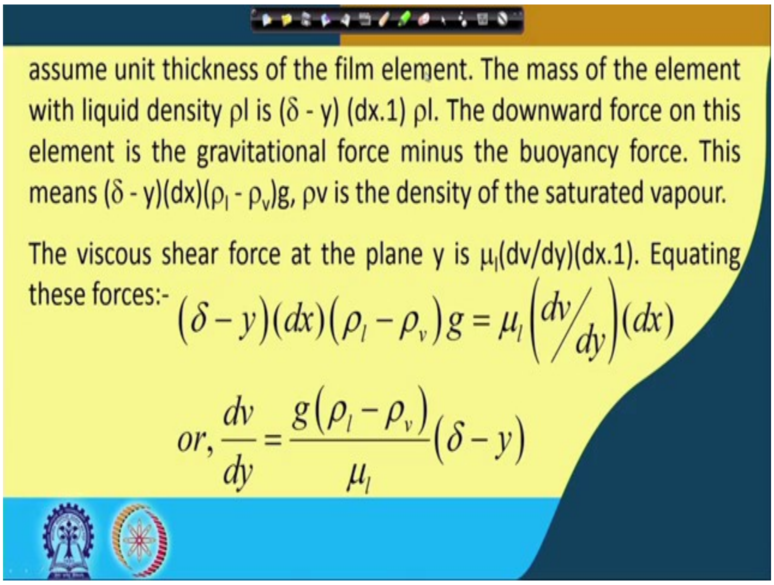
Thermal Operations in Food Process Engineering: Theory and Applications

Prof. Tridib Kumar Goswami
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

Lecture - 42
Condensation (Contd.)

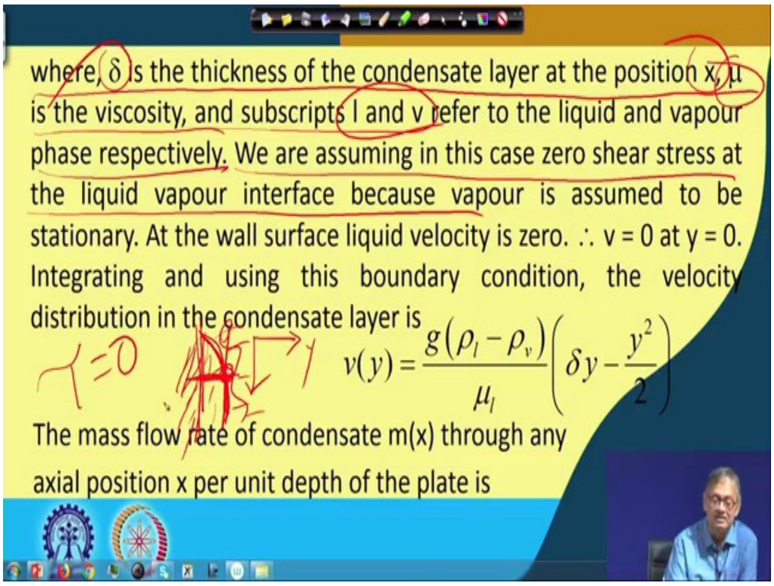
Good morning. So, in that Condensation class, the last class we had come to the point that dv dy was $(\rho_l - \rho_v) / (\delta - y)$ and that expression, 'right'. And we will start from there ok. So, this is our condensation continued class and lecture number 42, this is sorry lecture number 42, 'right', which we can underline like this, ok.

(Refer Slide Time: 01:04)



So, in lecture number 42, we will start with that the expression which we had given earlier as this, 'right' which we had given earlier as this that dv dy is $g \times (\rho_l - \rho_v) / \mu_l \times (\delta - y)$.

(Refer Slide Time: 01:30)

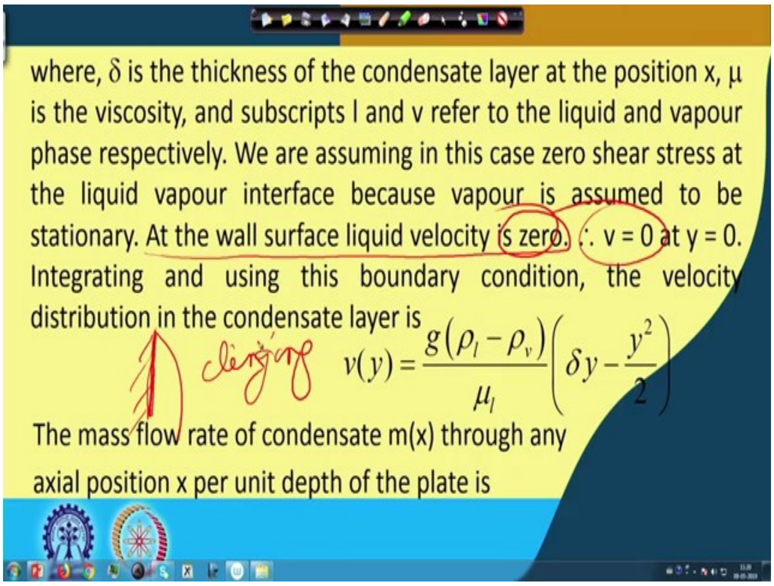


Start from there of course, the individual terms we know new term came up with that is the δ is the thickness of the condensate layer at the position x at any position if you remember we said that this was wall and this was the flow of the condensate, 'right' at any position x, 'right', this is the x and y direction, this is y, this is x. So, at any position x, 'right', the thickness is δ that is what we are saying the thickness is δ. And μ is the viscosity of the liquid and subscripts l and v refers to liquid and vapor phase respectively, 'right'.

So, and we are assuming in the case of zero shear stress, 'right' that is $\tau = 0$, in the case of zero shear stress at the liquid vapor interface, what is the liquid vapor interface? Liquid vapor interface is this, 'right', this is that because this side it is vapor, this is liquid which is coming in contact with this cold surface, 'right'. So, this is vapor.

So, at this liquid vapor interface say at x, so we can take it here, 'right'. So, at the liquid vapor interface like if we take that shear stress is zero, 'right' and this is assumed to be a stationary. So, we are assuming in this case that zero shear stress at the liquid vapor interface because vapor is assumed to be stationary at that point or as such. At the wall surface liquid velocity is zero, so, v is 0 again this is the wall, 'right' this was our fluid or the condensate.

(Refer Slide Time: 04:10)

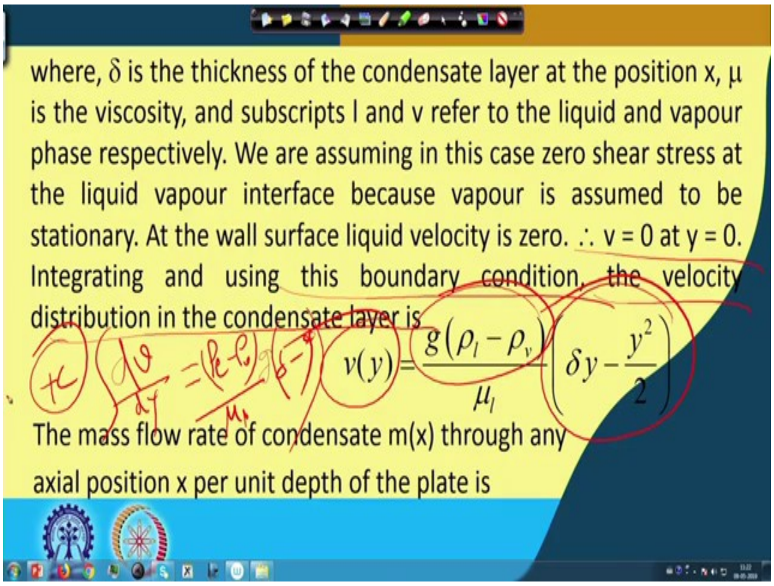


So, the liquid which is at the wall so we normally say that it is clinging to the surface clinging it is clinging to the surface, 'right' attached fully, 'right'. It is now allowing to move, so that is what we call clinging to the surface, 'right'. So, if that be true, then the velocity v is 0 at y is 0, which one is y?

Again the wall so this was the fluid or condensate and we said this was x-direction this was y-direction. So, y is 0 is this, 'right'. So, v is 0 at y is 0, then that equation which just now we had shown that equation which just now we had shown, 'right' like this one that dv/dy g (ρ_l - ρ_v) / μ_l × (δ – y).

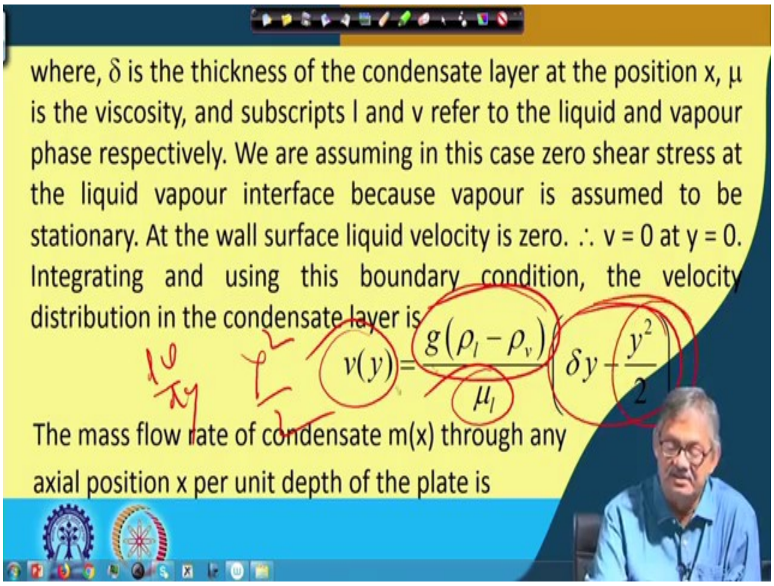
So, if we look at this equation, if we integrate that equation/ 'right' with the boundary condition that boundary condition is what v = 0 at y = 0, then the velocity distribution in the condensate that can be written as v(y) = g (ρ_l - ρ_v) / μ_l ×(δy – (y^2 / 2)), 'right'.

(Refer Slide Time: 06:02)



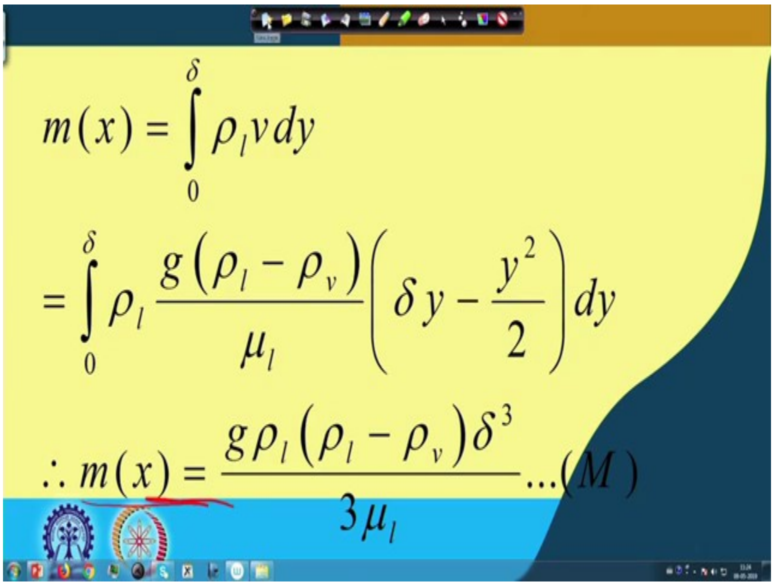
From there we can say that the equation which we had, it was dv dy and it was (ρ_l - ρ_v), 'right' over μ_l, 'right' into $\delta \times g$, 'right' times ($\delta - y$), 'right', ($\delta - y$), this was our expression, 'right'. So, when we integrated this, 'right', when we integrated this we have integration constant c, 'right' and the value of the integration constant and subsequently using that we get the velocity distribution v(y) is g (ρ_l - ρ_v)/ μ_l $\times$($\delta y - (y^2 / 2)$), 'right', because it was dv/dy.

(Refer Slide Time: 07:04)



So, dy was integrated. So, this side it would have been y^2 / 2 that is what is coming, 'right'. And this is nothing, but; this is nothing but expression for a parabolic flow of the fluid. So, v (y) is a parabolic in nature and that is how it is coming like this, 'right'. So; the mass flow rate of the condensate m(x) through any axial position x per unit depth of the plate that can be written as.

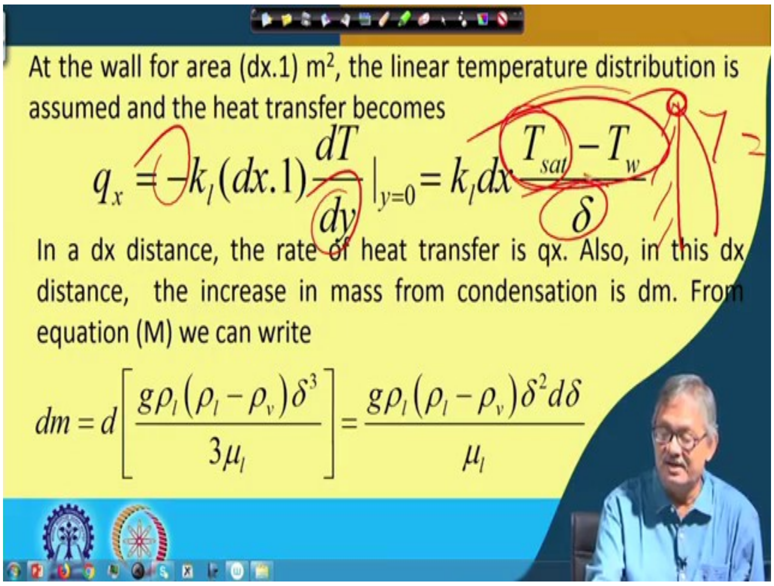
(Refer Slide Time: 07:54)



This can be written as m (x) = 0 to δ m (x) = 0 to δ ρ_l v dy 'right' 0 to δ ρ_l v dy, 'right', which we can rewrite is equal to 0 to δ ρ_l v already got in this form g × (ρ_l – ρ_v) / μ_l into (δ y – (y^2/2)) × dy, 'right'. So, dy was there, v we substituted, ρ_l we take took and this is between 0 to l integration, 'right'.

So, it is a definite integral and we get m(x) = g ρ_l × (ρ_l – ρ_v) × δ^3 / 3 μ_l which we turn to with the equation m, 'right'. So, we got the expression m (x) for in terms of δ and rho's in terms of del rho's and g we got the expression for m (x).

(Refer Slide Time: 09:23)



At the wall for area dx times 1 at the wall for the area dx times 1 m^2, the linear temperature distribution is assumed and the heat transfer becomes, because we have said that dx times 1 that is the wall area, 'right'. So, this was wall and this is x and the other side this side is taken unity, 'right'. So, that means we can say that this unit is like that.

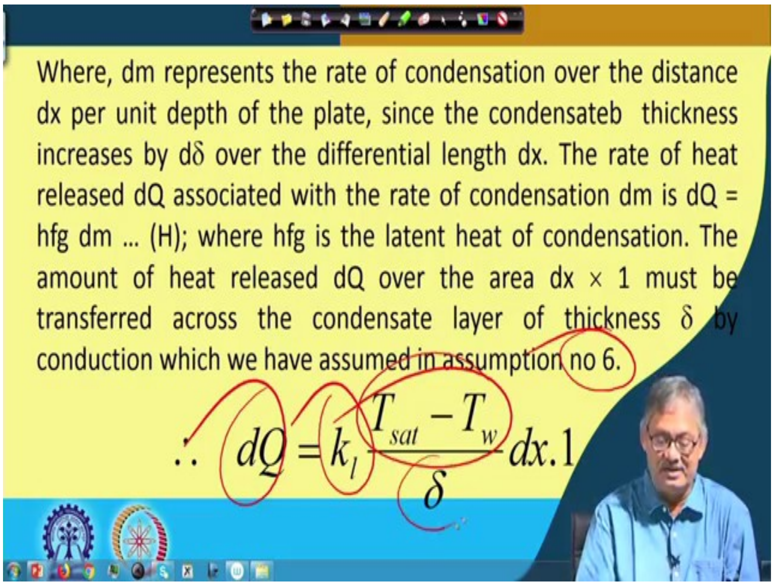
And here we say that in this area, the linear temperature distribution is assumed and the heat transfer then we can write q_x = -k dx × 1 that is the area dT dy at the position y = 0, 'right'. So, this we can rewrite that this is k_l k_l dx × ($T_{saturation}$ - T_w) / δ this dT dy is $T_{saturation}$ at the point y = 0. I repeat where the point y = 0 is this point, 'right'.

So, there we are saying that this is $T_{saturation}$ dT means ($T_{saturation}$ - T_w) since y is 0. So, we can write here it is equal to δ, 'right'. And since it would have been T_w - $T_{saturation}$, but with the negative, so ($T_{saturation}$ - T_w), 'right', so it is k_l dx ($T_{saturation}$ - T_w) / δ.

In a dx distance that is vertical this was our x direction so in a dx distance the rate of heat transferred is q_x. Also, in this dx distance the increase in mass from the condensation is dm, 'right', because we saw that this curve was like that. So, whatever mass was here mass is more at this place and mass is more at this place, so it is gradually changing, 'right'. So, we are saying in a dx distance, the rate of heat transfer q_x. Also, in this dx distance, the increase in mass from the condensation is dm, 'right'.

So, earlier equation which we wrote as m we can rewrite that dm is equal to d of m; m is $g \times \rho_l \times (\rho_l - \rho_v) \times \delta^3 / 3 \mu_l$, 'right'. So, this we can write that $g (\rho_l - \rho_v)$ into since it is d of that d dm of this, then this becomes δ^2 d δ / μ_l, 'right', differentiating d with δ is a δ if we differentiate it becomes δ^2 d δ, on integration this gives δ^3 'right'. So, this is δ^2 d δ is dm, 'right'. So, again we got equation which we need to integrate, 'right'.

(Refer Slide Time: 13:33)

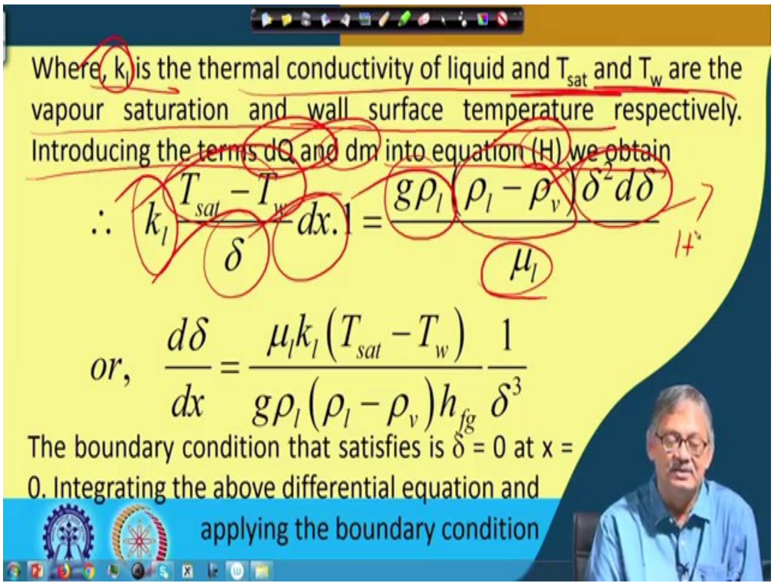


Where, dm is the representation of the rate of condensation over the distance dx per unit depth of the plate, 'right', again unit depth is that third dimension which you are referring to. So, this is y and this is that third dimension is unit, this is x, 'right', so that we are referring to.

Since the condensate b thickness since the condensate thickness not b, since the condensate thickness increases by d δ over the differential length of dx, the rate of heat released dQ associated with the rate of condensation dm is dQ equal to h_f g into dm, 'right'. This we write with the equation H. Of course, h_f g has to be said where h_f g is the latent heat of condensation, 'right' this we have we also used in the previous class h_fg for latent heat of condensation or vaporization whatever we call.

The amount of heat released dQ over the area dx times 1 must be transferred across the condensate layer of delta thickness by conduction which we have assumed, 'right' in the assumption number 6, 'right'. So, therefore, we can write that dQ is k_l ($T_{saturation}$ - T_w) / δ times the area that is dx × 1, 'right'. So, we got also dQ in terms of dx, 'right'.

(Refer Slide Time: 15:53)

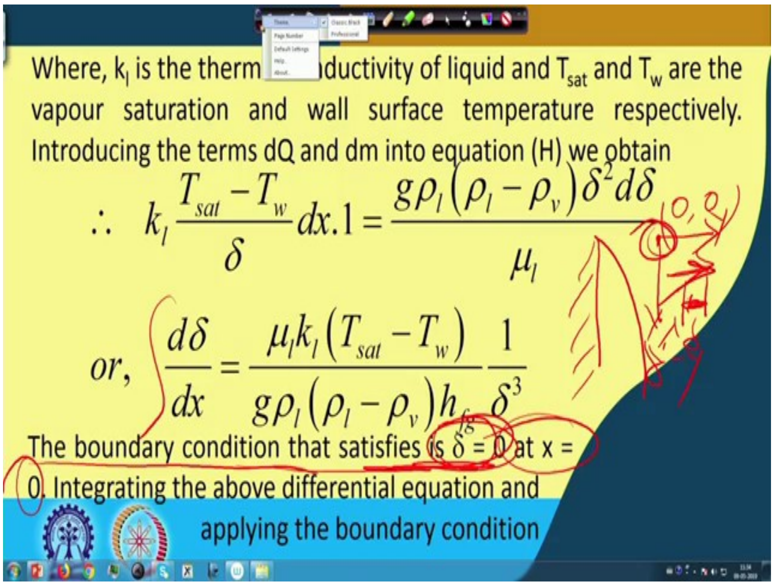


So, we can say of course, the new term which has come up in this is the k that is the k_l is the thermal conductivity of the liquid, T_{sat} and T_w we have said earlier also are the vapor saturation and walls surface temperature respectively, 'right'. So, introducing the term dQ and dm into the equation earlier we have defined as H we can write that k_l ($T_{saturation}$ - T_w) /δ times the area dx × 1 that is equal to g $\rho_l \times (\rho_l - \rho_v) \times \delta^2$ δ / 4 μ_l, this equation we wrote as equation H, 'right'.

So, we can rewrite this equation as d δ dx because we have here δ and we have dx. So, we can rewrite that d dx d δ dx is equal to μ_l q_l × (T_{sat} - T_w)/ g × ρ_l × ($\rho_l - \rho_v$) × h_f g, 'right' times $1/\delta^3$, 'right'. And the boundary form because this is if we want to integrate we need a boundary.

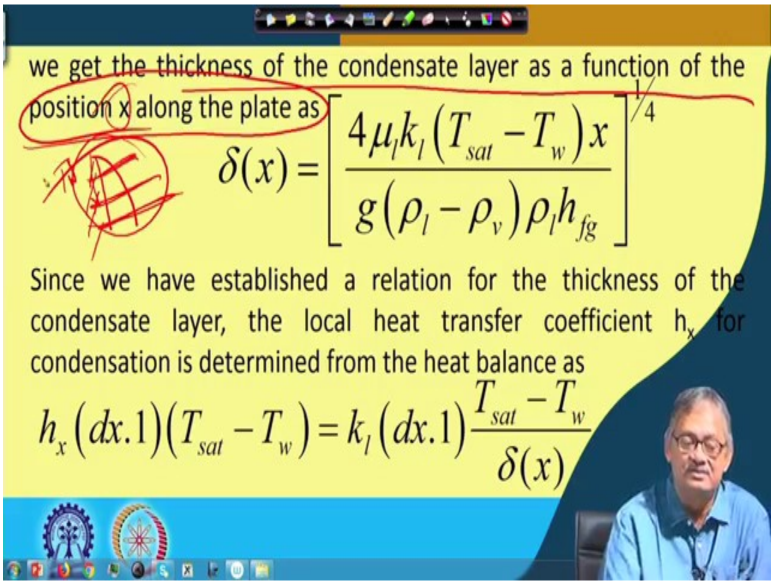
So, that boundary condition that satisfies is δ = 0 at x = 0, 'right', δ = 0 x at x = 0 which is what this was our wall and this was our that fluid which is condensing we said our y is in this direction, our x is in this direction.

(Refer Slide Time: 18:14)



So, and since y is it that this direction, we are taken that element, 'right' of thickness 1 and that was y δ – y, 'right', this our thing. However, here since it is in the y direction, so at δ = 0 is x = 0, here it is δ is 0 at x = 0 that is the origin, 'right'. So, if we take that, then we can say the boundary condition that satisfies is this δ is 0 at x = 0 and integrating this equation and applying the boundary condition, we can write.

(Refer Slide Time: 19:11)

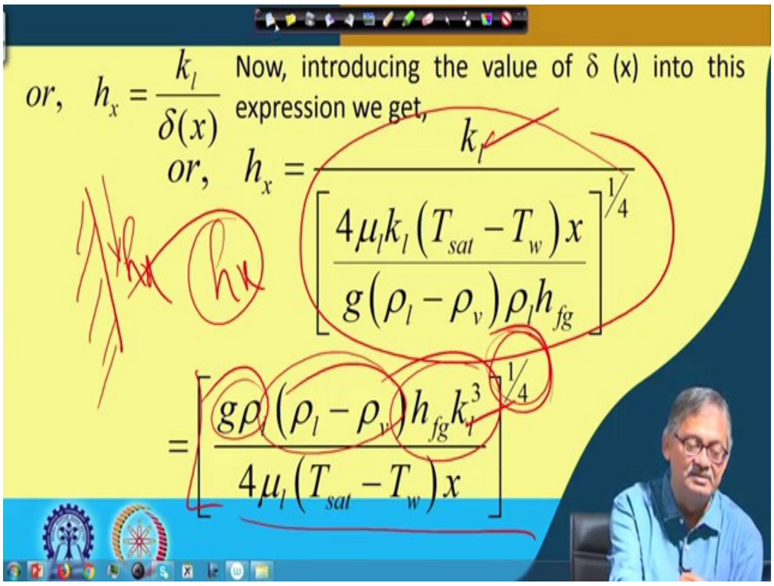


So, if we apply boundary and integrate, we can write that the thickness δ that we can write in this form in this way that the thickness of the condensate layer is a function of the position x along the plate that is also true. We have shown that this is the T wall T_w, and this was the; this was the fluid which was condensing or liquidity is condensing, 'right'. So, if this is the x direction, so whatever thickness is here is not the same thickness here is not the same thickness here. So, this thickness is a function of x, 'right', so, that is what we are saying.

That $\delta x = [4 \times \mu_l k_l \times (T_{saturated} - T_{wall}) \times x] / [g \times (\rho_l - \rho_v) \times \rho_l \times h_f g]^{1/4}$, 'right', because it came that your δ was cube and so when it make you it was integrated so it became 4 and that 4 δ 4 has now become 1/4 in this power, 'right'. So, the thickness at any position x in that film is $[4 \mu_l k_l (T_{saturated} - T_{wall}) \times x] / [g \times (\rho_l - \rho_v) \times \rho_l \times h_f g]^{1/4}$.

Now, since we have established a relation for the thickness of the condensate layer, the local heat transfer coefficient h_x or condensation is determined from the heat balance as $h_x \times dx \times 1 \times (T_{saturated} - T_{wall})$ is $k_l \times dx \times 1 \times (T_{saturated} - T_{wall}) / \delta x$, 'right'.

(Refer Slide Time: 22:09)

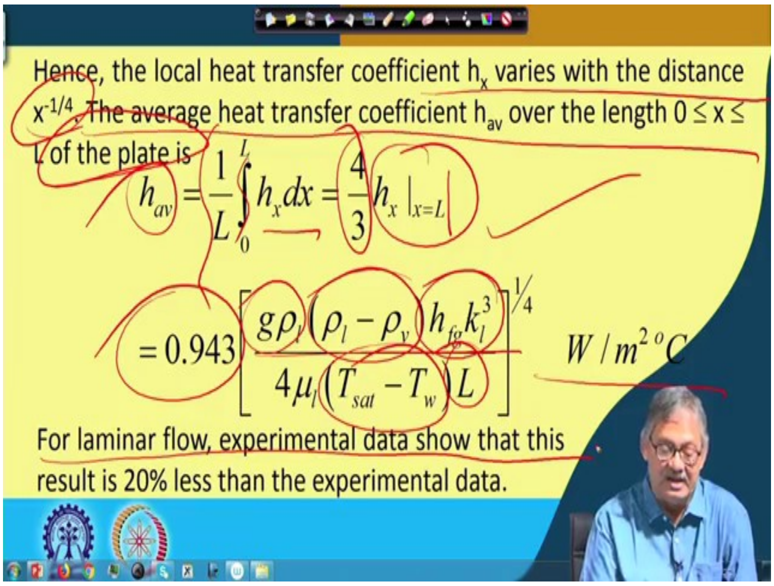


So, if this be true, then by changing the value of δ we can write that $h_x = k_l \times$ a k_l over δx, 'right'. So, this we can rewrite that now introducing the value of del x into this expression, we get h_x is $k_l / [[(4/g) \mu_l k_l (T_{sat} - T_{wall}) \times x] / (g \times (\rho_l - \rho_v) \times \rho_l \times h_f g)]^{1/4}$ that is the h_x that is the heat transfer coefficient or at a localized heat transfer coefficient h_x at any position x. We had again reminded you that this was the direction of x.

So, h at position any position x is the localized heat transfer coefficient which can be rewritten k_l taking inside as $\{[g \times \rho_l \times (\rho_l - \rho_v) \times h_f g\, k_l^3] / [4\, \mu_l\, (T_{sat} - T_w)\, x]\}^{1/4}$, 'right' this to the power 1/4 is the rewritten of this 'right'. This goes up, so 4 μ_l $(T_{sat} - T_w)$, 'right' 4 μ_l this was $g \times (\rho_l - \rho_v) \times h_f g \times$ ρ_l $h_f g$ of course ρ_l has been taken into that $h_f g$ into this k_l that k_l has come down.

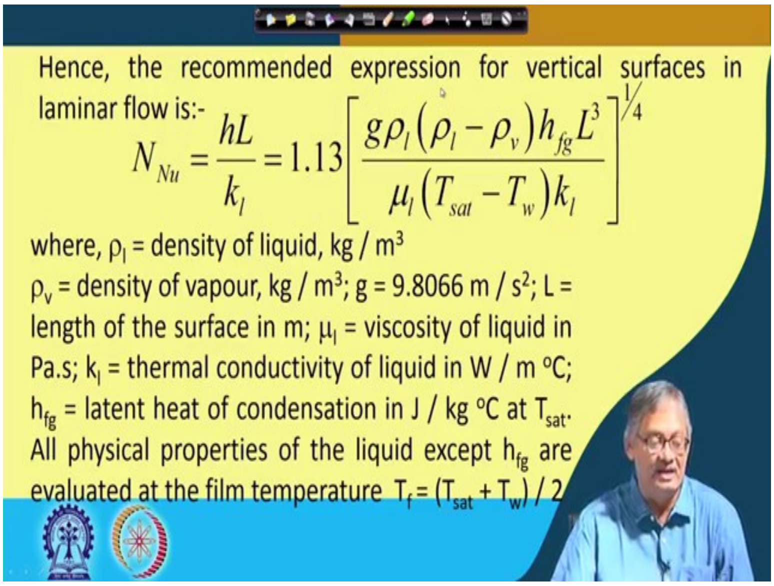
So, it is k_l, earlier we had 1 k_l here to the power 1/4. So, when this k_l is going inside, so it has to be $k_l^3/4\, \mu_l\, (T_{sat} - T_w)$ and times k because k has been taken inside k_l, 'right'. So, this is the h_x the local heat transfer coefficient.

(Refer Slide Time: 24:52)



Now, if we have the local heat transfer coefficient we can have an average heat transfer coefficient like this. The local heat transfer coefficient h_x varies with the distance x as $x^{-1/4}$. The average heat transfer coefficient h average over the length 0 to l is in the plate rather is h average is 1/l 0 to l h_x dx or it is 4/3 h_x at any x = l and that can be rewritten as 0.943 into $[g \times \rho_l \times (\rho_l - \rho_v)\, h_f\, g \times k_l^3] / [4\, \mu_l\, (T_{sat} - T_w) \times l]$ so much $W/m^2.°C$.

(Refer Slide Time: 26:13)

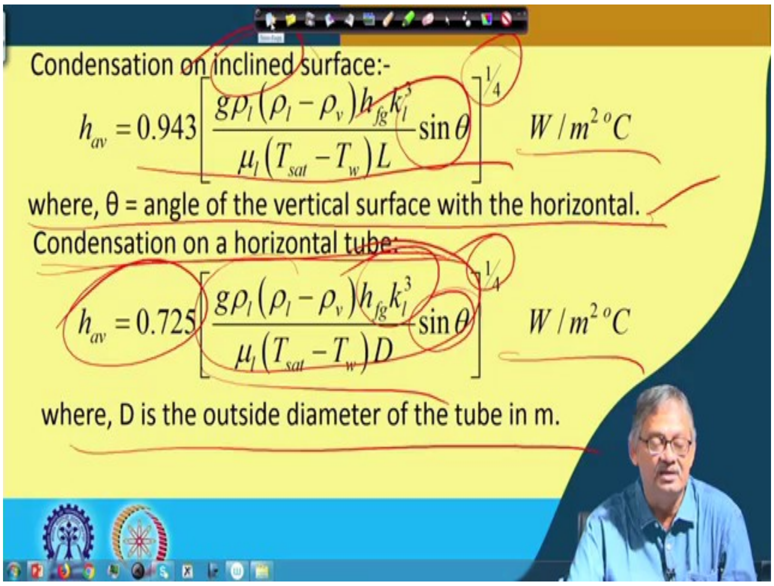


For laminar flow, experimental data show that this result is around 20% less than the experimental data for which a new proposal has come up from the researcher saying that Nusselt number can be related as this. Hence, the recommendation expression for the vertical surface in laminar flow is like this $N_{Nu} = hL/k_l = 1.13\ [g\ \rho_l \times (\rho_l - \rho_v)\ h_f\ g \times L^3]\ /\ [\mu_l\ (T_{sat} - T_w) \times k_l]^{1/4}$.

So, h now is average, 'right', h is the average heat transfer coefficient not the local and that is related to Nusselt number as hL/k_l this is again not the in the wide number, it would have been the solid, but here it is the fluid k_l liquid. So, hL/k_l is the Nusselt number expressed like this, where obviously, ρ_l density of liquid in kg/m^3 ρ_v density of the vapor in kg/m^3, g in 9.8066 m/s^2.

L length of the surface in meter; μ_l viscosity of liquid in Pa.s; k_l thermal conductivity of the liquid in W/m.°C; h_f g latent heat of condensation in J/kg.°C at T_{sat}. All physical properties of the liquid and h_f except h_f g are evaluated at the film temperature T_f is $(T_{sat} + T_w) / 2$ that is the average temperature of the saturated temperature of the vapor and the wall temperature, 'right'.

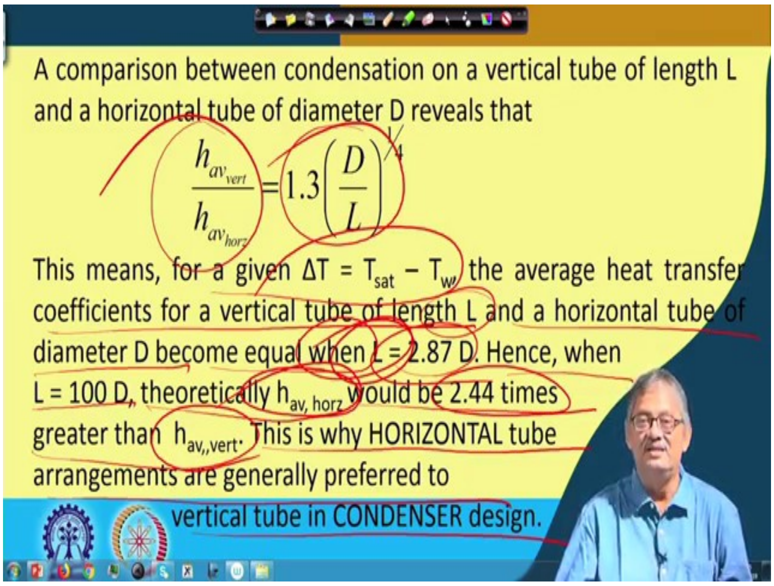
(Refer Slide Time: 28:05)



So, we can now say the condensation on inclined surface that can be written as $h_{average}$ is 0.943 $\{[g\ \rho_l\ (\rho_l - \rho_v)\ h_f\ g\ k_l^3]\ \sin\theta\ /\ [\mu_l\ (T_{sat} - T_w)\ L]\}$ depending on whether what is the inclination to the power 1/4 in W/m^2, where θ is the angle of vertical surface with the horizontal condensation on a horizontal tube, and condensation of course, horizontal surface that is the angle.

Now, condensation on the horizontal T will be h a average is 0.725 $\{[g\ \rho_l\ (\rho_l - \rho_v)\ h_f\ g\ k_l^3]\ \sin\theta\ /\ [\mu_l\ (T_{sat} - T_w)\ D]\}^{1/4}$ so much W/m^2, where D is the outside diameter of the tube in meter.

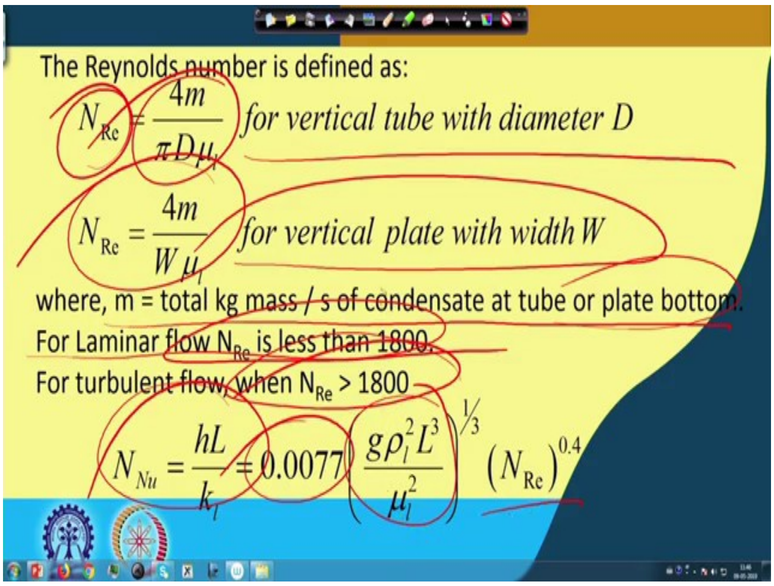
(Refer Slide Time: 29:15)



And since our time is very low we can say that h average over h_{av} vertical / h_{av} horizontal is 1.3 $(D/L)^{1/4}$. So, this means, that ΔT that is $T_{saturation}$ - T_{wall} there is the average heat transfer coefficient for vertical tube of length L and horizontal tube of diameter D become equal only when L = 2.7 time 2.8 times D.

So, when L = 100 D theoretically h_{av} h_{av} horizontal would be 2.44 times greater than that of vertical edge. So, this is y horizontal tube arrangements are generally preferred over vertical in the condensers, 'right'.

(Refer Slide Time: 30:21)



So, hopefully we have come to the end of the class. And we say some more here also a little just for few seconds that is $N_{Re} = 4m / (\pi D \mu_l)$, for vertical tube with diameter D. N_{Re} is $4m / (W \mu_l)$ for vertical plate with width W. And m is the total kg mass / s of the condensate active or plate bottom. For laminar flow $N_{Re} < 1800$; for turbulent flow $N_{Re} > 1800$. So, Nusselt number we can write $0.0077 (g \rho_l^2 L^3 / \mu_l^2)^{1/3}$ and $N_{Re}^{0.4}$. So, with this we come to the end of the condensation.

And we thank you.