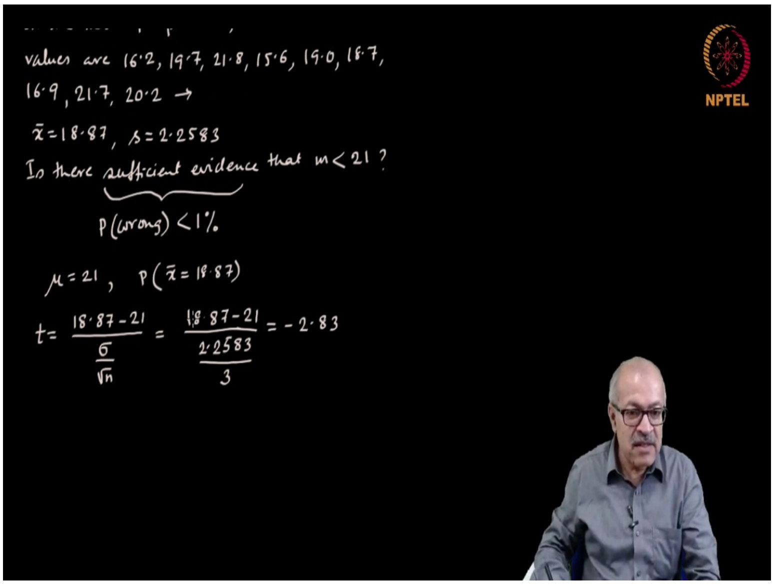

So, is there sufficient evidence that mass of the particle is less than 21? Now, what do you mean by sufficient evidence? It is basically that the probability that the statement is wrong is less than 1 percent. Sufficient evidence means, probability of 'wrong' is less than 1 percent. So, how do we proceed? We have already calculated the x bar and s. Now, the situation is the same as 'what is the probability of getting this value of x bar if the mass was actually 21?'

So, there is a range. It can be above 21 also. We are stating that 'it is not 21 or above' therefore, its lowest value is 21. So if the value is 21 then, what is the probability of getting, what I am getting, 18.87? What is the probability of getting that? So, our approach will be to assume that mu is 21, and check for the probability P that x bar is equal to 18.87. That is what we are trying to figure out.

Now, if this probability turns out to be less than 1 percent, then I can make that statement. If that probably turns out to be more than 1 percent, then I do not have sufficient evidence. Ok? So, this is how we proceed.

Now, in this case we cannot consult the z table. We have to consult the t table instead. First let us calculate the t value. t is, again, x bar minus mu: x bar is 18.87 minus 21 divided by sigma by square root n.

In this case, here I have square root of 9. But sigma I do not know. Therefore I have to substitute by the measured value here. So, this is 18.87 minus 21 by this. I will substitute by the measured value. Again it will incur a bit of error, but this is all we can do really. Square root of 9 is 3. This is equal to minus 2.83. Now, with that in hand, we have to consult the t table. Let us consult the t table now.

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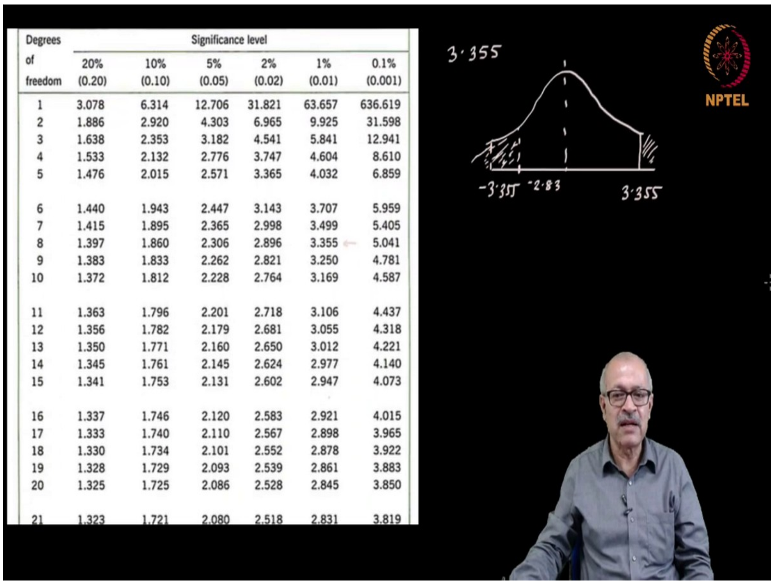

Degrees of freedom	Significance level					
	20% (0.20)	10% (0.10)	5% (0.05)	2% (0.02)	1% (0.01)	0.1% (0.001)
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819

Here is the t table. Notice here, this is organized in a way that is different from the way the z table is organized. You notice that there is something called 'significance level'. The significance level is 1 minus the confidence level. So, for 99 percent confidence level the significance level is 1 percent; for 95 percent confidence level the significance level 5 percent; and so on and so forth.

So, in this case, the demand was to have 1 percent significant level. So we have to look at this column. Now, here is the degrees of freedom. Degrees of freedom, as we know, is the number of data points minus 1. Number of data points was 9 and minus 1 is 8.

So, we have to go along this row and this column and we get this value. So, this is 3.355. That means, earlier we had the normal distribution. It will not be normal distribution curve; it will be a different curve. But nevertheless, the 1 percent area will be available outside 3.355. Since it is symmetric, 1 percent area will be available outside minus 3.355 also. So, 1 percent area is to the outside. The t value that we got was minus 2.83. So, minus 2.83 is somewhere here, which is to the right of this. Therefore the area to the outside of this will definitely be bigger than 1 percent.

Therefore, for this particular problem, we cannot have sufficient evidence to state that the mass of the particle is smaller than 21.

Why? Again let me repeat. Why? Because, we our demand was that the probability that this statement is wrong should be less than 1 percent. We assumed the limiting value 21, and we calculated what are the odds of getting this value of x bar in an experiment and we found that the odds, the probability, will be more than 1 percent. And therefore, the probability that I am wrong in making this statement is more than 1 percent. Therefore, we do not have sufficient evidence to make that claim.

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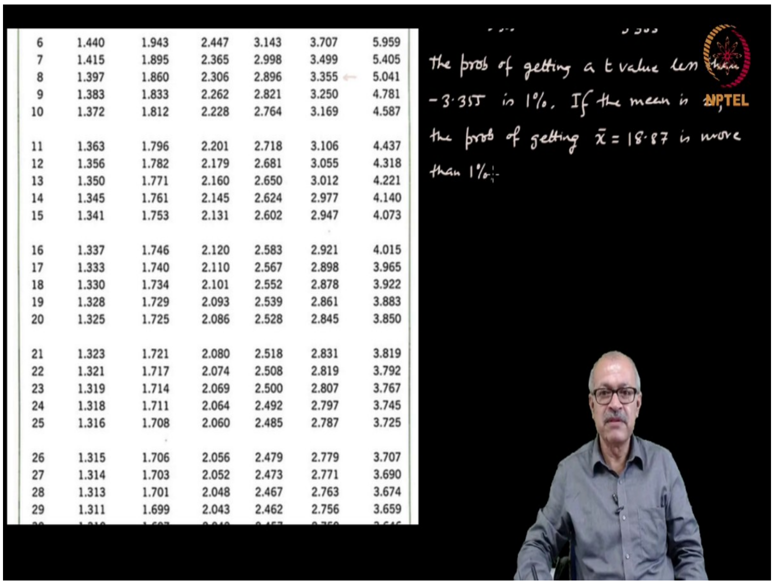

So the probability of getting a t value less than minus 3.355 is 1 percent. That is what the table says, and then, because of this, if the mean is 21, the probability of getting x bar as 18.87 is more than 1%. Therefore, we cannot state from the data that the mass of the particle is less than 21. That is the conclusion from this experiment. I will stop here and we will continue later.