Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 22
Transient Heat Transfer (Contd.)

So, in our 22nd class we were covering Transient Heat Transfer in the previous class, 'right'. And we said what is the meaning of transient heat transfer that is unsteady state heat transfer; if you remember we gave the example that you are putting in a pot hot water and then you dip some spherical ball and the temperature in the ball should be uniform; then only the heat transfer in the form of lumped system can be analyzed and we began with that, 'right'. So, we are continuing that transient heat transfer in the class 22nd.

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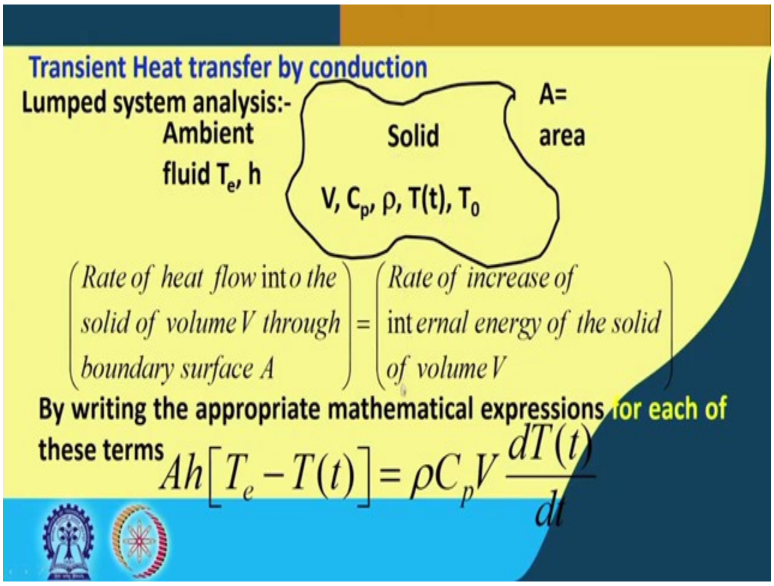

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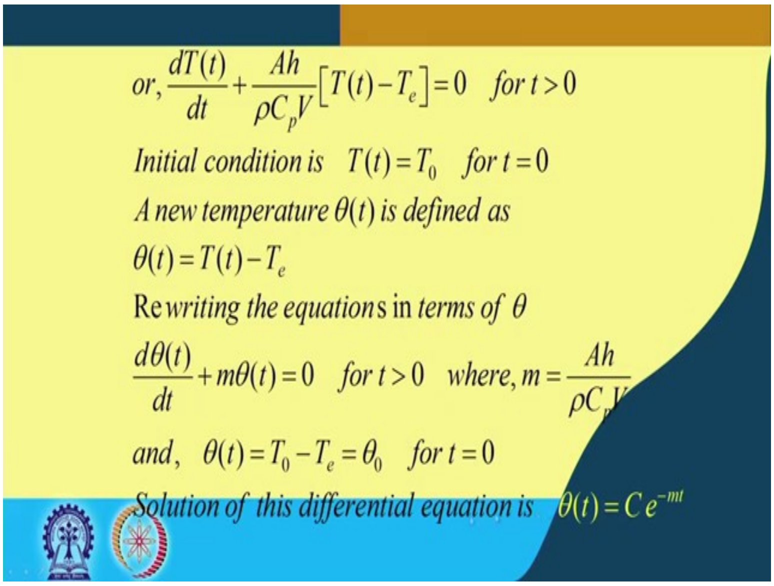

We had taken this we had analyzed and we came to the solution that θt is equal to Ce^{-mt} where m was supposed to n m was made equivalent to by ρC_p V, 'right'.

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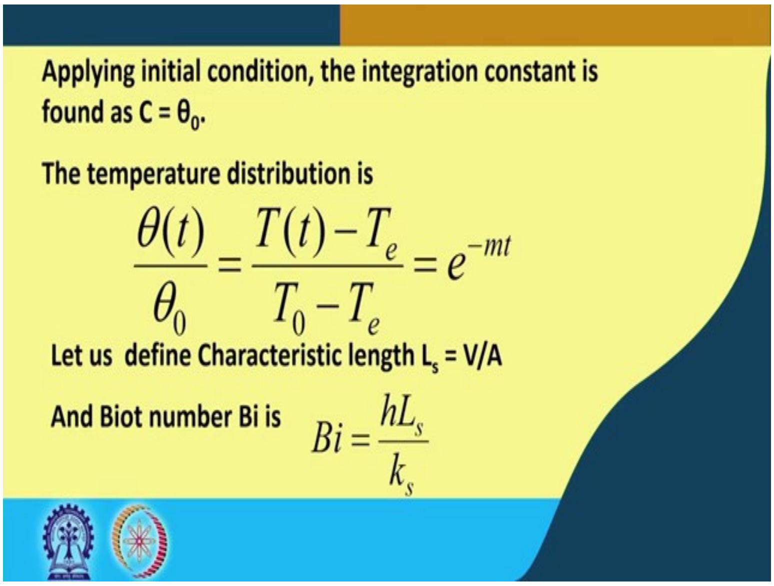

And this we then converted into the non dimensional temperature like theta t over theta 0 that was defined to be T_t -T_e over T_0- T_e and this was made equivalent to e to the power minus mt, 'right'.

And we said that this is dependent on non-dimensional parameter called Biot number or Bi which is $Bi = \frac{hL_s}{k_s}$ where L_s = V/A, or it is known as characteristic length or length or characteristic dimensional parameter. And h is the heat transfer coefficient and k_s is the conductivity of the solid, 'right'; we give the example of your that ball, so that is conductivity of the solid, 'right'.

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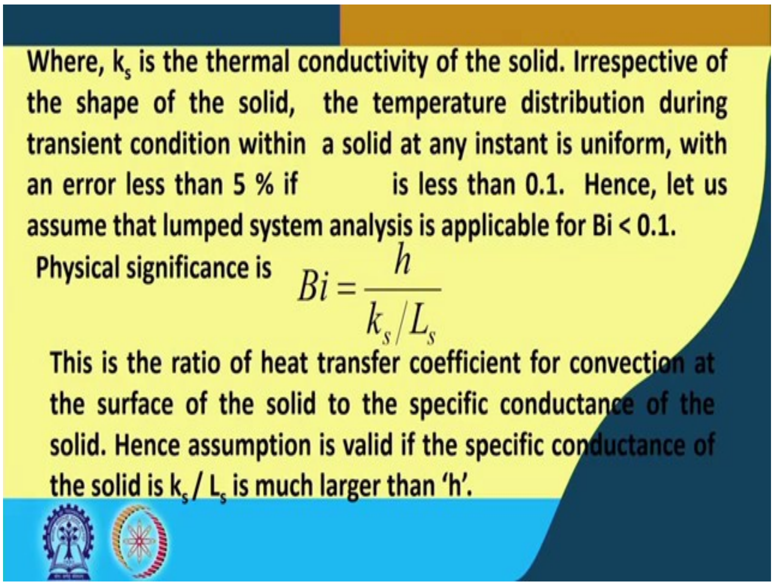

So, if these we true then we said that Bi and the k_s the thermal conductivity of the solid irrespective of the shape of the solid, temperature distribution during transient condition within a solid at any instant is uniform with an error less than 5 percent if we said Biot number is equal to $Bi = \frac{hL_s}{k_s}$ and is less than 0.1.

Hence, let us assume that the lamped system analysis is applicable for Biot number less than 0.1. And the physical significance we get like that $Bi = \frac{h}{k_s/L_s}$ h meaning that this is the ratio of heat transfer coefficient for convection at the surface of the solid to the specific conductance of the solid. Hence assumption is valid if the specific conductance of the solid is k_s over L_s much larger than that of h, 'right'.

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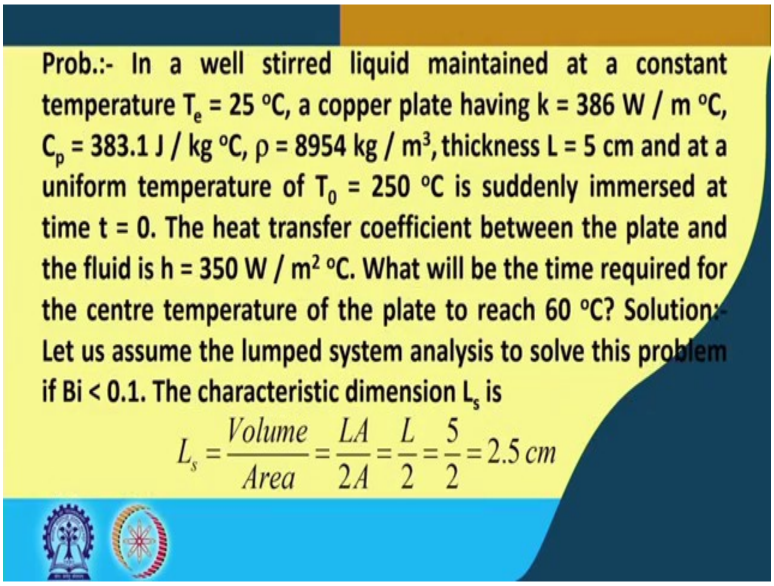

So, now we started we can go for a solution of a problem, 'right'. And the problem is like this in a well stirred liquid maintained at constant temperature T_e is equal to 25 degree centigrade a copper plate having conductivity k_s equal to 386 W/m°C; specific heat C_p equal to 383.1 J/kg°C, density ρ = 8954 kg/m^3, thickness L is equal to 5 centimeter and at the uniform temperature of T_0 equal to 250 °C is suddenly immersed at time t is equal to 0.

The heat transfer coefficient between the plate and the fluid is h and this is equal to 350 W/m^2°C. What will be the time required for the center temperature of the plate to reach 60 degree centigrade, 'right'.

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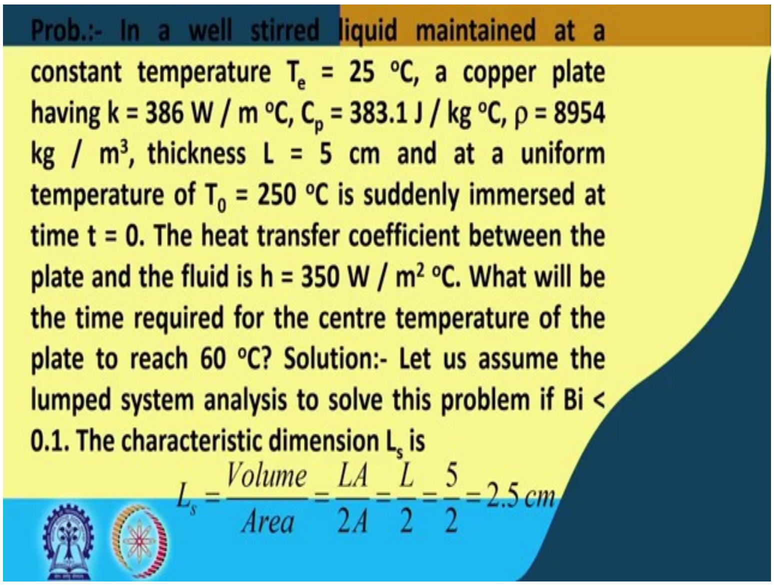

And this can be solved very easily like; solution of this problem can be written. But before we go to the solution we read out the problem once more that in a well stirred liquid maintained at a constant temperature T_e to be 25 °C, a copper plate, 'right'.

So, a well stirred liquid as we said earlier; so there you are putting a copper plate. So, that copper plate is under this new condition that is suddenly it is immersed into that liquid, 'right'. So, if that be true that is the liquid is maintained at the constant temperature of T_e equal to 25 degree centigrade, a copper plate having conductivity k = 386 W/m°C specific heat C_p = 383 J/kg°C.

And specific heat C_p to be 383.1 J/kg°C having a density of 8954 kg/m^3, thickness L = 5 centimeter and uniform temperature of T_0 250 °C is suddenly immersed at time t is equal to 0; that copper plate is suddenly immersed into this liquid at 25 °C, 'right'.

Now, this immersion is done in the liquid where the heat transfer coefficient between the plate and fluid is or h is 350; 350 W/m^{2}°C; what will be the time required for the center temperature of the plate to reach 60 degree centigrade? So, from that 250 °C to 60 °C how much time is required, 'right'? Now to solve it if we do it with the help of lamped system, then first we must look into what is the value of Biot number. Because for application of lamped system Biot number or Bi that is $Bi = \frac{hL_s}{k_s}$ that number should be less than 0.1, 'right'.

So, let us look into what is that first let us find out that what is the characteristic length dimension, 'right'. So, that L_s of that is solid length or characteristic length because to volume of the solid over the area of the solid that is LA over 2A; since it is a plate, so normally it is taken both the side the plate.

So, 2 is the area and L is the volume; so we can say it is $L_s = \frac{Volume}{Area} = \frac{LA}{2A} = \frac{L}{2} = \frac{5}{2} = 2.5\,cm$

So, the characteristic length we got to be 2.5 centimeter, 'right'.

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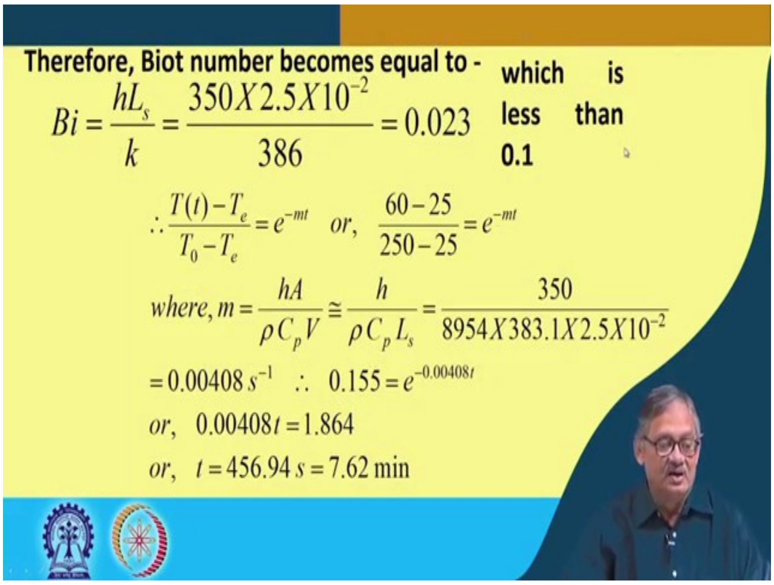

Now, before therefore, we can find out the Biot number and that Biot number is equal to Bi equal to $Bi = \frac{hL_s}{k} = \frac{350X2.5X10^{-2}}{386} = 0.023$ so this is less than 0.1.

So, we can use the lamped system analysis and in that lamped system analysis we have we remember that our solution was

$$\therefore \frac{T(t)-T_e}{T_0-T_e} = e^{-mt} \quad or, \quad \frac{60-25}{250-25} = e^{-mt}$$

$$where, m = \frac{hA}{\rho C_p V} \cong \frac{h}{\rho C_p L_s} = \frac{350}{8954X383.1X2.5X10^{-2}}$$

$$= 0.00408\,s^{-1} \quad \therefore \quad 0.155 = e^{-0.00408t}$$

$$or, \quad 0.00408t = 1.864$$

$$or, \quad t = 456.94\,s = 7.62\,\min$$

So, we have seen that a hotplate when suddenly immersed into a liquid with a heat transfer coefficient and the characteristic length and all other parameters are known; then how much time it will take from one initial temperature to a final temperature say 250 to 60 degree. So, what will be the time required to do this change under the given condition; this is what is transient heat transfer and this is what is the beauty of the lamped system.

So, easily you can solve the situation, 'right', but we have to check whether the Biot number was less than 0.5 or not, 'right'. Sorry, Biot number has to be less than 0.1; then only this is applicable, 'right'.

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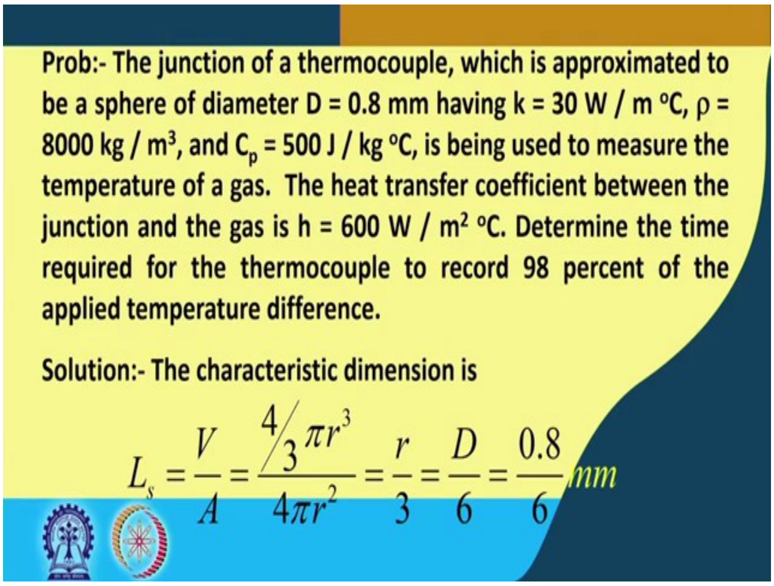

So, let us look into another problem which is like this that the junction of a thermocouple which is approximated to be a sphere of a diameter; of diameter D equal to 0.8 millimeter having k conductivity to be 30 W/m°C. Density ρ 8000 kg/m^3 or specific heat 500 J/kg °C is being used to measure the temperature of a gas.

The heat transfer coefficient between the junction and the gas is h equal to 600 W/m^2°C. Determine the time required to for the thermocouple to record 98 percent of the applied temperature difference, 'right'.

So, here you have to be careful because while reading some things are given which are very useful and very thoughtful also, 'right'. So, quickly if we look at the junction of a thermocouple which is approximated to be a sphere of diameter D 0.8 millimeter; having conductivity cap 30 W/m°C, density 8000 kg/m^3.

And specific heat C_p 500 J/kg °C; it is being used to measure the temperature of a gas, 'right' thermocouple you know. So, this is what junction one metal another metal; so make the junction and this point is used as the thermocouple and the other end of these two points are given to the measuring device, 'right'.

So, the heat transfer coefficient between the junction and the gas is h is equal to 600 W/ m^2°C. Determine realized that importance; determine the time required for the thermocouple to record 98 percent of the applied temperature difference. That means, you have already measured 98 percent that is accomplished 98 percent; unaccomplished 2 percent, 'right'. So, that is the crux of the problem; that is the key point of the problem 'right'.

So, again we have to first find out the Biot number to; find out the Biot number first we have to find out the characteristic length. So, characteristic length of this is a sphere, so it is

$$L_s = \frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{D}{6} = \frac{0.8}{6} mm$$

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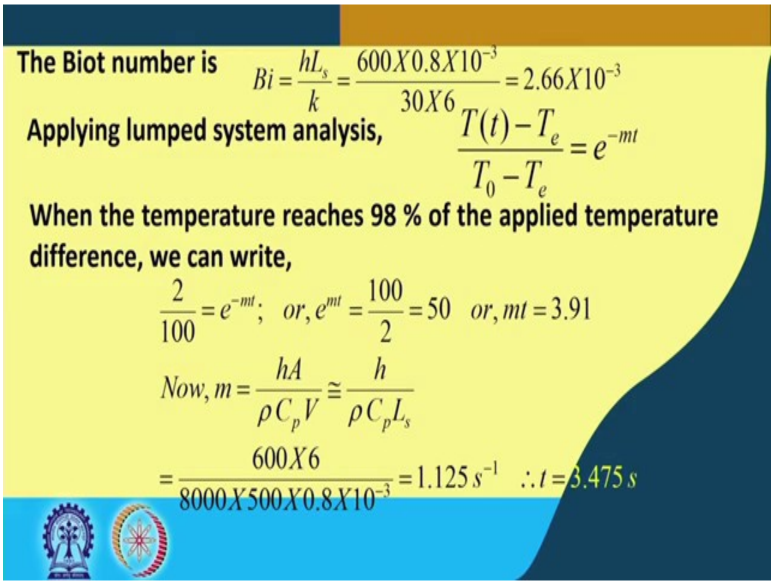

If that be true then the Biot number we can determine that is

$$Bi = \frac{hL_s}{k} = \frac{600X0.8X10^{-3}}{30X6} = 2.66X10^{-3}$$

it is that is 0.00266; that means, it is less than 0.1 'right'. So, since it is less than 0.1, so we can apply the lumped system analysis; so applying the lumped system analysis we can write that $\frac{T(t)-T_e}{T_0-T_e} = e^{-mt}$

And θ_t/θ_0 was equal to $\frac{T(t)-T_e}{}$ T temperature at any time T minus temperature of the environment or of the fluid where the it is getting transferred; over the initial temperature minus the environmental temperature, that is what the medium is getting heat transferred.

So, $$\frac{T(t)-T_e}{T_0-T_e}=e^{-mt}$$

$$\frac{2}{100}=e^{-mt};\quad or, e^{mt}=\frac{100}{2}=50\quad or, mt=3.91$$

$$Now, m=\frac{hA}{\rho C_p V}\cong\frac{h}{\rho C_p L_s}$$

$$=\frac{600X6}{8000X500X0.8X10^{-3}}=1.125\,s^{-1}\quad \therefore t=3.475\,s$$

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3.475 second was the time. So, the crux of the problem was how to make; how to make the use of that 98 percent factor, 'right'. So, that 98 percent factor we have used that 98 percent has been already accomplished; we have remaining 2 percent; so these 2 percent has to be measured. So, that is why it became 2 percent equivalent to 2 by 100, 'right'. So, we found out m and from there from the relation theta t by theta 0; e to the power minus mt, we have found out what is the time required.

It can be the other way around that in some problem, you may have been given all the 4 temperatures and you know you might not have been given all the 4 temperature you have been given on 3 temperatures. And you are asked that within this time how much will be the temperature drop or how much will be the temperature gain or what will be the environmental temperature etcetera; that combination you have to do with different problems, 'right'.

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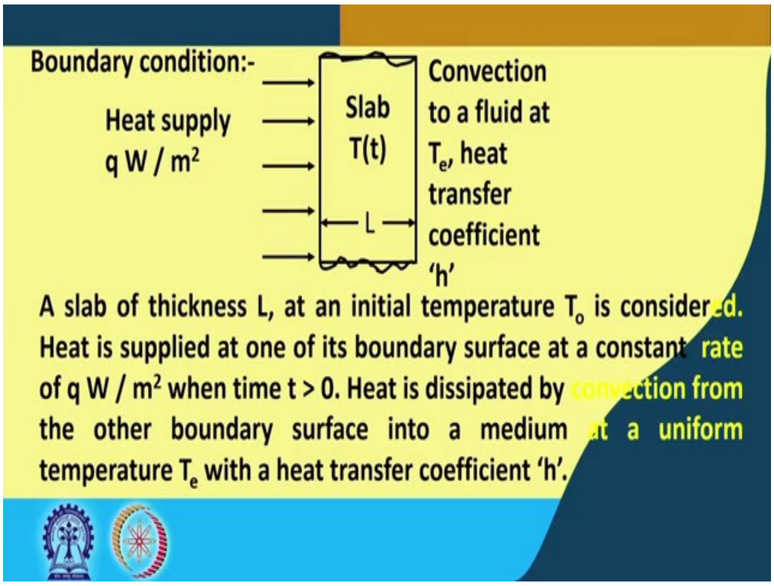

So, let us look into that to solve this kind of problem boundary conditions are very much necessary; boundary conditions of any slab that is fundamental to be looked into, 'right'. So, here we have given one boundary that you have a slab of thickness L and heat is being supplied q quantity. So, much Watt per meter square in one side and another side, a convection to a fluid that is T the heat transfer at the coefficient of h the slab has a thickness of L.

So, it is a slab of thickness L at an initial temperature of T_0 is considered heat is supplied at one of its boundary, surface at a constant rate of q W/m^2; when time is greater than 0. Heat is dissipated by convection from the other boundary surface into a medium at a uniform temperature T e; with a heat transfer coefficient h, 'right'; if that be true that this is a problem which is to be solved. So, q is given that is heat flux in one side, other side is convective heat transfer at time T is a greater than 0 constant rate of it flux is given another side is an the boundary, 'right'.

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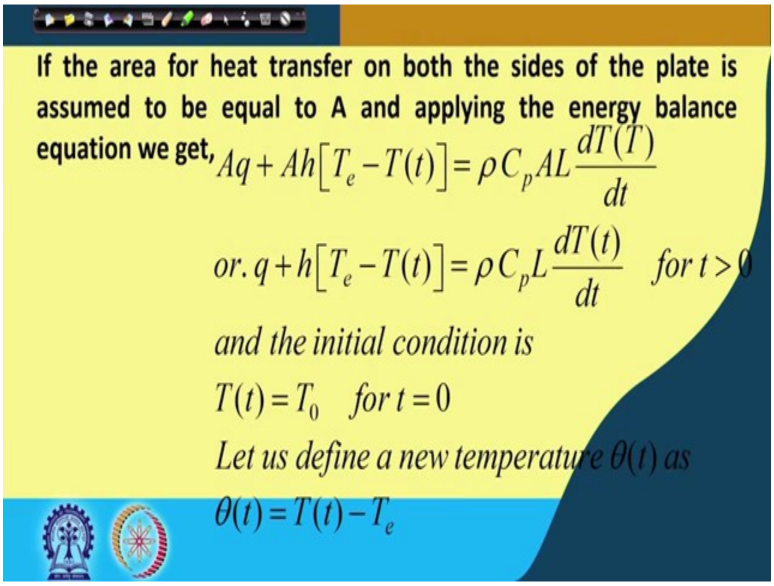

$$Aq + Ah\left[T_e - T(t)\right] = \rho C_p AL \frac{dT(T)}{dt}$$

$$or.\ q + h\left[T_e - T(t)\right] = \rho C_p L \frac{dT(t)}{dt} \quad for\ t > 0$$

and the initial condition is

$$T(t) = T_0 \quad for\ t = 0$$

Let us define a new temperature $\theta(t)$ *as*

$$\theta(t) = T(t) - T_e$$

And let us define a new temperature theta t; as theta t is T_t-T_e, 'right'. So, if this be true then we can find out that the solution of it, 'right'. So, next class we will do this solution because today now we are running out of time. So, we will solve it in the next class. So, next class before solving we will first have this problem again and then solve it ok.

Thank you.