

Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 32
Heat Transfer by Convection (Contd.)

So, you are handling non-dimensional parameters which are useful, maybe mostly for heat transfer and some also could be for mass transfer, 'right'. So, we come to that lecture number-32 with some more relations are under different situations, 'right'.

(Refer Slide Time: 00:43)

Annular Passage:- Stefan's correlation based upon the results of Hausen is $D_h = D_o - D_i$, $Re = UD_h / \nu$, and $Nu = hD_h / k$, and

$$Nu = Nu_{\infty} f(D_i/D_o) \frac{0.19(Pe \times D_h / L)^{0.8}}{1.0 + 0.117(Pe \times D_h / L)^{0.467}}$$

$0.1 < Pr < 10^3, 0, D_i/D_o < 1, Re < 2300$

$Nu_{\alpha} = 3.66 + 1.2 (D_i / D_o)^{0.8}$: Case I:- Outer wall of annulus is insulated

$Nu_{\alpha} = 3.66 + 1.2 (D_i / D_o)^{0.5}$: Case II:- Inner wall of annulus is insulated

: Case III:- None of the walls is insulated

$$Nu_{\alpha} = 3.66 + \left[4 - \frac{0.102}{(D_i/D_o) + 0.2} \right]$$

For example, in this case if it is an annular passage in many cases you may need it, 'right', 'right'. Like, you have a double tube heat exchanger, 'right'. So, in that case, a fluid which is flowing in through this, it is annular, 'right'. So, there you need to know how the heat transfer to be calculated? So, here such situation we are bringing are that is called Stefan's correlation based upon the results of Hausen that is where it is $D_h = D_o - D_i$, hydraulic diameter is $D_o - D_i$.

And Reynolds number is U or v whatever we call, this is U that is the UD_h/μ , 'right', UD_h/μ . And Nusselt number is hD_h/K , 'right'. In that case, Nusselt number is a

function of like that a $Nu_{\infty} = f(D_i / D_o) \frac{0.19(Pe \times D_h / L)^{0.8}}{1 + 0.117(Pe \times D_h / L)^{0.467}}$, 'right'. Where $0.1 < Pr < 10^3$ and $0 < D_i / D_o < 1$ and $Re < 2300$ this is valid.

But other relation, where this Nu_{∞} or the way it is symbolized $Nu_{\infty} = 3.66 + 1.2(D_i / D_o)^{0.8}$ for outer wall of annulus is insulated. If inner wall of annulus is insulated, then this $Nu_{\infty} = 3.66 + 1.2(D_i / D_o)^{0.5}$, where it was 0.8, 'right'.

And third case is like this, none of the wall is insulated. So, this is simply one case it was outer one is insulated, second case it was inner wall is insulated and in third case none of them are insulated, it is simply annulus this as it is. So,

$Nu_{\infty} = 3.66 + \left[4 - \frac{0.102}{(D_i / D_o) + 0.2} \right]$, 'right', this is if none of the walls are insulated, 'right'.

(Refer Slide Time: 05:02)

The function $f(D_i / D_o)$ is as follows for the three cases:

Case I : $f(D_i / D_o) = 1 + 0.14 (D_i / D_o)^{-0.5}$

Case II : $f(D_i / D_o) = 1 + 0.14 (D_i / D_o)^{1/3}$

Case III : $f(D_i / D_o) = 1 + 0.14 (D_i / D_o)^{0.1}$

for **Turbulent Flow**: **Dittus-Boelter equation**:-

Valid for fully developed turbulent flow. Properties are evaluated at the bulk temperature: $Nu = 0.023 Re^{0.8} Pr^n$ valid for $Pr = 0.6$ to 100 . For heating : $n = 0.4$, and for cooling : $n = 0.3$, and $Nu = hD_i / k$, $Re = \rho D_i v / \mu$, $Pr = \frac{C_p \mu}{k}$

Sieder and Tate correlation :- Variation of viscosity with temperature is included: $Nu = 0.036 Re^{0.8} Pr^{1/3} (\mu / \mu_w)^{0.14}$ where, μ_w is viscosity at wall temperature.

So, similarly some more correlation if we look at, if it is a $f(D_i / D_o)$, then there it is as follows for the three cases. $f(D_i / D_o) = 1 + 0.14(D_i / D_o)^{-0.5}$ i.e., where the outer annulus was insulated, then outer wall; then case II, where the inner wall was insulated that is this one was insulated. So, where, $1 + 0.14(D_i / D_o)^{1/3}$.

And third case was none of the walls were insulated, 'right', then it is $1 + 0.14(D_i / D_o)^{0.1}$. So, you see these only the power that is varying, 'right', all other things are, 'right'. So, for turbulent flow that was for laminar flow Dittus-Boelter equation is utilized, 'right', in most of the cases this is very useful. And valid for fully developed turbulent flow and properties are evaluated at the bulk temperature; and the relation is $Nu = 0.023 Re^{0.8} Pr^n$ rather Pr^n , this is valid for Prandtl number between 0.6 and 100.

For heating; for heating n is 0.4, for cooling n is 0.3 and Nusselt number is hD_i/K , 'right', as well that Reynolds number is $\rho D_i v / \mu$ and Prandtl number is $\mu C_p / K$, 'right'. So, it is very important, I were like to highlight once more, this side it is fully utilized that is, 'right' not visible. However, this for turbulent flow this Dittus-Boelter equation is very useful, valid for fully developed turbulent flow.

As a regarding this fully developed turbulent flow, let me also tell you that what we mean by fully developed turbulent flow? That when a flow is happening; for example, in your house you have put the; you have put that tap on, 'right', at least maybe in your lifetime once or twice it might happen that during your maybe you are taking bath and that time from the shower suddenly there was no water, no flow of water, 'right' that may happen.

And when the new water came, the waters when it was being coming to your shower; it was not a continuous, maybe a little and then again a little, then again a little, like that gradually it is flowing, 'right', this situation we called that this is not the fully developed, 'right'. This situation we call it is not fully developed that when if it is like this there is no development, but we take that this is fully developed and until the flow is fully developed or turbulent flow is there, this is not valid.

So, it is valid for fully developed flow and properties are evaluated at the bulk temperature. And the relation is $Nu = 0.023 Re^{0.8} Pr^n$. Of course, generally for gaseous Prandtl number or for cooling this 0.3 or if it is heating n is 0.4. And Prandtl number this is valid between 0.6 to 100 and Nusselt number is related as hD_i/K and Reynolds number

is $\rho D_i v$ over μ and Prandtl number is $\frac{\mu C_p}{K}$. So, this is the Dittus-Boelter equation.

Similarly, this we had already said once Sieder-Tate equation, the difference between as I said earlier that Dittus-Boelter equation and Sieder-Tate equation is that here you are not taking care of that viscous effect on the wall, 'right', that viscous effect on the wall you are not considering. So, in Sieder-Tate this is also taken care of and the variation of viscosity with temperature is included and there you are getting Nusselt number is here it was 0.023, it is 0.036; $Re^{0.8}$; Prandtl number was with 0.3 for cooling, 0.4 for heating whereas, here Prandtl number is 0.33 one-third.

And the viscosity correction is $(\mu / \mu_w)^{0.14}$, where μ_w is the viscosity at the wall, 'right'. So, this is the difference between the Sieder-Tate and Dittus-Boelter equation both is valid for turbulent flow.

(Refer Slide Time: 12:35)

Heat Transfer Coefficient from flow over a horizontal flat plate:-

Laminar flow:-

$$Nu_x = 0.332 Re_x^{0.5} Pr^{1/3} \quad \text{: valid for isothermal plate}$$

$$Nu_x = 0.332 Re_x^{0.5} Pr^{1/3} \quad \text{: valid for constant heat flux}$$

Turbulent flow:- $Nu_L = \frac{hL}{k} = Pr^{1/3} (0.037 Re_L^{0.8} - 850)$: Isothermal

Critical Reynolds Number has been taken to be 500,000. For constant heat flux, Nu_L is 4% more than that of isothermal.

Free Convection heat transfer coefficient:- **Isothermal case** :

Vertical flat plate and cylinder

$$Nu = 0.55 (Gr Pr)^{1/4} \quad \text{: } 10^4 < Re < 10^9$$

$$Nu = 0.1 (Gr Pr)^{1/3} \quad \text{: } 10^9 < Re < 10^{13}$$

For air, \rightarrow

$$Nu = 1.42 (\Delta t / L)^{1/4} \quad \text{: } 10^4 < Re < 10^9$$

$$Nu = 1.31 (\Delta t)^{1/3} \quad \text{: } 10^9 < Re < 10^{13}$$

But in one case it is valid, rather it is it utilized without the viscosity correction and the other case it is utilized with the viscosity correction factor, (μ / μ_w) . Some other cases are also very important for example, if heat transfer coefficient from the flow over a horizontal plate; if the flow is laminar, so we have a horizontal plate like this so and fluid is flowing over it, what is the heat transfer coefficient.

If the flow is laminar, this relation is given as $Nu_x = 0.332 Re_x^{0.5} Pr^{1/3}$ and valid for isothermal plate that means, this plate is isothermal at constant temperature, 'right', so this is one.

Another $Nu_x = 0.332 Re_x^{0.5} Pr^{1/3}$ and this is also valid for constant heat flux, 'right'. So, heat flux earlier we have said in many cases many times, so I am not repeating.

Turbulent flow, if it is then Nusselt number L, $Nu_L = \frac{hL}{k} Pr^{1/3} (0.037 Re_L^{0.8} - 850)$, 'right'. This is valid when the situation is isothermal for constant temperature.

Critical Reynolds number has been taken to be 500000. For constant heat flux, Nusselt number with respect with L, with respect to L is 4 percent more than that of isothermal condition. Free convection heat transfer coefficient, we are now shifting from the forced convection till now, because turbulent flow was there, so that was under forced convection to free convection that is natural convection by buoyancy.

So, if it is isothermal case, then vertical flat plate and cylinder. In that case, so $Nu = 0.55(Gr.Pr)^{1/4}$; valid between $10^4 < Re < 10^9$. And it's $0.1(Gr.Pr)^{1/3}$ valid for Reynolds number between 10^9 to 10^{13} .

For air, this $Nu = 1.42(\Delta t/L)^{1/4}$ and valid before 10^4 to 10^9 Reynolds number. And is equal to $1.31(\Delta t)^{1/3}$ is also valid for Reynolds number between 10^9 and 10^{13} , hopefully.

(Refer Slide Time: 16:41)

Horizontal cylinder:

$Nu = 0.53 (Gr Pr)^{1/4} : 10^4 < Re < 10^9$
 $= 0.13 (Gr Pr)^{1/3} : 10^9 < Re < 10^{12}$

For air, $Nu = 1.32 (\Delta t / d)^{1/4} : 10^4 < Re < 10^9$
 $= 1.24 (\Delta t)^{1/3} : 10^9 < Re < 10^{12}$

Upper surface of heated plate or lower surface of cooled plate,
 $Nu = 0.54 (Gr Pr)^{1/4} : 2 \times 10^4 < Re < 8 \times 10^6$
 $= 0.15 (Gr Pr)^{1/3} : 8 \times 10^6 < Re < 10^{11}$

For air, $Nu = 1.32 (\Delta t / L)^{1/4} : 2 \times 10^4 < Re < 8 \times 10^6$
 $= 1.52 (\Delta t)^{1/3} : 8 \times 10^6 < Re < 10^{11}$

Lower surface of heated plate or upper surface of cooled plate,
 $Nu = 0.27 (Gr Pr)^{1/4} : 10^5 < Re < 10^{11}$

For air, $Nu = 0.59 (\Delta t / L)^{1/4} : 10^5 < Re < 10^{11}$

A lot we have given some a little more are there. Like situation is like this if it is horizontal cylinder, then Nusselt number is 0.53, $Nu = 0.53(Gr.Pr)^{1/4}$, where Reynolds number is between 10^4 to 10^9 . And is $0.13(Gr.Pr)^{1/3}$, again it is valid between 10^9 to 10^{12} Reynolds number.

And if for air, $Nu = 1.32(\Delta t/d)^{1/4}$ valid between Reynolds number is between 10^4 to 10^9 . And $1.24(\Delta t)^{1/3}$, this is valid between 10^9 to 10^{12} Reynolds number.

Another case where, upper surface of heated plate or lower surface of cooled plate. So, you have two plates; in that case this upper surface is heated or the lower surface is cooled. So, in that case the for plate $Nu = 0.54(Gr.Pr)^{1/4}$ valid for 2×10^4 and 8×10^6 Reynolds number is equal to $0.15(Gr.Pr)^{1/3}$, this is also valid between 8×10^6 to 10^{11} Reynolds number.

For air, this $Nu = 1.32(\Delta t/L)^{1/4}$, this is valid for $2 \times 10^4 < I$ mean, Reynolds number is between 2×10^4 and 8×10^6 . And this is valid for or is equal to $1.52(\Delta t)^{1/3}$, where Reynolds number varies between 8×10^6 and 10^{11} , rather 10^{11} , 'right'.

For lower surface of heated plate or upper surface cooled. So, earlier it was this upper was heated, lower was cooled; now the upper one is cooled and the lower one is heated, 'right'. So, for air $Nu = 0.27(Gr.Pr)^{1/4}$, valid between 10^5 to 10^{11} Nusselt Reynolds

number and is related as $0.59(\Delta t/L)^{1/4}$, where Reynolds number varies between 10^5 to 10^{11} .

(Refer Slide Time: 20:06)

Constant heat flux:-

Vertical plate: $Nu_x = 0.6 (Gr^* Pr)^{1/5} : 10^5 < Gr^* < 10^{11}$

Horizontal cylinder: $Nu_d = 0.6 (Gr^* Pr)^{1/4} : 10^6 < Gr^* < 10^{13}$

where, $Gr^* = \frac{(g\beta q_w x^4)}{(kv^2)}$

Then a little perhaps this is the last that constant for constant heat flux under the vertical plate, $Nu_x = 0.6(Gr Pr)^{1/5}$, this is true for 10^5 Grashof number between 10^5 to 10^{11} .

Horizontal cylinder, under diameter Nusselt number under diameter defined under diameter rather $0.6(Gr Pr)^{1/4}$ valid between 10^6 to 10^{13} Grashof numbers. And Grashof

number is $\frac{(g\beta q_w x)^4}{kv^2}$, 'right'.

(Refer Slide Time: 21:17)

Prob. 1:- Air at the normal pressure passes through a pipe having an internal diameter of 25 mm and is heated from 30 to 90 °C. What is the film heat transfer coefficient h_i between air and the pipe wall if the average velocity of air is 15 m/s? Given, properties of air at the average temperature of 60 °C as: density = 10.5 kg / m³, viscosity = 0.02 cp, conductivity = 0.03 W / (m °C), specific heat = 1.0 kJ / (kg °C).

Solution:- $Re = \frac{\rho v d}{\mu} = \frac{(10.5 \times 15 \times 25 \times 10^{-3})}{(0.02 \times 10^{-3})} = 196895$ $Pr = \frac{\mu \times 10^{-3} \times 1}{0.03}$

$= 0.00067$; $Nu = 0.023(196895)^{0.8} \times (0.00067)^{0.4}$

$= 21.25$; $Nu = \frac{hd}{k}$ $h = 25.504 \text{ W/m}^2\text{°C}$

So, let us now at least look at some problems which can be solved, 'right' with this. For example, air at the normal pressure of thermal pressure air at the sorry, air at the normal pressure passes through a pipe having an internal diameter of 25 mm and is heated from 30 to 90 °C. What is the film heat transfer coefficient h_i between air and the pipe wall; if the average velocity of air is 15 m/s?

And to solve the problem given properties of air at the this should be said of given properties of air at the average temperature of 60 °C are like this density 10.5 kg/m³, viscosity equal to 0.02 centipoise, conductivity 0.03 W/m.°C and specific heat 1.0 kJ/kg. °C.

Here one thing at least we should explain, because for the long term we are saying air film heat transfer coefficient why not only heat transfer coefficient why, film heat transfer coefficient what is that? It is like that we have said here that normal flow or pressure, blah blah this thing, so we have a pipe, 'right'. I am not that confident that if I change the color whether that will be a permanent or not; however, this is good enough we can see.

So, this is a pipe, 'right' and air is flowing through this, 'right'. Now, as we said earlier also that when this air or any fluid is flowing whether it is the air or water or any fluid is flowing; if this is the pipe, 'right'; the layer which is close to the pipe, 'right', the layer

which is close to the pipe that will not move, this is called clinging to the surface, 'right'. So, it is clinging to the surface it is not moving, because the surface is stationary, 'right'.

So, the heat transfer coefficient at this and the heat transfer coefficient whether it is turbulent or natural does not matter; obviously, turbulent will have more value, natural will have less, but still the film heat transfer coefficient which is on this is much lower than that at this, 'right', because here you have some; here you have some turbulence or some movement. Here there is no such so, the film heat transfer coefficient has to be found out, because that is equivalent to giving the resistance, 'right', so this we have to keep in mind.

Many cases we may have to find out, this type of heat transfer coefficient and we will be finding out depending on the situation as we have given many relations, some are other relation may be very much effective or useful and will be utilized, 'right'.

So, let us solve this problem at least in this class that air at the normal pressure passes through a pipe having an internal diameter of 25 mm and is heated from 30 to 90 °C. What is the film heat transfer coefficient h_i between the air and the pipe and here also one h is missing and the pipe here f is missing and the pipe wall if the average velocity of air is 15 m/s. So, h_i between air and the pipe wall, so that is to be determined.

Given, property values of air at the average temperature of so 30 and 90 average is 60, so at the average temperature of 60 °C as density equal to 10.5 kg/m³, viscosity is equal to 0.02 centipoise, conductivity is equal to 0.03 W/m.°C, specific heat equal to 1.0 kJ/kg.°C.

So, if this is given, then first thing which we should do is to find out the Reynolds number, 'right'. So, Reynolds number is $\rho dv/\mu$, 'right', $\rho dv/\mu$ whatever form we call it, 'right'. And Reynolds number must be over the tongue, 'right', $\rho dv/\mu$ is the Reynolds number. If you are an engineering student and if you are not able to tell anywhere that Reynolds number is $\rho dv/\mu$, and then obviously the other side will feel very bad.

So, whether it is in heat or mass or any heat transfer of any energy or in any normal class other, you must know at least Reynolds number as non-dimensional parameter, so that is $\rho dv/\mu$. So, we find out $\rho dv/\mu$ as $10.5 \times 15 \times 25 \times 10^3$, 'right', this is what is given; 25

our density is given 10.5 and $\rho d v$ is given, we have been given 15 m/s, 'right', so v is 15, d is 10.5 and sorry, ρ is 10.5, v is 15 and d is 25×10^{-3} , 25 mm, 'right'.

So, $25 \times 10^{-3} / \mu$ given 0.02 centipoise, one centipoise is 10^{-3} Pa.s. So, 0.02 into 10 to the power minus 3, so that means it is 196895; 1 9 6 8 9 5 Reynolds number. So, it is under high turbulence.

Similarly, Prandtl number let us find out and that comes to be equal to $(\mu \times 10^{-3} \times 1) / 0.03$, where this is $C_p \mu / k$, 'right'; C_p given is 10 kJ oh sorry, 1 kJ/kg.K or kJ/kg.°C that is 1, 'right' and μ is 0.02 centipoise, $C_p \mu / k$, k is 0.03. So, Prandtl number is this is not mu, this should be 0.02, 'right', 0.02 into 10 to the power minus 3 into 1 by 0.03 that comes 0.00067.

So, from the Sieder-Tate equation we can find out or Dittus Dittus-Boelter equation we can find out that $Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$, because it is being heated. So, this comes to be equal to 21.25. So, from the definition of $Nu = hd/k$ where h is can be found, k is known, Nusselt number is known 21.25 and d is known.

So, it comes to be 25.504, 'right', so much W/m.°C. So, this way try to do application of these formulae and solved as many problems as you can ok. So, time is up.

Thank you.