Course Name: Watershed Hydrology

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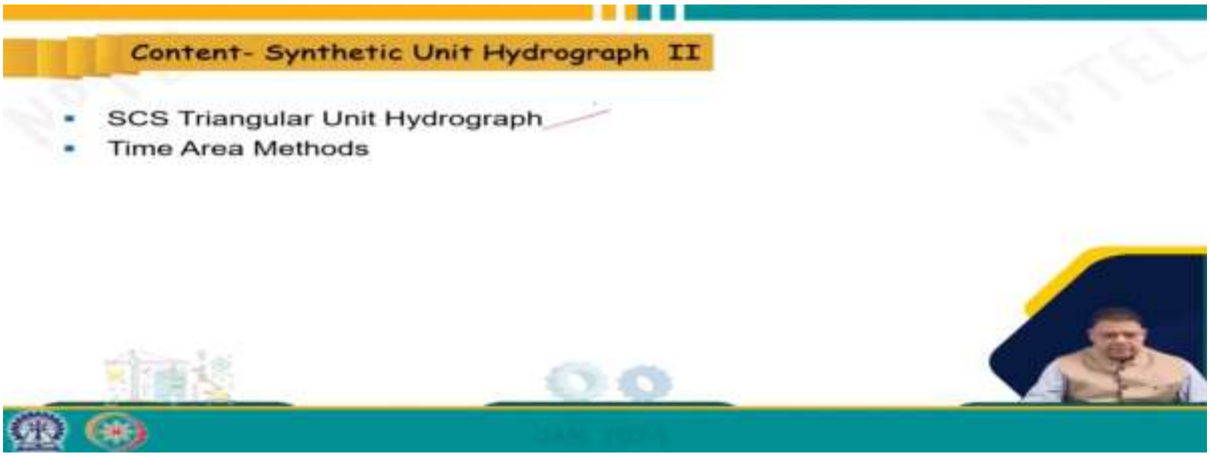
Institute Name: Indian Institute of Technology Kharagpur

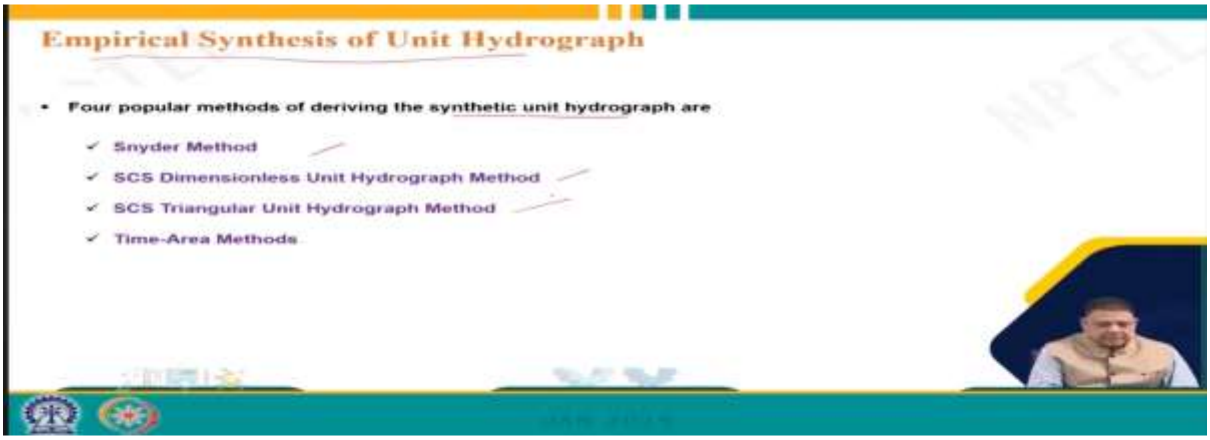
Week: 06

Lecture 27: Synthetic Unit Hydrograph II

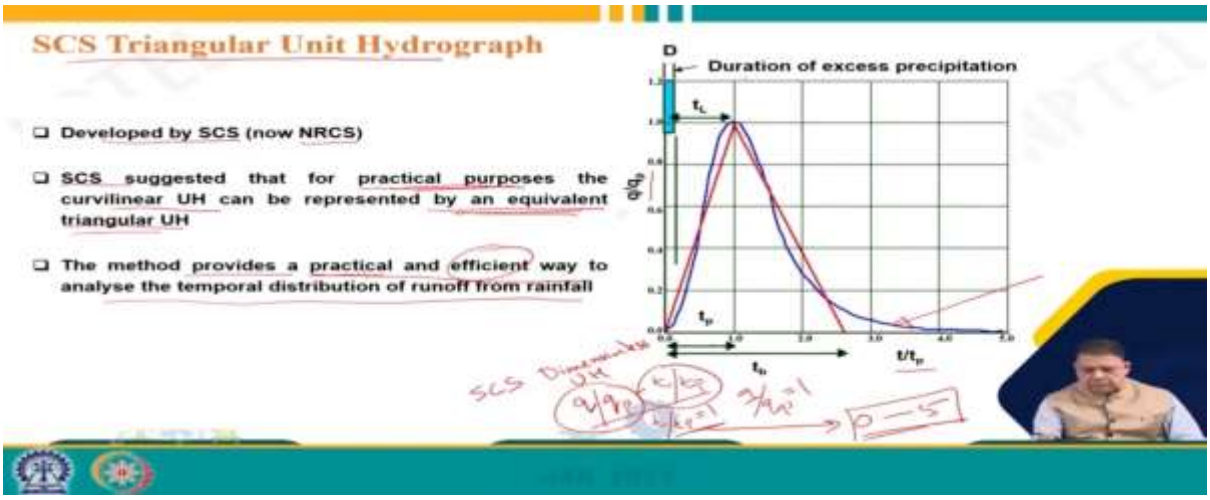
Hello friends, welcome back to this online certification course on Watershed Hydrology focusing on Synthetic Unit Hydrograph. I am Rajendra Singh, a Department of Agricultural and Food Engineering professor at the Indian Institute of Technology Kharagpur.



We are currently in Module 6, Lecture 2, which covers Synthetic Unit Hydrographs, specifically focusing on the SCS Triangular Unit Hydrograph and the Time-Area Methods for developing synthetic unit hydrographs.

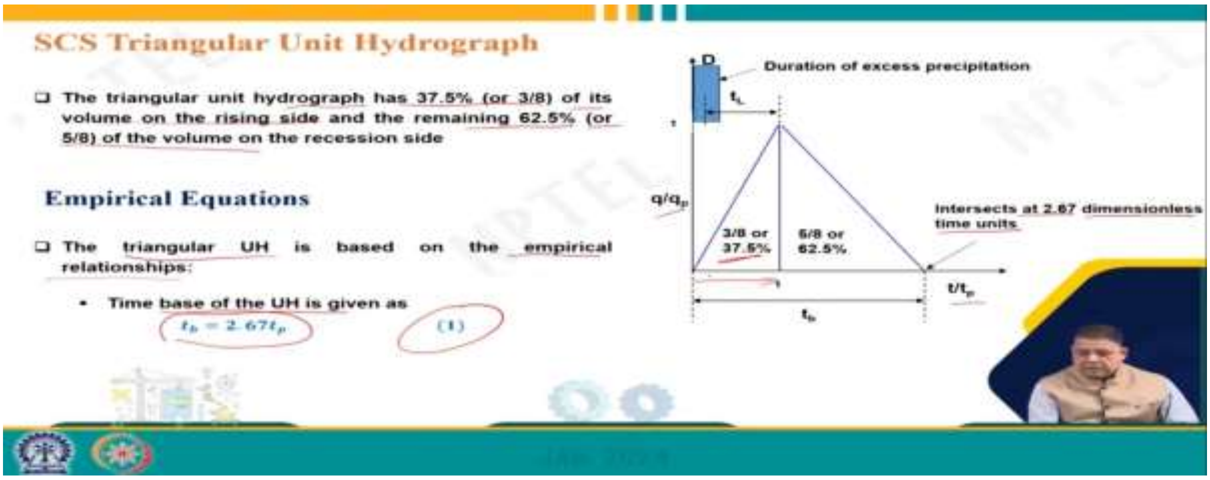


To recap our previous lecture, we delved into the empirical synthesis of unit hydrographs, particularly useful for small, ungauged watersheds where data may be limited. Synthetic unit hydrographs establish a relationship between hydrograph characteristics and basic watershed characteristics. We discussed four methods: the Snyder Method, the SCS Dimensionless Unit Hydrograph Method, the SCS Triangular Unit Hydrograph Method, and the Time-Area Method. In the last lecture, we covered the Slider Method and the SCS Dimensionless Unit Hydrograph Method. Now, we will proceed with the remaining methods in this session.



Starting with the SCS Triangular Unit Hydrograph Method, as the name suggests, it was developed by the Soil Conservation Service (SCS), now known as the NRCS (Natural Resources Conservation Service). SCS proposed that the curvilinear unit hydrograph can be approximated by an equivalent triangular unit hydrograph for practical purposes. In our previous discussion on the SCS Dimensionless Unit Hydrograph, we explored how it relates discharge (q) to peak discharge (q_p) and time (t) to peak time (t_p). We established that at $t/t_p = 1$, $q/q_p = 1$, and t/t_p ranges between 0 and 5.

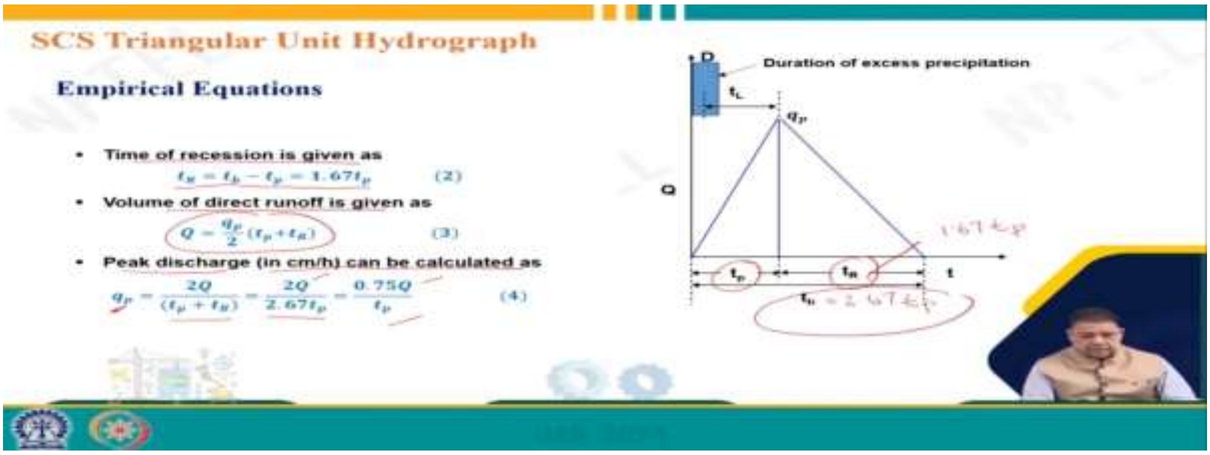
SCS provided a dimensionless unit hydrograph table, allowing us to find corresponding q/q_p values based on t/t_p. However, for practical efficiency, SCS proposed using an equivalent triangular unit hydrograph. This method offers a straightforward approach without reliance on tables, making it efficient for analyzing temporal runoff distribution from rainfall.



The triangular unit hydrograph distributes 37.5% or 3/8 of its volume on the rising side and the remaining 62.5% or 5/8 of the volume on the recession side. As depicted in the graph, the triangular unit hydrograph's volume distribution shows 37.5% on the rising side and 62.5% on the recession side, delineated by the time base (t) and discharge ratio (q/q_p) axis.

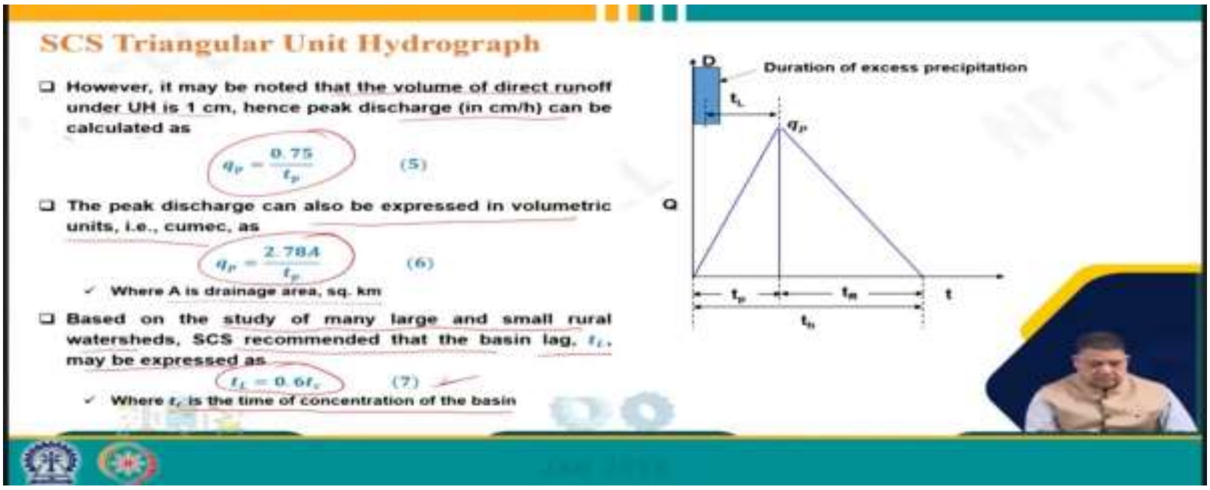
This triangular unit hydrograph is designed to ensure that the recession takes much longer, giving it a skewed distribution despite its triangular shape. As you can visualize, it has a longer right tail. Soil Conservation Service (SCS) suggested several empirical relationships for developing this triangular unit hydrograph method.

Starting with the empirical equations, the time base of the unit hydrograph is given by the relationship $t_b = 2.67t_p$. Here, t_b represents the time base, and t_p is the time to peak. These interact at 2.67 dimensionless time units, which is the maximum limit reached in the SCS dimensionless curve where t/t_p goes up to 5. This limitation arises due to the triangular shape of the hydrograph.



The time of recession is given as $t_r = t_v - t_p$, where tr represents the time of recession. It is equal to 1.67 times t_p, as t_v equals $2.67t_p$.

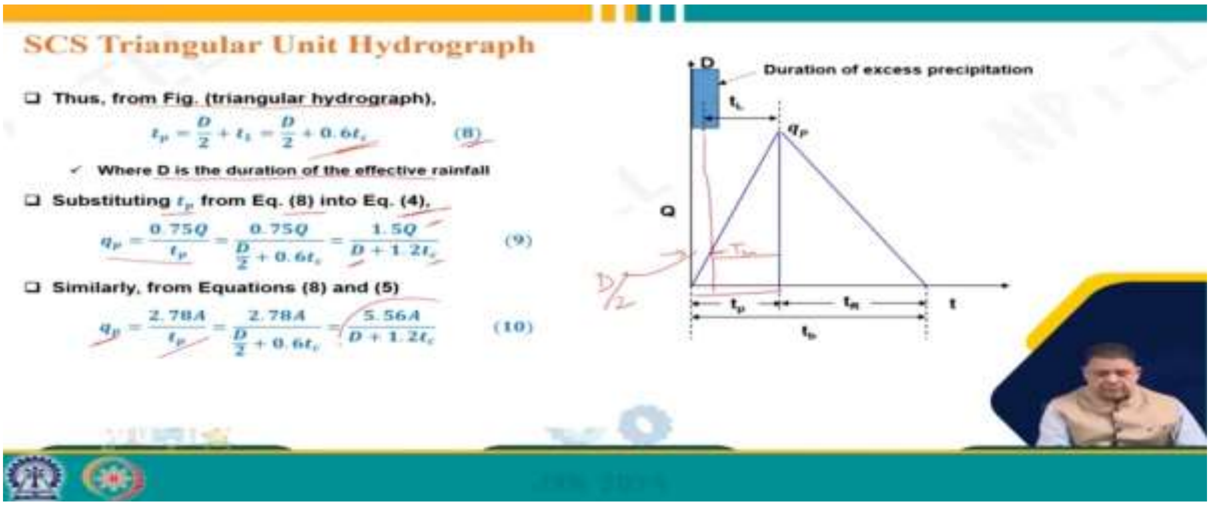
The volume of direct runoff can be calculated using the formula: Volume = 0.5 * q_p * (t_b or t_p + t_r). Peak discharge in centimeters per hour can be derived from the relationship q_p = 0.75 * Q / t_p, considering t_b equals 2.67 times t_p.



However, it's important to note that the volume of direct runoff under unit hydrography, which represents the unit depth of effective rainfall, is 1 centimeter. Hence, peak discharge can also be calculated as 0.75 * Q when q = 1.

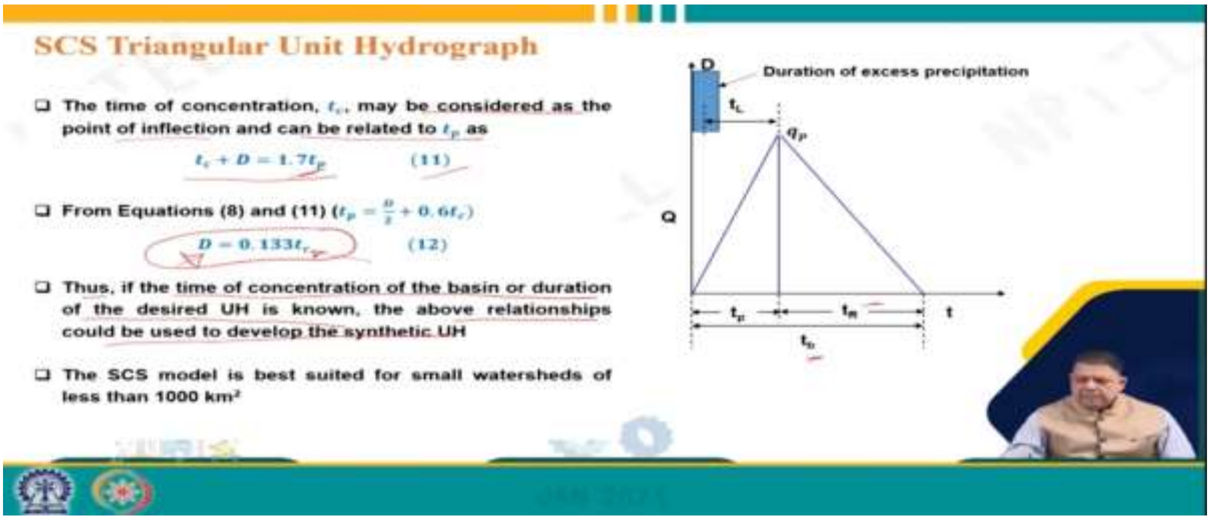
Peak discharge can also be expressed in volumetric units using the formula $q_p = 2.78 * A / t_p$, where A is the drainage area in square kilometers and t_p is in hours.

SCS recommended that the basin lag t_L may be expressed as $t_L = 0.6 * t_c$, where t_L is the basin lag and t_c is the time of concentration of the basin.



From the triangular hydrograph figure, we can deduce that t_p is the duration from the leftmost point to the peak. It has two components: t_L, which is measured from the center of mass of the unit, and half of the duration (d/2). Therefore, $t_p = D/2+t_L$, or in terms of t_c, $t_p= D/2+0.6 * t_c$.

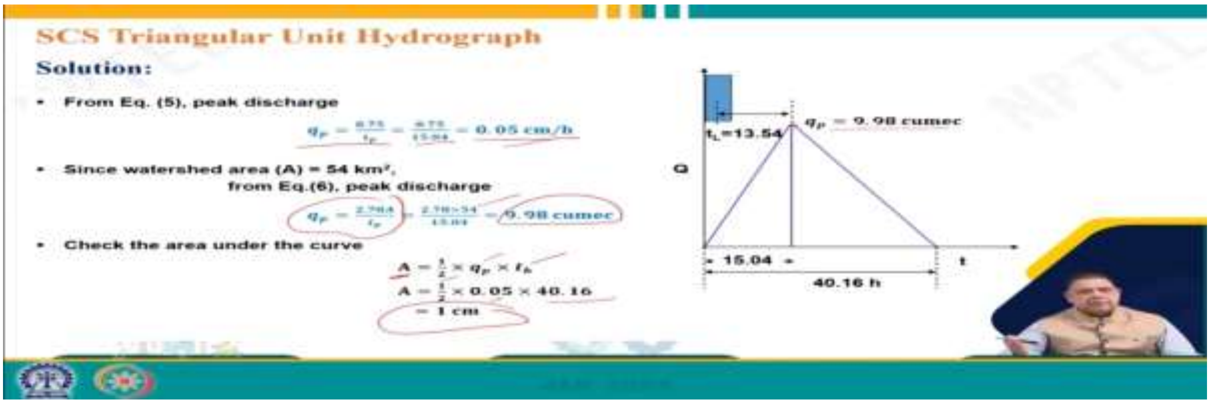
So, this equation relates t_p to t_c, where d is the duration of the effective rainfall. Substituting t_p from equation number 8 into equation number 4, which gives q_p equals to 0.75 Q by t_p, and manipulating because everything else is known, we get $q_p=1.5 Q/(D + 1.2 t_c)$. This means q_p can be related to the volume and duration of the effective rainfall and the time of concentration of the basin. Similarly, we can also express q_p in terms of discharge units, which come out to be $5.56*A/(D+1.2 T_c)$, as shown in equation number 10 for q_p.



The time of concentration t_c may be considered as the point of inflection and can be related to t_p, $(t_c+D) = 1.76t_p$. Earlier, we mentioned in the SCS dimensionless unit hydrograph that one of the points of inflection lies at 1.67 times t_p, which is taken as 1.7 t_p. So, we can write it like this, and from equations 8 and 11, we find that $D=0.133\ t_c$, meaning D is also related to the time of concentration.

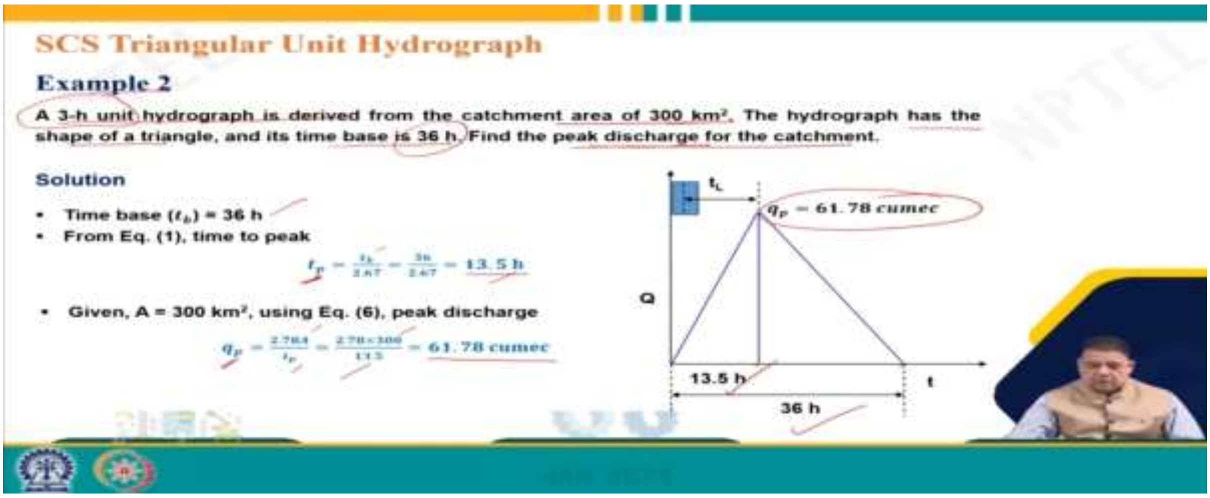
Thus, if the time of concentration of the basin or the duration of the desired unit hydrograph is known, the above relationships could be used to develop the synthetic unit hydrograph. This means that if we know the time of concentration, we can calculate D, or if we have a predetermined D, then we can find out the time of concentration. Once we know the time of concentration as D, we can find out t_p, and then with backward calculation, we can find out all other parameters, like q_p. Of course, once we know t_p, we can find out t_b and t_r, and then we can draw the unit hydrograph quite easily. This is how, based on these relationships, the SCS triangular unit hydrograph can be developed, which essentially comes from the SCS dimensionless unit hydrograph, providing a simple representation equivalent to a triangle.

Let's consider an example: deriving a 3-hour unit hydrograph using the SCS triangular unit hydrograph method for a watershed of 54 square kilometers. We've been given d=3 hours. Using the relationship with the time of concentration, that $D=0.133\ t_c$, we can calculate t_c, which comes out to be 22.56 hours. We also know that t_c is related to t_L by the relationship 0.6 t_c. Using that, we can find out t_L, which comes out to be 13.54 hours. Additionally, we know that $t_p=t_l + D/2$. Now, we need D, and we know t_L, so by putting those values, we can find out t_p to be 15.04 hours. Also, the time base is related to 2.67 t_p, so once we know t_p, we can find out t_b. Thus, we can locate these three points.



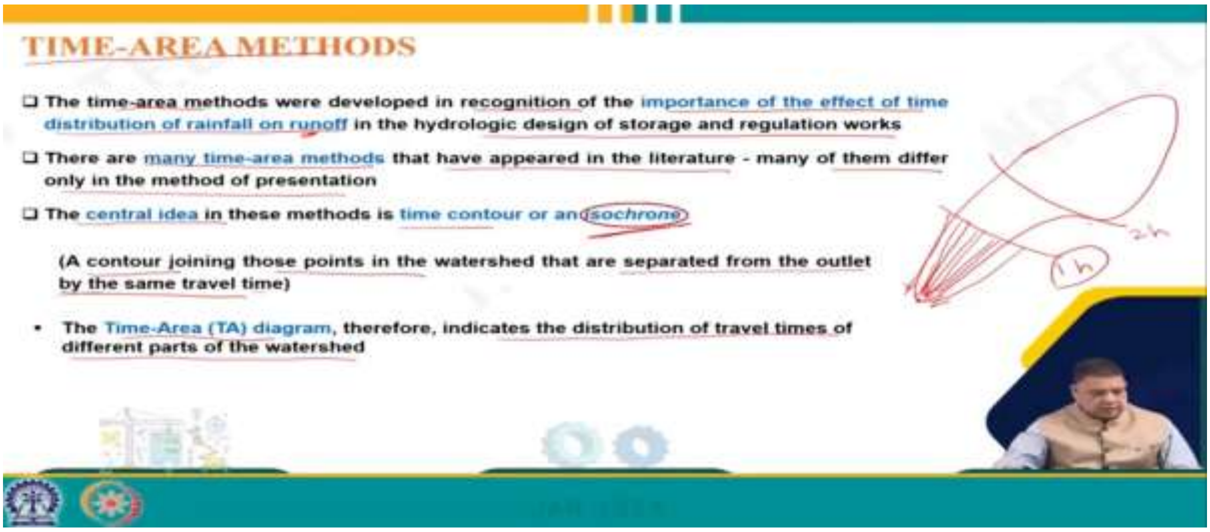
The only thing left to calculate is q_p to draw this hydrograph. We know that q_p=0.75/ t_p, and by putting that value, we find out it's 0.05 centimeters per hour. Or, in volumetric units, by this relationship, we know the area is 54 square kilometers. So, putting that, we find out q_p is 9.98 cumec, so q_p is also known.

So, we can plot this unit hydrograph, and just to ensure that this is perfectly alright, we can calculate the area under the curve, which is (0.5* q_p* t_b). So q_p= 0.05 centimeters per hour, and t_b is 40.16 hours. If we calculate the area, it comes out to 1 centimeter, meaning the area under the curve is 1. This represents a unit depth of effective rainfall or runoff volume. In this case, there is one unit depth of runoff volume from the basin. So, with these principles, we can say that the hydrograph we have developed is perfectly alright.



Now, let's consider example number 2: a 3-hour unit hydrograph is derived from a catchment area of 300 square kilometers. The hydrograph has the shape of a triangle, and its time base is 36 hours. We need to find the peak discharge of the catchment. So, in this case, we know D and we also know the time base, which is 36 hours. We know that t_p is 2.67 times t_p. So, we can calculate t_p as 13.5 hours. Our goal is to find the peak discharge.

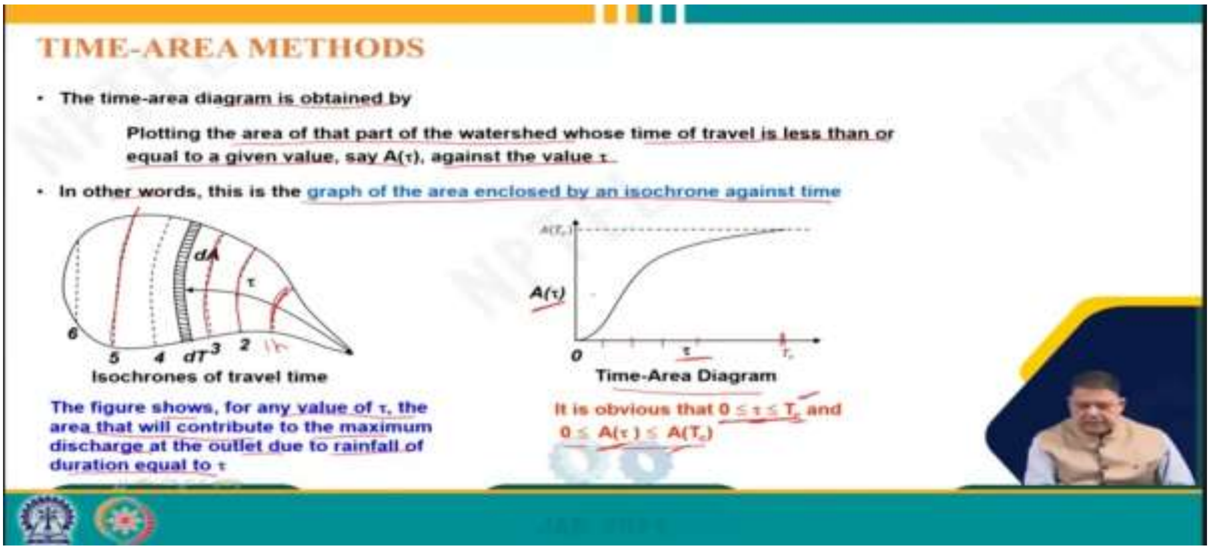
q_p =2.7 A / t_p, where the area is given as 300 square kilometers. t_p has already been calculated as 13.5 hours. By putting these values, we find q_p to be 61.78 cumec. So, that's the discharge of this particular 3-hour unit hydrograph. Of course, we can plot it because t is already given as 36 hours, and t_p has already been calculated. So, we know all the requisite points for drawing this unit hydrograph.



Now, let's move on to the next method: the time area method. These methods were developed in recognition of the importance of the effect of time distribution of rainfall on runoff in the hydrologic design of storage and regulation structures. As we know from the hydrological cycle with all its definitions and explanations, rainfall distribution over the basin impacts the runoff. Time area methods take this into account.

Many time-area methods have appeared in the literature. Many of them differ only in the manner of presentation. So, several time-area methods vary a little in how they represent the theory, but otherwise, they are the same. The central idea of all these methods is a time contour or an isochrone. An isochrone is a contour joining those points in the watershed that are separated from the outlet by the same travel time. So, if we identify the area from where the water travels at the same time, let's say 1 hour, it forms a 1-hour isochrone, meaning a contour joining those points of water separated from the outlet by the same travel time. Any area represented by this will contribute water in 1 hour. That's what an isochrone means. Similarly, a 2-hour isochrone means the entire area contributes in 2 hours. That's how isochrones work.

The time area diagram indicates the distribution of travel times of different parts of the watershed. We can always find out for different times what the area is, and then draw a time area diagram that indicates the distribution of travel times of different parts of the watershed. This is obvious because isochrones are defined in that way.

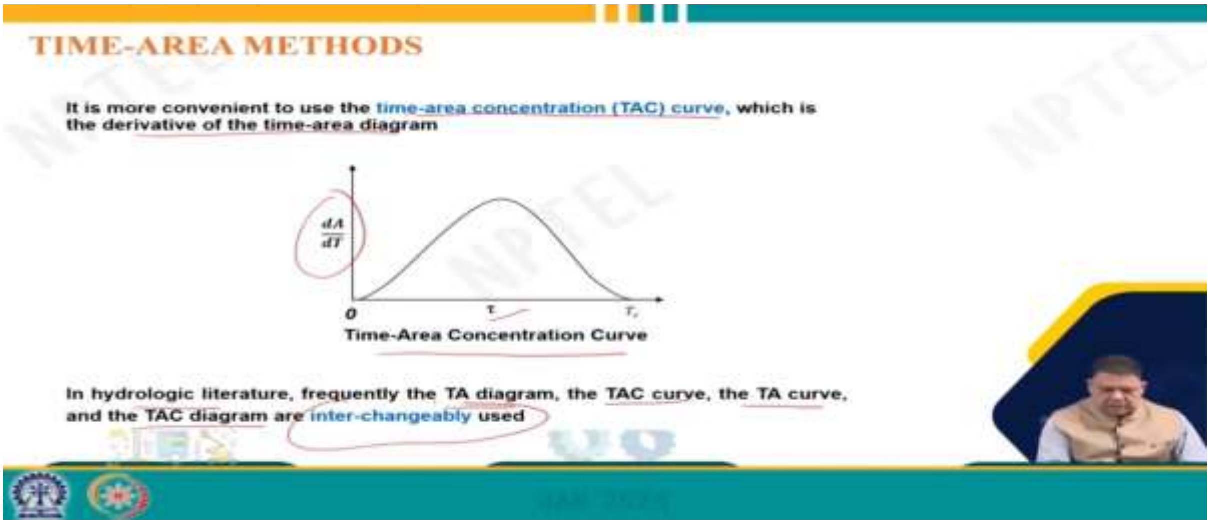


The time area diagram is obtained by plotting the area of that part of the watershed whose time of travel is less than, more than, or equal to a given value, say Aτ, against the value τ, where τ represents time. In other words, this is a graph of the area enclosed by the isochrone against time. So, for any value of tau, the area that will contribute to the maximum discharge at the outlet due to rainfall of duration equal to τis represented. That simply means if rainfall of τ duration occurs, then this entire area will contribute to the outlet.

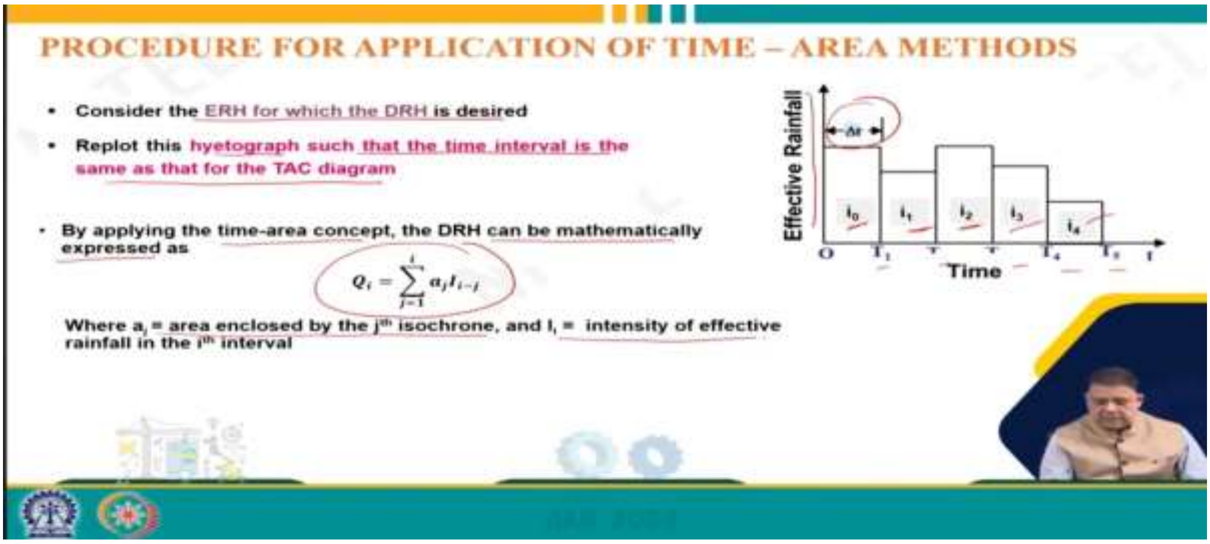
So, it is here that if it is 1, 2, 3, 4, 5, or 6 hours, it simply means, as we discussed just now, that this area will contribute in 1 hour, this area will contribute in 2 hours, this in 3 hours, this in 5 hours, and so on. That's how we can identify the area once we can draw isochrones. We can find out the magnitude of the area, denoted as Aτ, which contributes water to the outlet due to the effective rainfall of τ duration. This is what the time area diagram looks like: it is Aτ plotted against tau. So, for 1 hour, we can find out what the area contributing is, for 2 hours, we can find out what the area contributing is, and so on, up to t_c. So, τ will vary between 0 and t_c, and the area will vary between 0 and the area for t_c.

Now, t_c is defined as the maximum time taken by water to travel from the remotest point in the watershed to the outlet. That's the maximum travel time from the remotest point. So, the time required for water to travel from the extreme-most point is t_c, and that's why tau, which represents the travel time, can only have an upper value of t_c. The same is true for the area: when t_c approaches, it means the entire basin contributes. This concept aligns with the rational formula, which is also based on this idea: that t_c intensity is defined for a duration equal to the time of concentration, presuming that the entire basin will contribute, as discussed earlier.

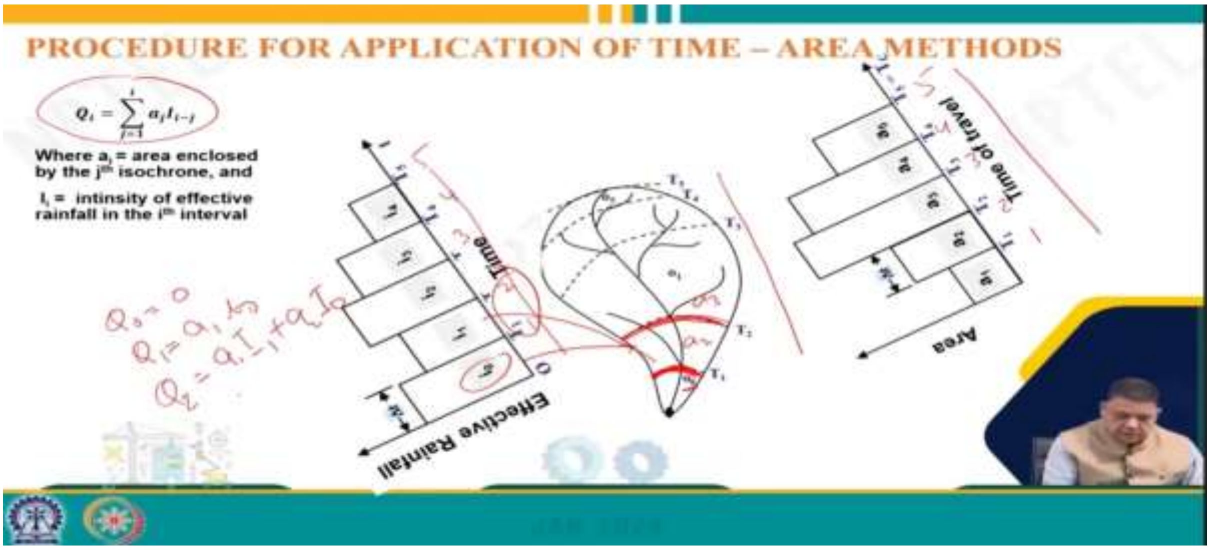
So, that's why the area has an upper limit of $A*T_c$, which occurs at T_c. This is how the time area diagram can be plotted.



Now, it's more convenient to use the time area concentration curve, which is the derivative of the time area diagram. If we plot the derivative dA/dT against ꞁ, it's called the time area concentration curve. In hydrologic literature, the TA diagram, TAC curve, TA curve, TAC diagram—are all interchangeably used, meaning the same thing: area contributing against time. The way the axes are represented may vary.



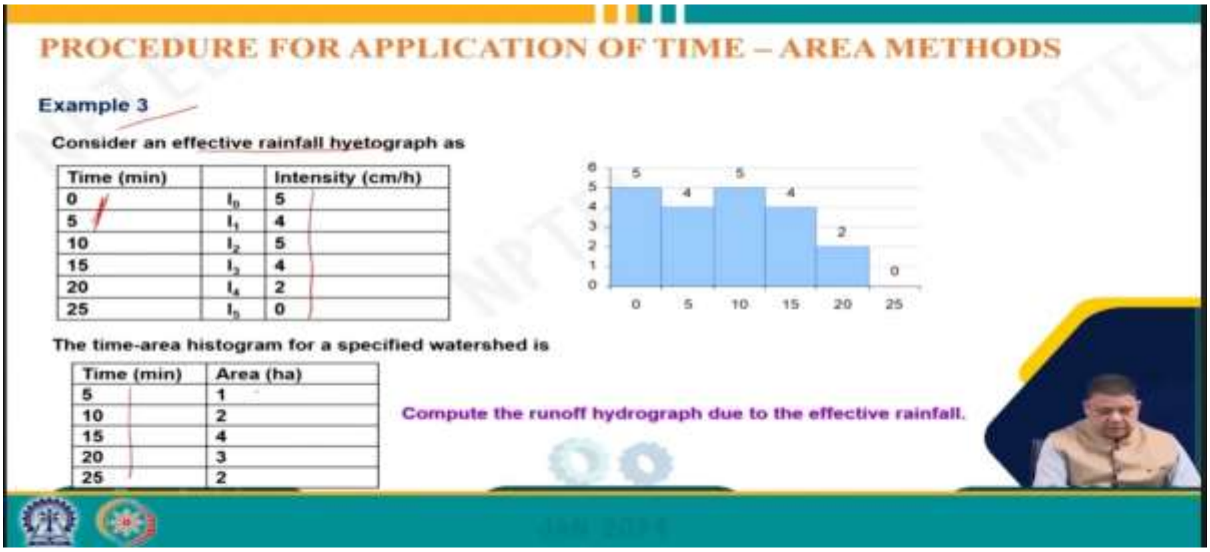
Now, coming to the procedure for the application of the time area methods: let's say we have a watershed. Determine its T_c, the time of concentration, and divide it into several time intervals of equal length. Then, draw isochrones to divide the watershed into the same number of strips as the number of intervals. So, if Tc is equal to, let's say, 5 hours, we can divide it into T_1, which is 1 hour, T_2, which is 2 hours, T_3, which is 3 hours, and so on. Then, we can identify the area that contributes to 1 hour, 2 hours, 3 hours, and so on. Then, we can plot this curve: a_1, T_1, T_2, T_3, T_4. So, those are the isochrones, and we can find out the area of these strips and plot the time area diagram. So, we can find out what area is represented by each isochrone, representing the time of travel to the outlet. Let's say that's a_1, a_2, a_3, a_4, a_5. Of course, the time interval ΔT, which is 1 hour in this case, could be any time ΔT.



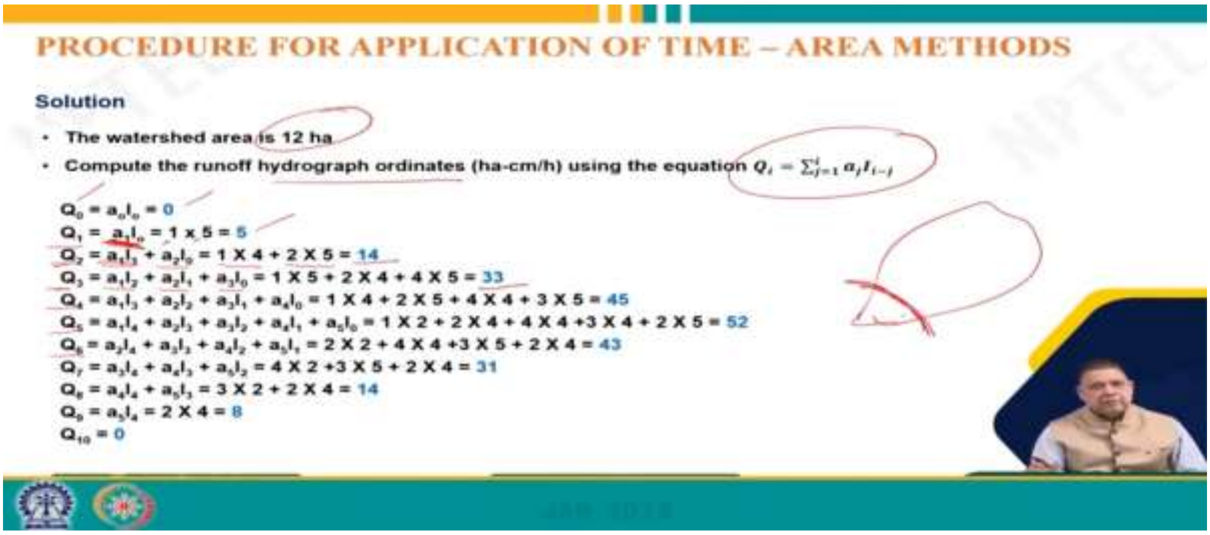
Now, what we do is consider the effective rainfall hyetograph for which we need to derive the direct runoff hydrograph (DRH). Then, we re-plot the hyetograph such that the time interval is the same as that for the TAC diagram. So, here, the effective rainfall hyetograph, which we have, will be re-plotted considering that the time duration between the slices becomes ΔT, which is the same as we saw in the previous diagram. Then, T_1, T_2, T_3, T_4, and T_5—the intensity could be i_0, i_1, i_2, i_3, i_4, and so on. Now, by applying the time area concept, the DRH can be mathematically expressed by this relationship: Q_i $\Sigma^i_{j=1}(a_j{*}I_{i-j})$, where a_j is the area enclosed by the j_{th} isochrone, and I_i is the intensity of effective rainfall in the i_{th} interval.

So, we can try to understand it like this: this is our watershed, this is how it is, and this is our time area diagram. This is how it is. Then, what we are saying is that if this effective rainfall is occurring.

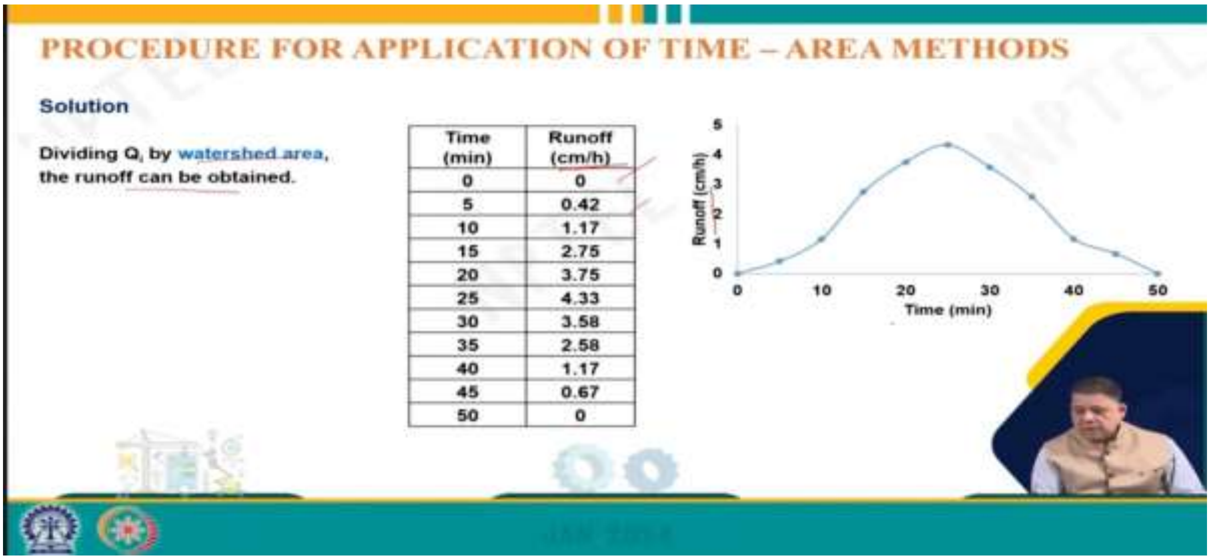
So, let's say we have 1 hour, 2 hours, 3 hours, 4 hours, and 5 hours here, and similarly, 1 hour, 2 hours, 3 hours, 4 hours, and 5 hours here, for example. Now, suppose that the entire rainfall occurs over this basin. Then, what happens is, let's say the rainfall occurred for 2 hours only. Now, because the time of travel from this part to the outlet is 1 hour, obviously, this entire water due to this I_1 which occurred in this area a_1 will flow out. So, in the first hour, Q_0 is 0, but Q_1, when I=1, means only the first one. So, j from 1 to 1, only one slice, a_j is 1, and i_1-1 is i_0. This is i_0 here, so it will only be a_1 and i_0. But, if you come to the second hour, at the end of 2 hours, this part as well as this part will contribute. But just imagine, only the rainfall which occurred in this period here, because the time of travel is there, though it occurred in the first hour, but we will take 2 hours. So, from here, this rainfall will come. So, when it is 2, then there is j from 1 to 2, then 2 slices, a1 (i_2 -1,) which is (i_2- 1) is 1, and then when it is 2, 2 minus 2 is 0. So, it is a_1, it will be a_1 i_1+a_2 i_0, and so on. Then more and more slices will contribute. That is how this relationship works.



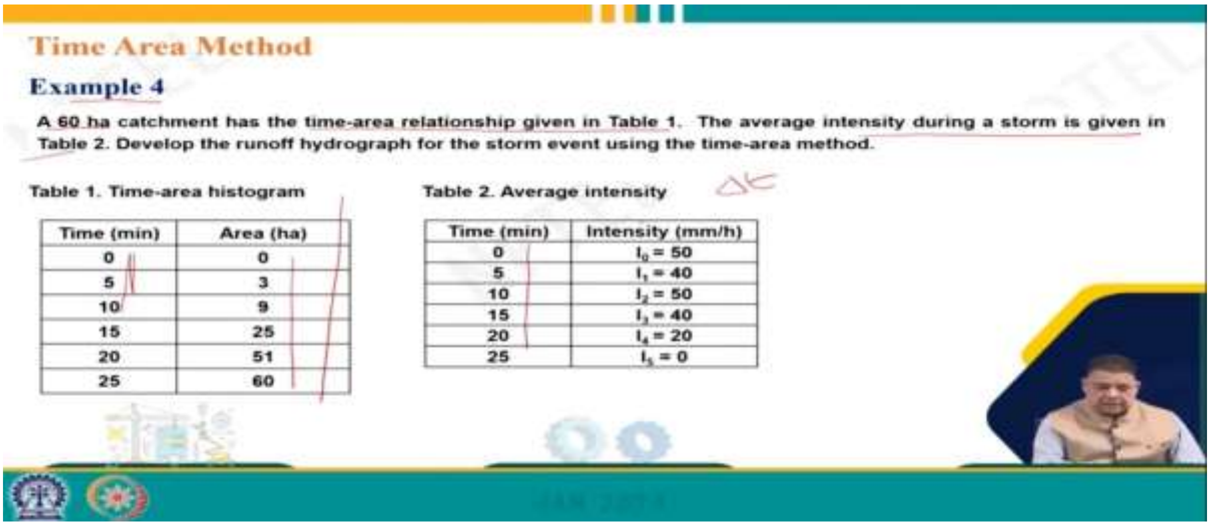
Let's take an example. Consider an effective rainfall hydrograph given in this way. I_0 is at 5-minute intervals, and I_0, I_1, I_2 values are given. And here, a time-area histogram, again at 5-minute intervals, and area is given. Compute the runoff hydrograph due to the effective rainfall, that's what we have to find out.



So, the watershed area is 12 hectares. We have to compute the runoff ordinates using this relationship. So, as we discussed, at Q_0 is 0, at Q_1, only the first slice, so only the first area will contribute, say a_1 i_0. So, a_1 i_0, 1*5 is 5. When 2 comes, then 2 areas will contribute, a_1 i_1 because of the second-hour rainfall, area 1, and because of the first rainfall, area 2. So, 1*4+2* 5 = 14. I mean, for 3 hours, 3 areas will contribute. Area 1 will contribute because of I_2, area 2 because of I_1, 8 because of I_0. I explained to you just now, that you can extend that. So, it is 33. So, I_4, 4 areas, I_5, 5 areas, I_6, then it will be again 4 areas, 3 areas, 2 areas, because in 5 hours, the entire rainfall from the first hour, the first slice, plus this area after 5 hours, this area will stop contributing, because all the 5-hour rainfall has already run off.



So, from 6 hours, area 1 will not contribute, and so on. This is how we will get the different magnitudes, and then this is runoff magnitudes by dividing the area in the cube by the watershed area, that is in centimeters per minute. We can use any unit we want, because we can cube meter per second also, depending upon what unit you choose and time in minutes. So, we can plot this synthetic hydrograph using the time-area method.



Let's take another example: a 60-hectare catchment with a time-area relationship given in Table 1. This is a time-area histogram, again at 5-minute intervals, where you have different areas given, and the average intensity during the storm is given in Table 2.

Time Area Method

Solution:

Time (min)	Area (ha)
0	0
5	3
10	9
15	25
20	51
25	60

Time (min)	Contributing area (ha)
0-5	$a_1 = 3$
5-10	$a_2 = 6$
10-15	$a_3 = 16$
15-20	$a_4 = 26$
20-25	$a_5 = 9$

Time (min)	Intensity (mm/h)
0	$I_0 = 50$
5	$I_1 = 40$
10	$I_2 = 50$
15	$I_3 = 40$
20	$I_4 = 20$
25	$I_5 = 0$

$Q_0 = a_0 I_0 = 0$

$Q_1 = a_1 I_0 = 3 \times 50 = 150$

$Q_2 = a_1 I_1 + a_2 I_0 = 3 \times 40 + 6 \times 50 = 420$

$Q_3 = a_1 I_2 + a_2 I_1 + a_3 I_0 = 3 \times 50 + 6 \times 40 + 16 \times 50 = 1190$

$Q_4 = a_1 I_3 + a_2 I_2 + a_3 I_1 + a_4 I_0 = 3 \times 40 + 6 \times 50 + 16 \times 40 + 26 \times 50 = 2360$

$Q_5 = a_1 I_4 + a_2 I_3 + a_3 I_2 + a_4 I_1 + a_5 I_0 = 3 \times 20 + 6 \times 40 + 16 \times 40 + 26 \times 40 + 9 \times 50 = 2430$

$Q_6 = a_2 I_4 + a_3 I_3 + a_4 I_2 + a_5 I_1 = 6 \times 20 + 16 \times 40 + 26 \times 50 + 9 \times 40 = 2420$

$Q_7 = a_3 I_4 + a_4 I_3 + a_5 I_2 = 16 \times 20 + 26 \times 50 + 9 \times 40 = 1980$

$Q_8 = a_4 I_4 + a_5 I_3 = 26 \times 20 + 9 \times 40 = 880$

$Q_9 = a_5 I_4 = 9 \times 20 = 180$

$Q_{10} = a_5 I_5 = 0$

Runoff hydrograph ordinates (ha-mm/h) can be estimated using the equation

$$Q_t = \sum_{i=1}^{t} a_i I_{t-i}$$

So, here again, the same time because we discussed that delta t has to be the same. So, delta t is 5 minutes here, 5 minutes, and intensities are given here. Develop the runoff hydrograph for the storm using the time-area diagram. So, again, time is given here, and the contributing area is known to us because here, the cumulative area was given. So, that is why we have to calculate the individual area there that is contributing in the first part, that is, the isochrone area, and I_0 values are already given. So, it is a time-area diagram. We know the contributing area, then time intensity is given, the relationship we already know.

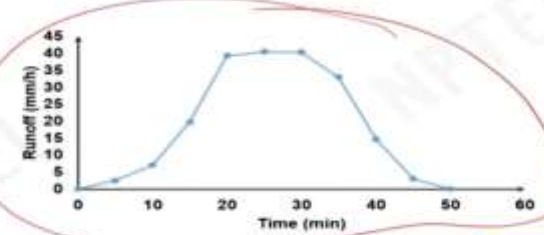
So, 0 is 0, but in the first hour, 1 will contribute, in the second hour, 2 will contribute, in the third hour, 3 will contribute, in the fourth hour, 4 will contribute, and in the fifth hour, the entire watershed will contribute. In the sixth hour, only from the second to the fifth will contribute, and it's like putting the values already we know the values; you just put a simple multiplication and addition.

Time Area Method

Solution: ❑ Catchment area = 60 ha

Time (min)	Runoff (ha-mm/h)	Runoff (mm/h)
0	0	0
5	150	2.5
10	420	7
15	1190	19.83
20	2360	39.33
25	2430	40.5
30	2420	40.33
35	1980	33
40	880	14.67
45	180	3
50	0	0

Col.2 divided by 60 ha



So, the values come like this, and the runoff in hectare millimeter per hour we calculated. Then we can divide by the area to get in hectare millimeter per hour; the area is 60 hectares. So, we divide that and we get that, and then we can plot the direction of the hydrograph here. So, it is a very simple way once you understand the concept. So, in this lecture, we extended our discussion of deriving the synthetic unit hydrograph and discussed two methods: the SCS triangular unit hydrograph method and the time-area method. And please, if you have any doubts, raise questions which can be answered on the forum. Thank you very much.