Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 33
Heat Transfer by Convection (Contd.)

Good morning. So, we are doing some problems on the application of the relations which we have given, 'right' and this is lecture number 33.

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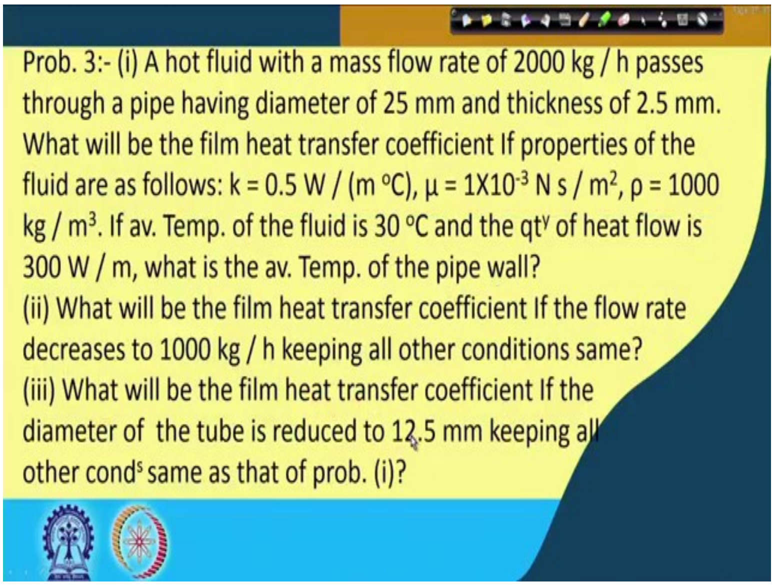

Yeah we are doing it; I think this one will give you the problem and I wish that you solve it. And if you are not able to solve then only on the next maybe next to next class or some other classes, we will come back to this solution and then you can and that time we will of course, do hurriedly. May not be directly with the convection by that time if it is over; then because I will show give you some time so that you can do it ok.

So, the problem is that it is a also similar to the relations which we have handled; that hot fluid with the mass flow rate of 2000 kg/h, passes through a pipe having diameter of 25 mm and thickness of 2.5 mm. What will be the film heat transfer coefficient if properties of the fluid are as follows; k is equal to 0.5 W/m.°C, viscosity mu is 1×10^{-3} N.s/m^2 ρ= 1000 kg/m^3; if average temperature of the fluid is 30 °C and the quantity of heat flow is 300 W/m; what is the average temperature of the pipe wall?

Second, what will be the film heat transfer coefficient if the flow rate decreases to 1000 kg/h; it was 2000 kg/h, now it is 1000 kg/h keeping all other conditions identical or same. Third what will be the film heat transfer coefficient if the diameter of the tube is reduced to 12.5 mm keeping again all other condition same, ‘right’. So, this solution you do and if you are not able to then we will come back to a solution after some time, ‘right’.

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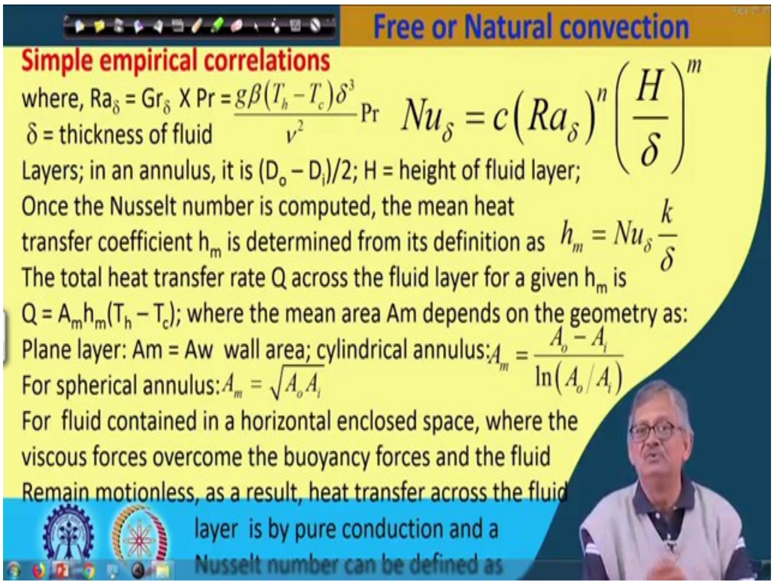

So, now let us go to free convection ok; free or natural convection till now we have handled with forced convection, ‘right’. And free or natural convection by definition it is due to the buoyancy or density difference; the flow is happening and there is a heat exchange, ‘right’.

So, generally the simple correlations are like this that there is a number called Rayleigh number which is Ra; may be in terms of del, i.e., thickness is equal to Grashof number; in terms of again del thickness into Prandtl number. So, this comes to be

$$Ra_\delta = Gr_\delta \times Pr = \frac{g\beta\beta(_h - T_c)\delta^3}{v^2}.$$

And Nu in terms of δ, Nu that is not Nu; Nu that is, Nusselt number in terms of δ is

$Nu_\delta = c(Ra_\delta)^n\left(\frac{H}{\delta}\right)^m$ where H and m deviously these are the two constants which are to be determined, 'right'. δ is the thickness of the fluid, 'right'; so this Nusselt number is $(Ra_\delta)^n\left(\frac{H}{\delta}\right)$, where Ra is Grashof number into Prandtl number as we said, δ is the thickness of the fluid layers in annulus; it is D_o/D_i thickness is $(D_o-D_i)/2$, H is the height of the fluid layer.

Once the Nusselt number is computed; the mean heat transfer coefficient h_m is determined from its definition as $h_m = Nu_\delta \frac{k}{\delta}$ 'right'. Nusselt number is equal to h δ / k or h_m / k; h_m δ / k. Total transfer of heat Q rate across the fluid layer for a given h m is; $Q = A_m h_m (T_h - T_c)$, where the mean area A_m depends on the geometry like plane layer A_m is A_w or wall area.

Cylindrical annulus, $A_m = \frac{A_o - A_i}{\ln(A_o/A_i)}$; outside area over in inner area. For spherical annulus it is $A_m = \sqrt{A_o A_i}$. For fluid continue contained in a horizontal enclosed space where the viscous forces overcome the buoyancy forces. And the fluid remained motionless as a result the heat transfer across the fluid layer is by pure conduction.

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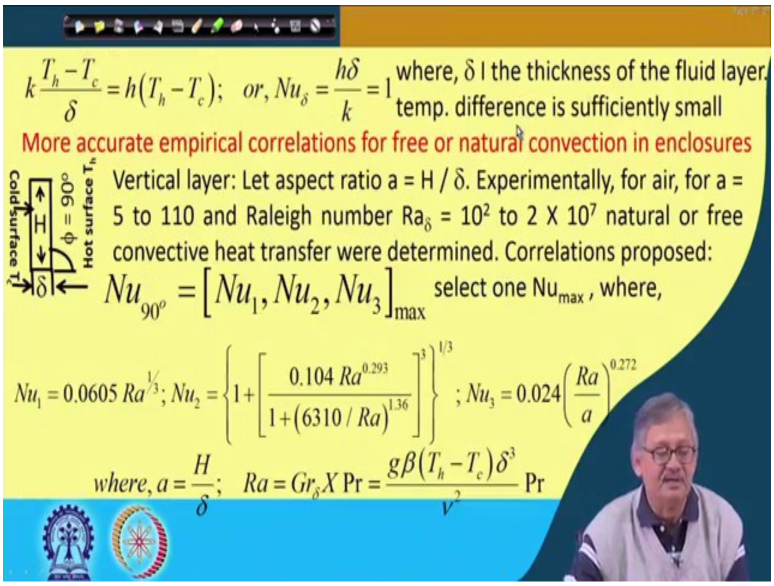

And the Nusselt number can be defined as; Nusselt number can be defined as this that

$k\frac{T_h - T_c}{\delta} = h(T_h - T_c)$ or $Nu_\delta = \frac{h\delta}{k} = 1$, 'right', where δ is the thickness; it should be I S is the thickness of the fluid layer and the temperature difference is sufficiently small.

For more accurate empirical correlations, for free natural convection in enclosures are like this. What we have? Here is that system; system is this we have an enclosure which has a height equal to H, thickness equal to δ. And with the plane this is 90°; phi is 90°, hot surface is at T_h this is the hot surface; at T_h and the cold surface is at T_c; this is the cold surface, 'right'.

So, under this condition; if it is a vertical layer that like this if it is a vertical layer, let aspect ratio is H/δ is equal to a, 'right' H/δ is the aspect ratio is a; where H is the height of the fluid and δ is the thickness of the fluid. Experimentally for air; a is equal to 5 to 110 and $Ra_\delta = 10^2$ to 2×10^7.

So, there natural or free convective heat transfer was determined or were determined. And correlations proposed are like this $Nu_{90^\circ} = [Nu_1, Nu_2, Nu_3]_{max}$ means any one of Nu_1, Nu_2 or Nu_3 maximum of that is the Nusselt number. So that means, you need to know Nu_1, Nu_2 and Nu_3, 'right'. So, from there you select one Nu_{max} or Nusselt number max

where, $Nu_1 = 0.065Ra^{1/3}$. $Nu_2 = \left\{1 + \left[\frac{0.104Ra^{0.293}}{1+(6310/Ra)^{1.36}}\right]^3\right\}^{1/3}$.

And $Nu_3 = 0.024\left(\frac{Ra}{a}\right)^{0.272}$,'right'; a is the aspect ratio, H/δ;

$Ra = Gr_\delta \times Pr = \frac{g\beta(T_h - T_c)\delta^3}{v^2}Pr$, 'right'.

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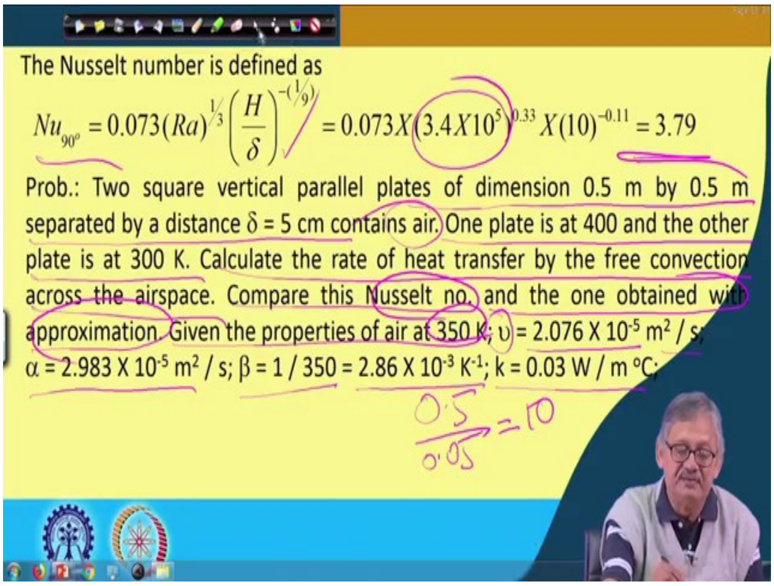

So, knowing this; this is a relation for free convection or natural convection. The Nusselt number is defined as $\mathrm{Nu}_{90^0} = 0.073(\mathrm{Ra})^{1/3}\left(\frac{\mathrm{H}}{\delta}\right)^{-1/9} = 0.073 \times 3.4$ perhaps this we have done as the solution, 'right'.

Two square vertical parallel plates of dimension 0.5 m by 0.5 m; separated by a distance δ is equal to 5 cm contains air. One plate is at 400 and the other plate is at 300 K; calculate the rate of heat transfer by the free convection across the airspace and compare this Nusselt number and the one obtained with approximation, 'right'.

So, given the properties of air at 350 K is. $\nu = 2.076 \times 10^{-5}\ \mathrm{m}^2/s$, $\alpha = 2.983 \times 10^{-5}\ \mathrm{m}^2/\mathrm{s}$. $\beta = 1/350 = 2.86 \times 10^{-3}\ \mathrm{K}^{-1}$. And k = 0.03 W/m.°C, 'right'. Now, H/δ ; H is 0.5 m, δ is 5 cm, 'right'. So, 0.5/0.05, 'right' is equal to 10 'right'. So, from this relation; if Rayleigh number can be found out I think this was found out. So, it came out to be Nusselt number to be 3.79 simply.

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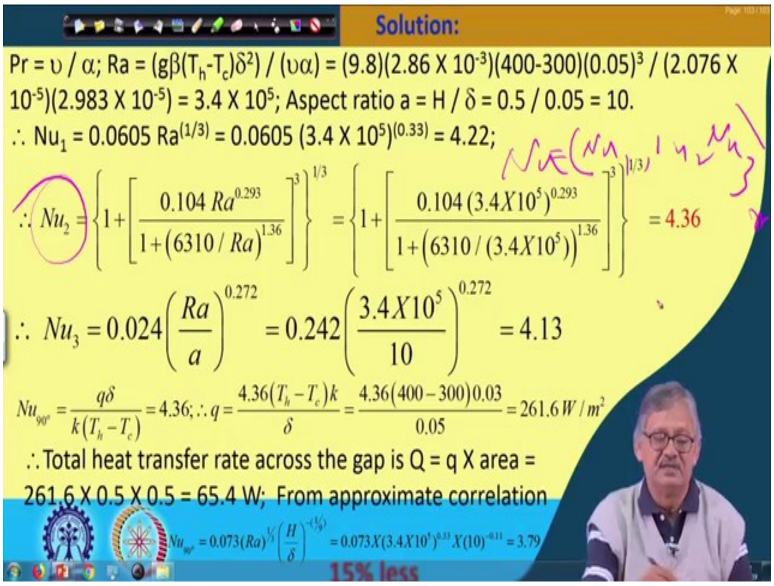

But if we want to see that Nu_1, Nu_2 which we have done if we want to see that, then we have to find out like this that the $Pr = \nu/\alpha$, $Ra = \left(g\beta\left(T_h - T_c\right)\delta^2\right)/\left(\nu\alpha\right) = (9.8\times 2.86\times 10^{-3})(400-300)(0.05)^3/(2.076\times 10^{5})(2.983\times 10^{-5})$. So, it comes to 3.4×10^5.

Aspect ratio or a found to be; a=H/ δ 'right'. So, H/ δ was the aspect ratio equal to 0.5/ 0.005 so; that means, it is 10. So, from 1; $Nu_1 = 0.0605Ra^{1/3} = 0.0605\left(3.4\times 10^5\right)^{0.33} = 4.22$, 'right'. So, Rayleigh number we have found out 3.4×10^5; so this came to be 4.22. Now, Nu_2 that is in the second these are more accurate 'right'; we said Nusselt number you find out either in terms of Nusselt number 1 or 2 or 3 in any three expressions and whatever is the max that you take, 'right'.

So, Nusselt number 2 was more complicated was 1 plus by substituting the values, expression already we had shown earlier; I am not repeating here. So,

$\left\{1+\left[\frac{0.104\mathrm{Ra}^{0.293}}{1+(6310/\mathrm{Ra})^{1.36}}\right]^{3}\right\}^{1/3}$. So, this on simplification gives

$\left\{1+\left[\frac{0.104\left(3.4\times10^{5}\right)^{0.293}}{1+\left(\left(3.4\times10^{5}\right)\right)^{1.36}}\right]^{3}\right\}^{1/3}$,'right'.

So, this on simplification gives 4.36, 'right' and the third one which would like to do is Nusselt number equal to $0.024\left(\frac{Ra}{a}\right)^{0.272}$ and this is equal to $0.024\left(\frac{3.4\times10^{5}}{10}\right)^{0.272}$ and the aspect ratio we found out to be 10; so by 10; so this is equal to 4.13. So, out of these three; 4.22, 4.36 and 4.13; we see that Nu_2 is equal to 4.36 is the max; so Nu_{max} is equal to 4.36, 'right'.

Now, once we have the Nu_{max}; we use Nu_{90} that is because you are set the vertical. So, $Nu_{90}=\frac{q\delta}{k(T_h-T_c)}=4.36$. So, Nusselt number we are taking 4.36 and $Nu_{90}=\frac{q\delta}{k(T_h-T_c)}=4.36$. From there $q=\frac{4.36\left(T_h-T_c\right)k}{\delta}$,'right'.

And this is equal to $q=\frac{4.36(400-300)0.03}{0.05}$ is 261.6 W/m^2. So, q the flux heat flux we got 261.6 W/m^2. So, to know the total Q; we must know q × A, 'right'. So, this is the flux and if area is multiplied; then we know the total heat flow or heat rate flow rate.

So, total transfer rate across the gap is capital Q = q × area and that becomes equal to this we have got 261.6 times; area we had a plate, 'right'. So, plate means it has two dimensions given 0.5 and 0.5,'right'. So, the area becomes 0.5 into 0.5; so 0.5^2.

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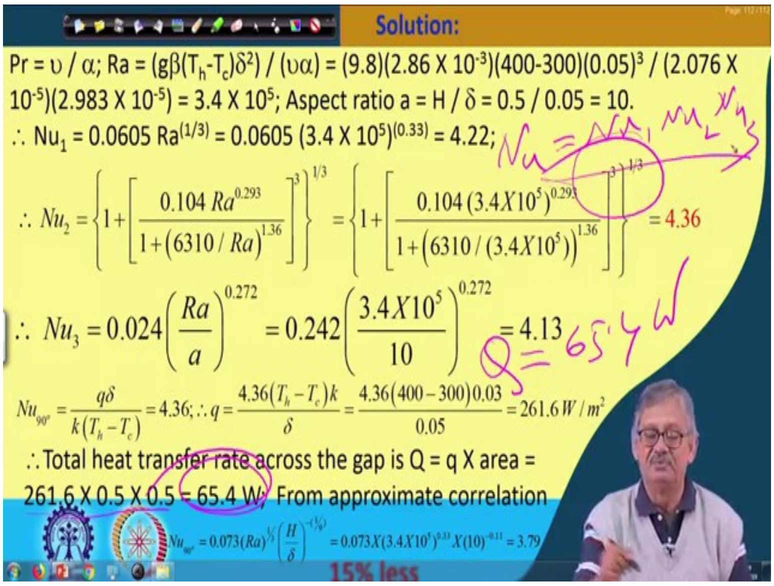

So, that becomes equal to 65.4 W; so Q we found out to be 65.4 W. So, this is according to the best relation we have, 'right'; we used Nu is equal to Nu_1, Nu_2, Nu_3; out of which the maximum we have taken we saw Nu_2 was the maximum, 'right'. And this is the accurate way of determining the heat transfer or rate or heat transfer coefficient or Nusselt number whatever we call.

Now, we had seen in the beginning; there is a very simple correlation for Nusselt, number for vertical plate like that. So, it is $Nu_{90^\circ} = 0.073(\mathrm{Ra})^{1/3}\left(\frac{H}{\delta}\right)^{-1/9} = 0.073\times\left(3.4\times10^5\right)$, 'right'; $\left(\frac{H}{\delta}\right)^{-1/9}$. And this becomes 0.073 Rayleigh number; 3.4; 10^5. We have already found out 1 by 3 means 0.33 times H by δ; that is 10 to the power – 1/9 is -0.11; which came out to be 3.79. So, 3.79 in this case and in this case we got 4.36, 'right'; so the difference is less than 15%.

So, the accurate one which we have done is more than 15% of the approximate one; whereas, in the approximate one you can do it and remember also the relation very easily as $0.073(Ra)^{1/3}\times\left(H/\delta\right)$; that is aspect H by a or h by δ that is aspect ratio to the power -1/9 'right', that came to be 3.79, 'right'.

So, this is less than 15% from the accurate one, 'right' and I; I think we had also seen in the previous slide something like this yeah. So, this was the more simple, 'right' the

$0.073(Ra)^{1/3} \times (H/\delta)^{-1/9}$. So, when we saw that we thought that what suddenly to us; so this was the solution of this problem. So, 0.073 × 3.4 that was found out after was in the next slide we got it that Rayleigh number is 3.4×10^5, to the power 0.33 and H/ δ that is aspect ratio to the power -0.11 or -1/ 9. So, it came out to be 3.79, 'right'.

So, this is how if we look at that this was for free convection; we can utilize the relations to find out free convection and the solution of the free convection, 'right'. So, I hope we have come across the convection more or less and conduction more or less very much thoroughly. And since we are also going very fast; I mean fast in the sense what time is also going out, so we have other things to cover. So, next class we will try to go into the radiation, 'right'.

Thank you.