Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 15
Thermal Resistance

So good afternoon, so we are continuing with the Thermal Resistance concept. And as in the previous class we are said that, till now we have done irrespective of cylindrical or rectangular or spherical coordinate. The resistance was seen series, all the resistances where in series, but it is not unnecessarily all the time you will have all the resistances in terms of series; it may be also in terms of a combination of series and parallel. So, we will look into those also, 'right'. So, we come to the class number lecture number 15, where we shall highlight on thermal resistance, 'right'. So, on thermal resistance will do this today that earlier we have already done, the all spherical, cylindrical and rectangular coordinate, 'right'. So, if we now take that thermal resistance concept where, if you remember we had said that equating equation number 1 and equation 3 and equation 4, things like that again it will come here, 'right'. So, the Q that quantity of heat is flowing through any resistance is

$$Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_b}{R_b}$$

That means, you have been one side T_a temperature and in another side T_b temperature, where in between different temperature ranges like, T_1, T_2, T_3, T_4 you can have at different locations, 'right'. And in this case as we have said earlier that Q is equal to delta T by resistance. So, here also that delta T each case delta T for the first case it is T_a minus T_1 and its resistance is R_a. So, R_a can be $1/h_a$ just a minute, so I that is what it is a composite medium, 'right'.

So, this is a composite medium and in that composite medium, we are doing this resistance concept, 'right'. So, environment is T_a, with a Q heat flow is going on and fluid is flowing at a temperature T_a, h_a and another this is temperature is going through a, another environment is, with the temperature T_b and it is flowing Q and heat transfer coefficient is h_b, 'right'.

So, at the surface we have temperature T_1, with conductivity of the material 1 to k_1 having the thickness L_1, coming to point 2, where it is T_2. And the second material is having conductivity of k_2, and the thickness L_2 third material with the conductivity of k_3 at the temperature of the interface T_3 with transfer with thickness L_3, 'right'.

If this be true then, we can write that Q quantity of heat is flowing where it is studying where the temperature T_a which are through a resistance R_a at temperature T_1, coming to a temperature T_1 through that resistance R_1, coming to the temperature T_2 through the temperature resistance R_2 coming to the temperature T_3 and resistance R_3 coming to the temperature T_4. And finally, through are we going out to that temperature T_b where to the Q flowing out, 'right'.

So, this we have then this picture we have finally, made it like this that Q is equal

$$Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_b}{R_b}$$

$$R_a = \frac{1}{Ah_a} \quad R_1 = \frac{L_1}{Ak_1} \quad R_2 = \frac{L_2}{Ak_2} \quad R_3 = \frac{L_3}{Ak_3} \quad and, R_b = \frac{1}{Ah_b}$$

$$\therefore \quad Q = \frac{T_a - T_b}{R} W$$

$$where, \quad R = R_a + R_1 + R_2 + R_3 + R_b$$

So, similar thing we have to find out, whether it is a series or parallel, in this case all the resistances, which we have shown where under resistance under series condition, 'right'. So, all the resistance is where under series condition. So, it can be that it will be series or it may be not in series, so that subsequently let us look into, 'right'.

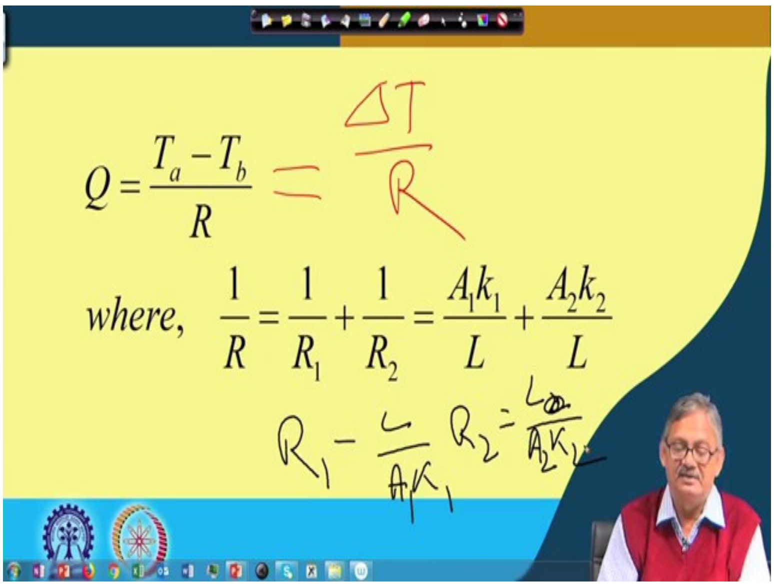
For this example, we have taken that on L length, 'right', L length, this side is insulated, this side is insulated and the other perpendicular to this is also insulated. So, only one dimensional heat transfer is occurring through this, 'right'. And these 2 sides are at the temperature T_1, these 2 sides are at the temperature T_2 and this is divided into 2 pieces that is this is one material, having area A_1 and conductivity k_1. This is another material having A_2 area and conductivity k_2. So, in this case it is appearing that the resistances are not in series.

So, Q quantity of heat is coming to the point temperature T_1 and then, a resistance of R is R_1 is equal to L/Ak_1. Because the L is the length for both the slabs, 'right'; both the slabs the thickness is L through which it is following so it is L and the resistance is R_1 is L/Ak_1, 'right'. And another resistance R_2 is equal to L/A_2k_2, A_1, k_1 was the first former this and L/A_2k_2 is this, 'right'. If this be true, then we can now use the parallel series or series and parallel this concept.

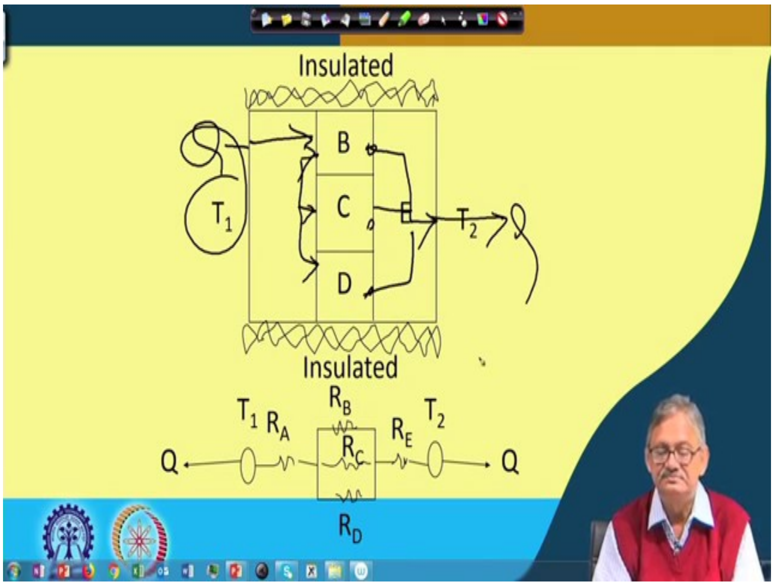
So, here it is; obviously, appearing this R_1 and this R_2 or not that is that Q is coming the to this point and getting this distributed one through this and another through this. And then getting conducted coming to this point T_2 this is also late coming to this point T_2 then, this through our again going out as a Q, 'right'. So, that is what pictorially we are demonstrated can this how? That Q is coming out this is here then through this resistance that is L/Ak_1 is equal to R_1 and through this resistance equal to R_2 is equal to L/A_2k_2 coming to the point T_2 and going out as Q, 'right'.

So, we have to solve this problem, 'right'. So, this problem can be easily solved by the thermal resistance, only we have to take care of the proper resistances. So, Q is $\Delta T_{total}/ R$, 'right' and in this case ΔT is equal to as it is written $T_a - T_b$, 'right' and R is the total resistance equal that is R.

And in this case that R is not in series, but in parallel, so for parallel resistance we know that $1/R$ is equal to $1/R_1$ plus, $1/R_2$, 'right' and R_1, R_2 are known. So, R_1 was L_1/Ak_1; and R_2 was L/A_2k_2, 'right'.

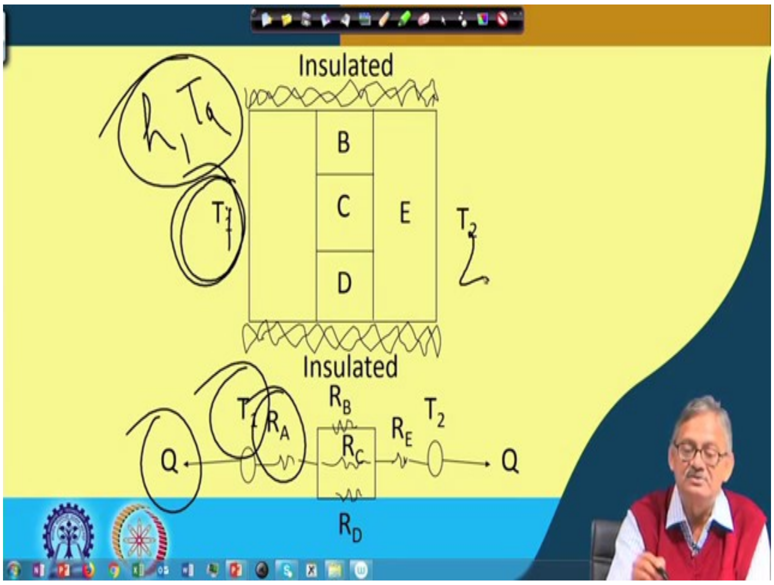


So, if we do that, then we can see that this 1/R is equal to $(1/R_1)+(1/R_2)$ which on simply which on substitution gives A_1k_1/L plus A_2k_2/L, 'right'. So, this on simplification gives us; this on simplification gives us, perhaps here we are not taken and this we can write this is equal to L, so A_1 which we are not further proceeded, but this can be written $A_1k_1+A_2k_2$, 'right'. So, if A_1, k_1; A_2, k_2, L are known, if T_a and T_b are known, then Q you can easily we found out, 'right' this is one example.

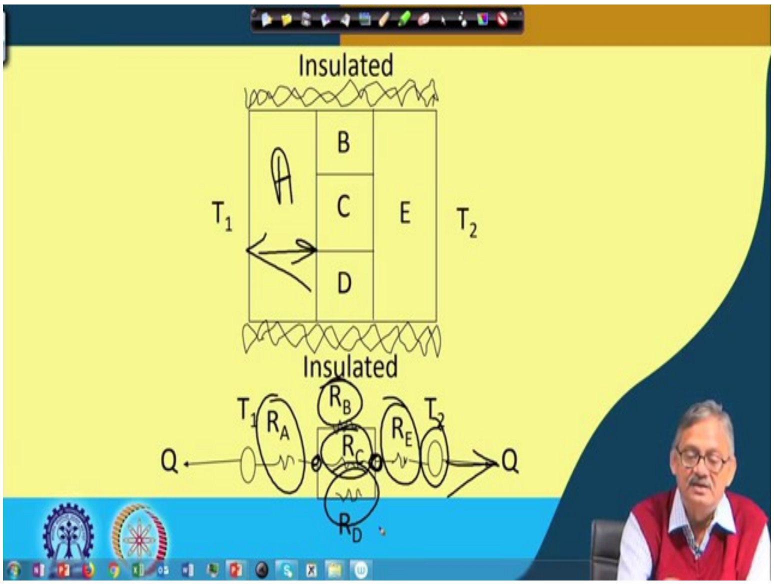


Another example can be; another example can be like this. So, you have T_1 as insulated, I not insulate T_1 on one side T_2 on another side and other 2 sides are insulated, 'right'; that means, there is no heat flow from the other two sides. So, in this case we have a slab like this another small slab like this, third is like this, fourth is like this and fifth one is like this, 'right'. So, 5 individual slabs of different length and conductivity are there and of different type of flow pattern, 'right'.

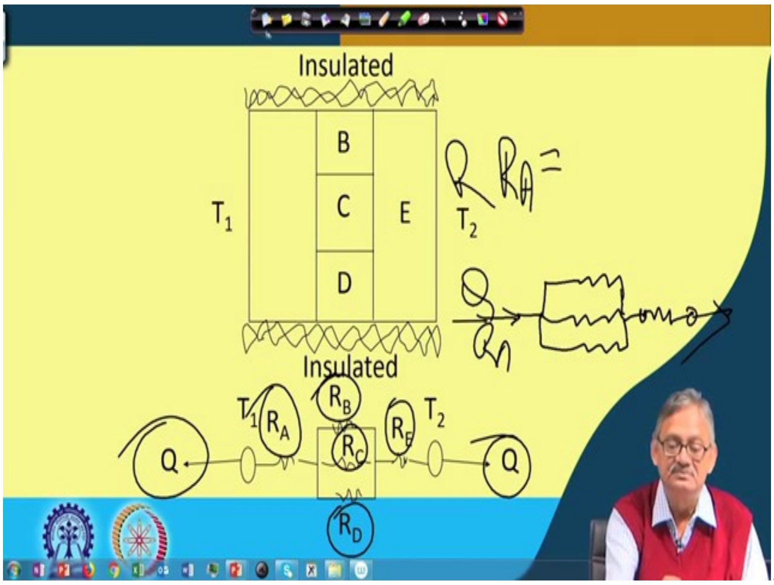
So, in one case, it is this T that is Q is coming through this T through this material at this point and then getting distributed as earlier, 'right', getting distributed like this, 'right'. Coming to this point and then again they are joining each other for being this T_2 and then going out as Q, 'right'. So, this is the concept which we have utilized here.



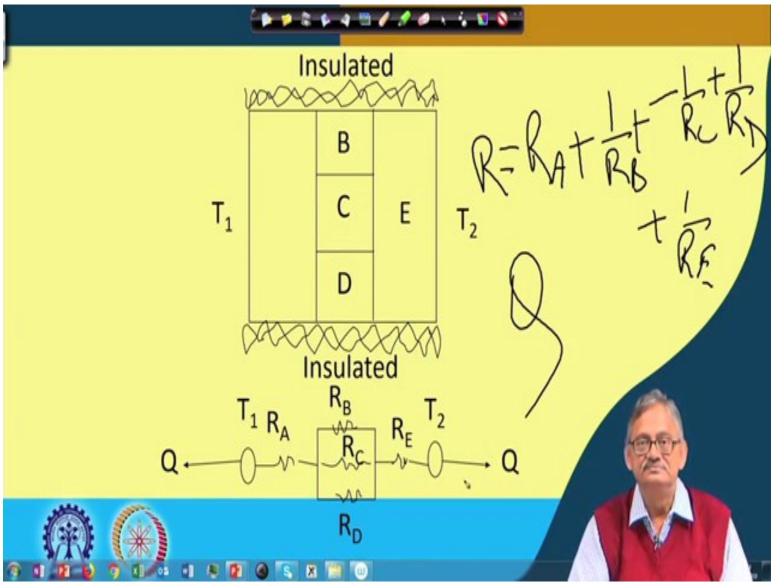
So, Q quantity of heat is coming to the point T_1, which the resistance of R_A corresponding to this T_1, because this T_1 can we corresponding to 1 h_1 and another T_a which will give us R_A. Then it has come to that point and that if it is assume that this is a constant temperature boundary, this is also a constant temperature boundary that this boundary is giving first resistance of this which is R_A, so this is the material A, 'right',



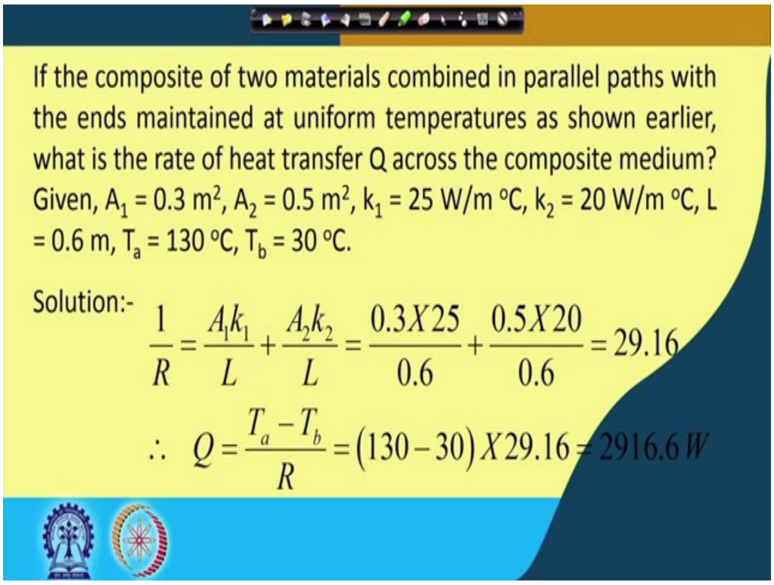
So, that gives us R_A and this again going through all these 3 one is R_B and these R_C and another is R_D there are in parallel. Then all these 3 parallel are coming to this point, from this point and then going to the R_E and to the temperature T_2 and then going out as Q, 'right'.



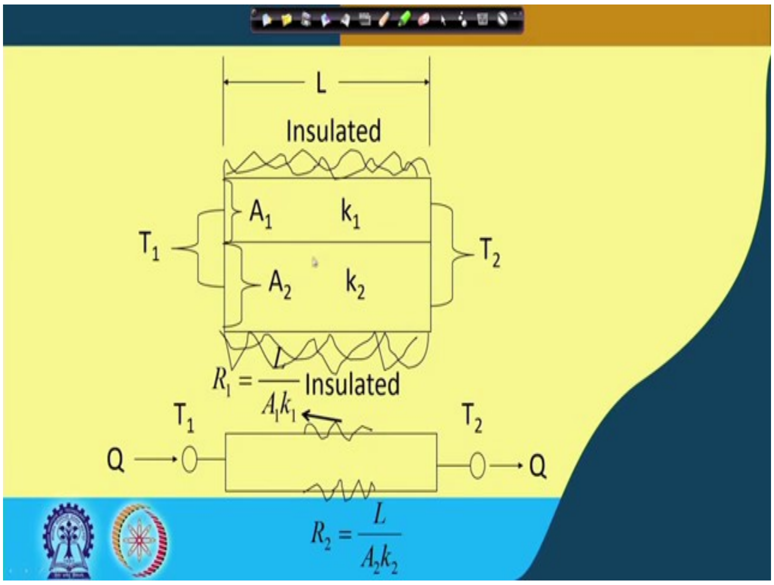
So, we know A; we know R_B, R_C, R_D, R_E, then we can easily find out Q that taking that R_A is in R_A is there. So, then we have 3 parallel resistances, R_B, R_C, R_D, 'right', which 3 are then going through R_E and then going out as Q, 'right', this is as Q.



So, R_A plus, total R will be R_A+1 /R_B+R_C +1 R_D+1/R_E, 'right' which we did in the previous one also. So, ones R_A, R_B, R_C, R_D, R_E are known we find out that value of Q, 'right', so this is another example of combination.

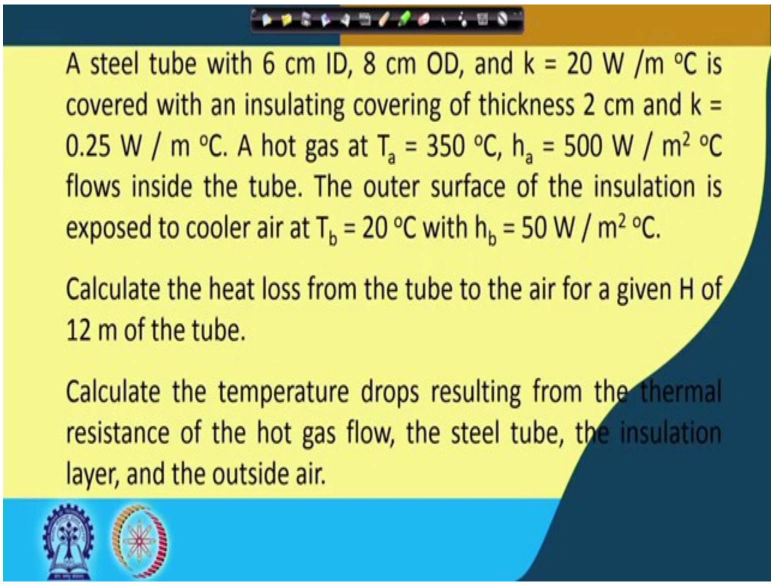


Similarly, we can see that if the composite of two materials combined in parallel paths, with the ends maintained at uniform temperature as shown earlier, what is the rate of heat transfer Q across the composite medium? Given A_1 is 0.3 m^2, A_2= 0.5 m^2, k_1 =25 W/ mºC, k_2=20 W/mºC, L 0.6 meter, T_a =130ºC, T_b =30 ºC, 'right'. So, from this problem it appears that the insulation will be like this, 'right' that insulation will be like this.



Or insulation can be also like this, it is A_1, k_1, A_2, k_2, L is constant. So, this is maintain that T_1 this is maintain that T_2, 'right'. So, similar this if it is then, we know that 1/R, equal to $A_1k_1/L + A_2k_2/L$, 'right'. So, this means A_1 given 0.3, and L is 0.6, k_1 given 25 and L 0.6, A_2 given 0.5, k_2 given 20, L same 0.6. So, this becomes 29.16 that is 1/R .

So, if it is so then Q is equal to delta T, i.e., T_a -T_b over R. So, delta T is 130 minus 30 and 1 by R has the value of 29.16; so 1 by R into 29.16 equal to 2916.6 watt, 'right'. So, this is another example which we have found out.

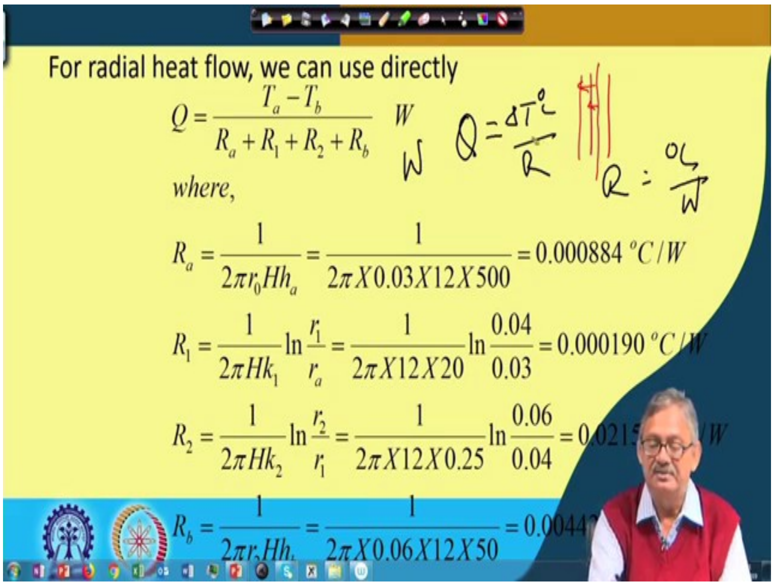


Now, it was proceed further and see a different type of solution, a steel tube with 6 centimeter ID, 8 centimeter OD and k W/m°C is covered with an insulating covering of thickness 2 centimeter and k conductivity point 0.25 W/m°C. A hot gas at T_a equal to 350 °C, h_a equal to 500 W/m^2°C flows inside the tube

The outer surface of the insulation is exposed to cooler air at T_b is equal to 20°C with heat transfer coefficient h_b is equal to 50 W/m^2°C. Calculate the heat loss from the tube to the air for a given H of 12 meter of the tube. Calculate the temperature drops during or resulting from the thermal resistance of the hot gas flow, the steel tube, the insulation layer and the outer air, 'right', so all the resistances we have to find out, 'right'.

So, if we read it once more perhaps this will be today is a last problem because the time which we have by that I hopefully we can complete this one. A steel tube with 6 centimeter, ID 8 centimeter, OD and k 20 W/m°C is covered with an insulating covering of thickness 2 centimeter and k conductivity 0.25 centimeter; k conductivity 0.25 W/m°C. A hot gas at T_a 350 °C, h_a 500 W/m^2°C flows inside the tube.

Outer surface of the insulation is exposed to cooler air at T_b is 20 degree centigrade with h_b equal to 50 W/m^2°C. Calculate the heat loss from the tube to the air for a given H of 12 meter of the tube. Also calculate the temperature drops during temperature drops resulting from, the thermal resistance of the hot gas flow the still tube the insulation layer and the outside here, 'right'.



$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_b} \quad W$$

where,

$$R_a = \frac{1}{2\pi r_0 H h_a} = \frac{1}{2\pi X 0.03 X 12 X 500} = 0.000884\ ^{o}C/W$$

$$R_1 = \frac{1}{2\pi H k_1}\ln\frac{r_1}{r_a} = \frac{1}{2\pi X 12 X 20}\ln\frac{0.04}{0.03} = 0.000190\ ^{o}C/W$$

$$R_2 = \frac{1}{2\pi H k_2}\ln\frac{r_2}{r_1} = \frac{1}{2\pi X 12 X 0.25}\ln\frac{0.06}{0.04} = 0.021510\ ^{o}C/W$$

$$R_b = \frac{1}{2\pi r_2 H h_b} = \frac{1}{2\pi X 0.06 X 12 X 50} = 0.004420\ ^{o}C/W$$

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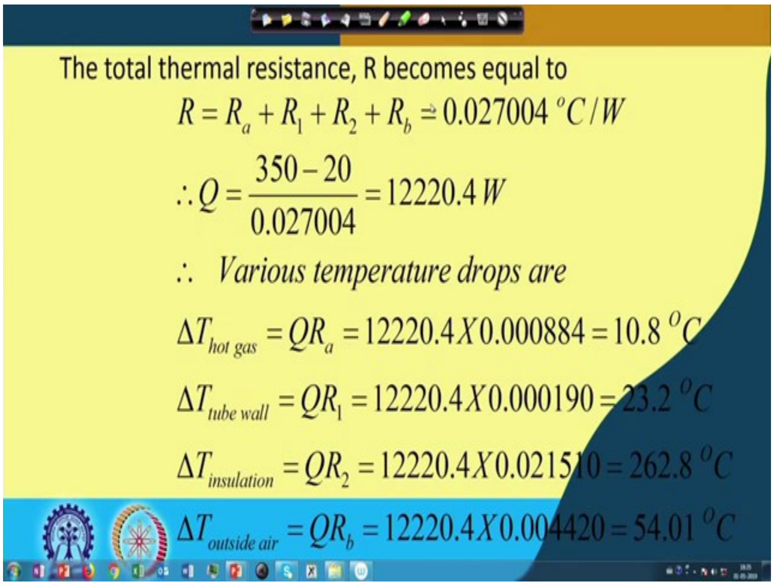

$$R = R_a + R_1 + R_2 + R_b = 0.027004\ {}^{o}C/W$$

$$\therefore Q = \frac{350-20}{0.027004} = 12220.4\,W$$

$\therefore$ *Various temperature drops are*

$$\Delta T_{hot\ gas} = QR_a = 12220.4X0.000884 = 10.8\ {}^{O}C$$

$$\Delta T_{tube\ wall} = QR_1 = 12220.4X0.000190 = 23.2\ {}^{O}C$$

$$\Delta T_{insulation} = QR_2 = 12220.4X0.021510 = 262.8\ {}^{O}C$$

$$\Delta T_{outside\ air} = QR_b = 12220.4X0.004420 = 54.01\ {}^{O}C$$

So, this means that if we have different then, we can find out delta T also and that is what is that we said the thermal resistance concept we will do it today and we will finish and you have done it. So, you do it at home, practice with different problems of different combinations. May be in terms of series in terms of single parallel or in terms of combination of series and parallel and then it will be very clear to you, ok.

Thank you, good day.