Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 12
One Dimensional Heat Transfer Through Cylinders (Contd.)

Good morning, so we were discussing about the steady state One Dimensional Heat Transfer in your cylindrical coordinate, 'right'. So, it is lecture 12, 'right', so it was by mistake it was written lecture 11, so now, it is 12, 'right'. So, we now go to that straightaway, where we were solving the problem. You remember, we were given a problem, so let us go directly to that.

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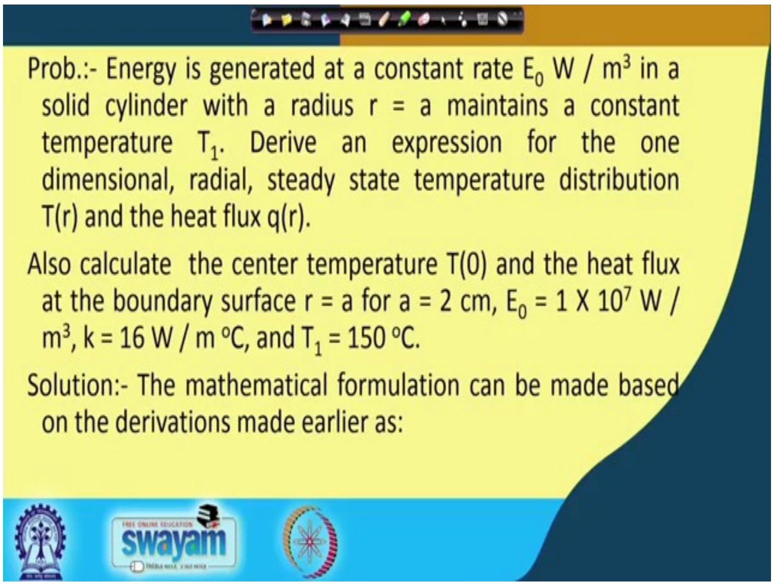

That this problem was given energy is generated at a constant rate of E_0 W/m^3 in a solid cylinder with a radius r is equal to 'a' maintains a constant temperature T_1. Derive an expression for one dimensional, radial, steady state temperature distribution T(r) and the heat flux q(r), 'right'.

So, this is to be done and also we are suppose to calculate for different E_0, r and T_1 what is the center temperature, 'right'. So, we had come to the level that we had integrated, 'right' and then, we had found out the two integration constant C_1 and C_2 by utilizing the boundary conditions; two boundary conditions; one boundary was dT/dr is equal to 0 at r is equal to 0, that is the it is similar to that there is no heat transfer at r is equal to 0, 'right' and the second boundary was at r is equal to a T(r) is equal to T_1, 'right'. So, with these two boundaries, we found out C_1 to be 0 and then the second boundary, where our time was over, second boundary, we found out C_2.

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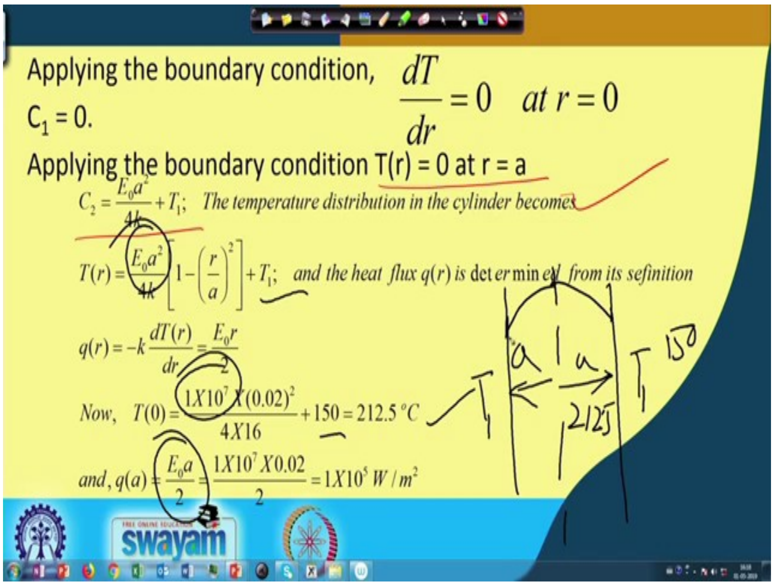


$$C_2 = \frac{E_0 a^2}{4k} + T_1; \quad \textit{The temperature distribution in the cylinder becomes}$$

$$T(r) = \frac{E_0 a^2}{4k}\left[1 - \left(\frac{r}{a}\right)^2\right] + T_1; \quad \textit{and the heat flux } q(r) \textit{ is determined from its sefinition}$$

$$q(r) = -k\frac{dT(r)}{dr} = \frac{E_0 r}{2}$$

$$Now, \quad T(0) = \frac{1X10^7 X(0.02)^2}{4X16} + 150 = 212.5\ ^oC$$

$$and, q(a) = \frac{E_0 a}{2} = \frac{1X10^7 X0.02}{2} = 1X10^5\ W/m^2$$

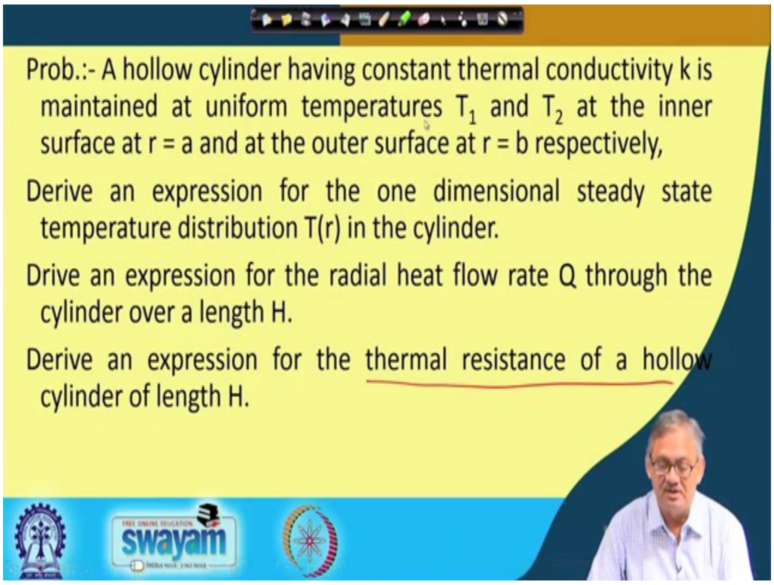
In the previous classes also I said whenever we are doing problems and you also please check that the whether the values coming like this or not. There may be some mistake calculation mistake or by as you have seen somewhere in some places by cut and paste there are some mistake.

So, mistakes can happen, so please do cross check and if it is different, then please bring to our notice that it is not, 'right'. Hopefully, it will not be, but still we cannot say that it will not be. So, please check, then if it is all right no problem, but if it is different, then bring to our notice we shall definitely do the correction again and then tell you, ‘right’.

there is an internal energy generation, your question may come in mind that you said that this is the axis and this is ‘a’, ‘right’; this is ‘a’, ‘right’ and this side is T_1 and this side is also T_1. sorry, this side is also T_1.

So, if this is true then and this T_1 value is 150 ºC, then how come this T_0 is 212.5, ‘right’? So, that may be a question arising in your mind, but the thing is that here we have said there is an internal energy generation. So, that is equivalent to 1×10^7 W / m^3, ‘right’. So, that is why the temperature distribution has become like this that here 150 sorry; here it was 150; rest to 212 and the other side is same because this is the mirror image, ‘right’.

So, this can happen because there is internal energy generation. So, this way if there is internal energy generation, we can solve the problem by analytical method and by integrating and then, putting the boundaries, we can solve it, ‘right’, so this is one way. Now, let us look into the next.



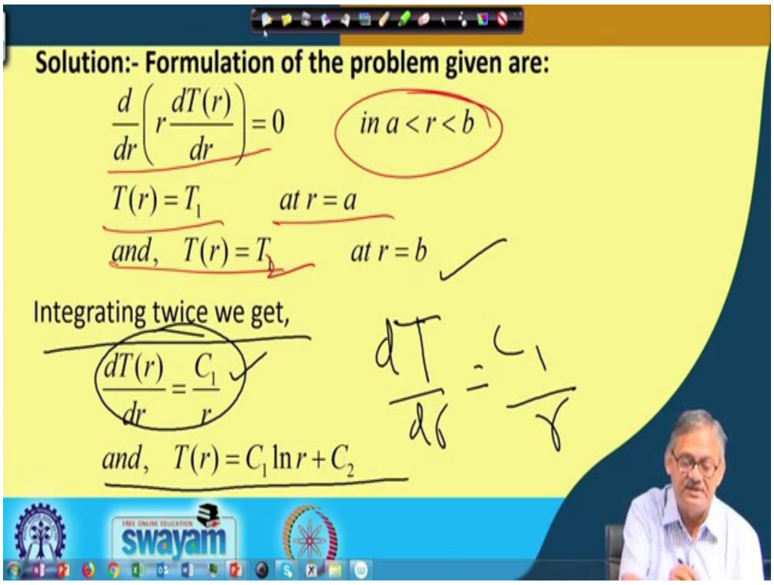
A hollow cylinder having constant thermal conductivity k is maintained at uniform temperature T_1 and T_2 at the inner surface r at r is equal to ‘a’ and at the outer surface at r is equal to ‘b’ respectively. Derive an expression for the one dimensional steady state temperature distribution T(r) in the cylinder. Also derive an expression for the radial flow radial heat flow rate capital Q through the cylinder over a length H. Derive an expression for the thermal resistance of a hollow cylinder of length H, ‘right’ and you remember that in the some previous class, we said also that to solve any problem please read at least twice, so that you first time you whenever you are reading.

So, that time only you can make what we what is given and what we done, but subsequently on the second time be sure that this is given and this is to be done, so that is fundamental. So, it here also it is like that a hollow cylinder having constant thermal conductivity k is maintained at uniform temperatures $T_1$and T_2, at the inner surface at r is equal to ‘a’ and the outer surface at r is equal to ‘b’ respectively.

Derive an expression for the one dimensional steady state temperature distribution T r in the cylinder as well. Derive an expression for the radial heat flow rate capital Q through the cylinder over a length H and also derive an expression for the thermal resistance, ‘right’; for the thermal resistance of the hollow cylinder of length H, ‘right’. Now, if we remember, we had said earlier that if you have; if you have a hollow cylinder like this, ‘right’ that is what is said. So, this is the axis; that means, if we take this side that is good enough to say the other side.

So, if we; that means, here the one radius is given that is at ‘a’ and the other radius is given at ‘b’. The same is true on this two sides also, this is ‘a’ and that is ‘b’, ‘right’. So, this means if we take only this one side, then this side has can be eliminated ‘right’. So, to do that what we can do? Here, we draw it again perhaps we have drawn that it appears to be like this and this is b and this is sorry rewritten, this is b and this was a, ‘right’; this is a and this is b. So, the net this one is b minus a, ‘right’. So, b minus a is the thickness through which the heat is flowing, either this way or this way depending on what is the temperature.

Now, the temperature given is also here it is say T_1 at r is equal to a. So, here it is T_1 and here it is T_2, ‘right’, then and there is no internal energy generation. Now, if you remember in Cartesian coordinate, we said if there is no a internal energy generation, then the solution becomes easier because then you can utilize the electrical resistance concept also or equivalent to thermal resistance concept, ‘right’.

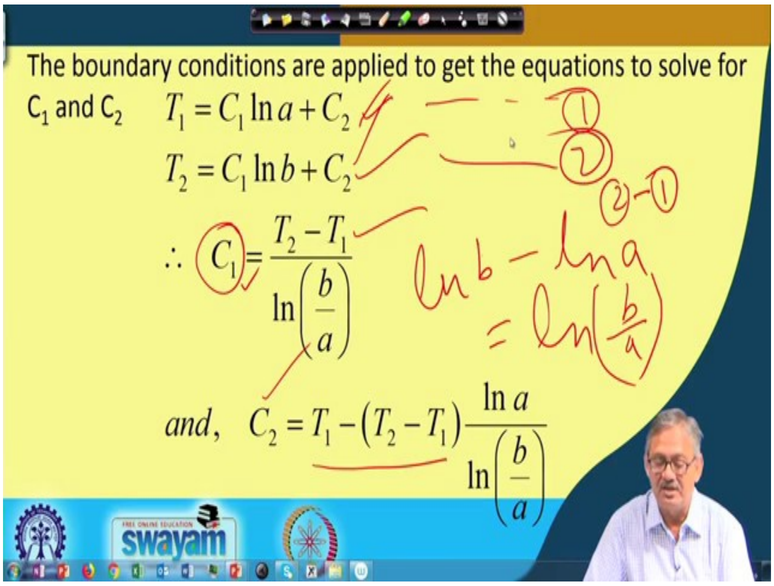


So, from the resistance concept, we can write we can solve it like this; we can solve it like this that this is the probable solution is

$$\frac{d}{dr}\left(r\frac{dT(r)}{dr}\right)=0 \qquad in\ a<r<b$$
$$T(r)=T_1 \qquad at\ r=a$$
$$and, \quad T(r)=T_1 \qquad at\ r=b$$

T(r) is equal to T_2 not T_1, So, if these be true, then we can say that integrating between that integrating the two integrations

$$\frac{dT(r)}{dr}=\frac{C_1}{r}$$
$$and, \quad T(r)=C_1\ln r+C_2$$

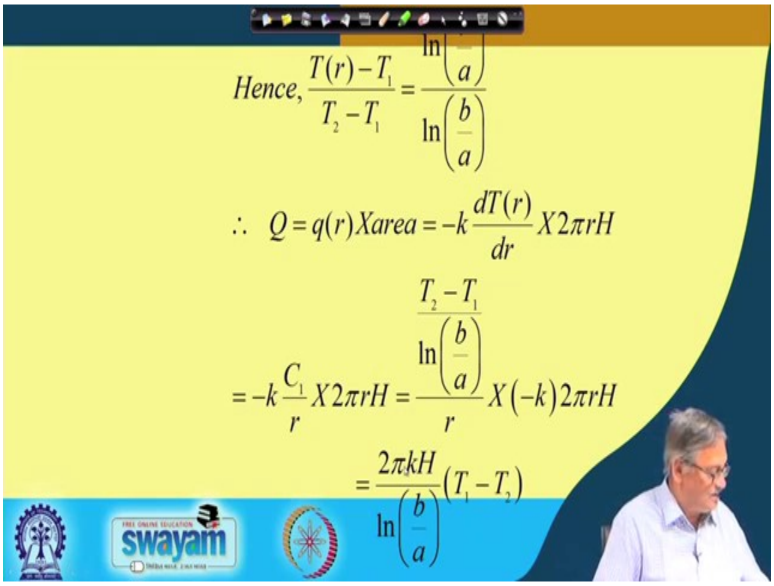


Then if this be true, we have 2 boundaries and we can solve it like the boundary conditions given are

$$T_1 = C_1 \ln a + C_2$$
$$T_2 = C_1 \ln b + C_2$$
$$\therefore \quad C_1 = \frac{T_2 - T_1}{\ln\left(\frac{b}{a}\right)}$$
$$and, \quad C_2 = T_1 - \left(T_2 - T_1\right)\frac{\ln a}{\ln\left(\frac{b}{a}\right)}$$

So, by I mean what we did we subtracted equation 2 from equation 1, 'right'. So, we are equation 1 which subtracted from equation 2 that is why it became $C_1 = \frac{T_2 - T_1}{\ln\left(\frac{b}{a}\right)}$.

Therefore, C_1 is known now we can substitute anywhere either in 1 or 2, this value of C_1 and we can get the value of $C_2 = T_1 - \left(T_2 - T_1\right)\frac{\ln a}{\ln\left(\frac{b}{a}\right)}$ So, C_2 ln becomes this becomes, 'right' like that and we can we can say that C_2 has become like this.

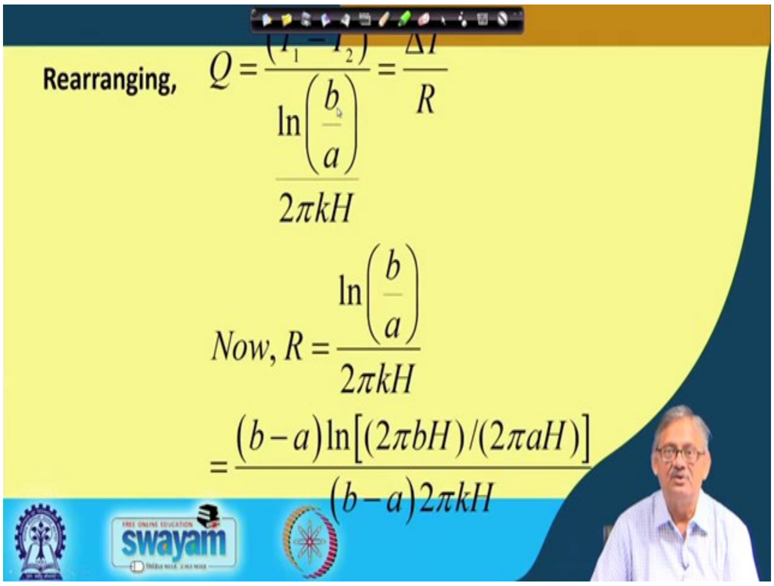


$$Hence, \frac{T(r)-T_1}{T_2-T_1} = \frac{\ln\left(\frac{r}{a}\right)}{\ln\left(\frac{b}{a}\right)}$$

$$\therefore \quad Q = q(r) X area = -k\frac{dT(r)}{dr} X 2\pi rH$$

$$= -k\frac{C_1}{r} X 2\pi rH = \frac{\frac{T_2-T_1}{\ln\left(\frac{b}{a}\right)}}{r} X(-k)\, 2\pi rH$$

$$= \frac{2\pi kH}{\ln\left(\frac{b}{a}\right)}(T_1-T_2)$$



So, this on rearrangement again, it is coming

$$Q = \frac{(T_1-T_2)}{\frac{\ln\left(\frac{b}{a}\right)}{2\pi kH}} = \frac{\Delta T}{R}$$

$$Now, R = \frac{\ln\left(\frac{b}{a}\right)}{2\pi kH}$$

$$= \frac{(b-a)\ln[(2\pi bH)/(2\pi aH)]}{(b-a)\,2\pi kH}$$

So, we have multiplied by 2 π a H both in the numerator with b and a, so it goes up. This is what we have explicitly made according to our future goal that we want to bring it to the tune, as we have shown delta T by R is Q and delta T, we have said T_1 - T_2, but are we as now coming to this that this is equal to that R is T/kA_m, 'right'.

$$where,\ A_m = \frac{A_0 - A_i}{\ln(A_0/A_i)}$$

A_m called log mean area, 'right'; A_m is the log mean area. So, R is

$$or,\quad R = \frac{t}{kA_m};\quad where,\ A_m = \frac{A_0 - A_i}{\ln(A_0/A_i)}$$

$$where,$$
$$A_i = 2\pi aH = area\ of\ inner\ surface\ of\ cylinder$$
$$A_0 = 2\pi bH = area\ of\ outer\ surface\ of\ cylinder$$
$$A_m = \log arithmic\ mean\ area$$
$$t = (b - a) = thickness\ of\ cylinder$$

A_m is the log logarithmic area or mean area, thickness is t; small t. Though, small t is generally denoted for time, but in many cases where time is not t because here it is time independent. So, that is why t is taken as the thickness, in many cases t is also taken as thickness and not only the time. Either time or thickness normally this is these are the two connotations used.

So, t is the thickness which is b minus a, that is thickness of the cylinder. So, we can write R= T/ kA_m and A_m we have already written, 'right'. So, again we have come to the end of the class. So, here we have seen today that we are able to do both for this solid cylinder analytical solution and for the hollow cylinder, we have done the not the analytical solution, again it is of course, I cannot say it is analytical solution, but not like that it is with respect to the thermal resistance concept.

And with the thermal resistance concept, we have found out and shown that you know anything is equal to delta T/R or driving force this is called driving force by resistance, ‘right’. Any parameter which can be written as driving force by resistance; driving force in this case is temperature this is delta T and resistance R that also you have shown, ‘right’. Hopefully, you will go through and in the next class we will do some problem solutions, ok.

Thank you.