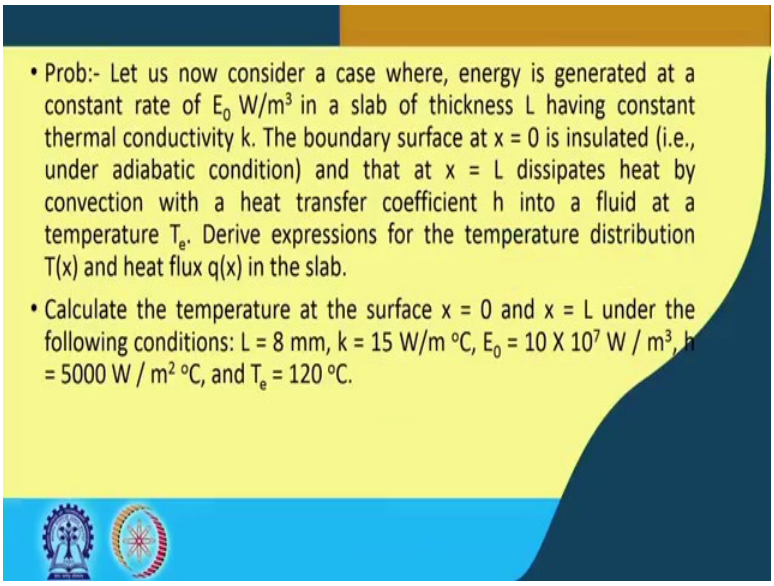
Thermal Operations in Food Process Engineering: Theory and Applications

Prof. Tridib Kumar Goswami
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

Lecture - 10
One Dimensional Steady State Heat Conduction (Contd.)

Hello. So, good morning today again we are continuing that One Dimensional Steady State Heat Conduction equation. We were solving problems and through h the solution of the problems we are coming across more and more how to solve different problems with different boundary conditions, ‘right’. So, now, let us go to another. So, this again one dimensional steady state heat conduction continuation lecture number 10.

(Refer Slide Time: 00:59)



So, we come to this that problem. Let us know how consider a case where energy is generated at constant rate of E_0 W/m^3 in a slab of thickness L having constant thermal conductivity k.

The boundary surface at x is equal to 0 is insulated that is under adiabatic condition. And, that at x is equal to L dissipates heat by convection with a heat transfer coefficient h into a fluid at a temperature T_e. Derive expressions for the temperature distribution T(x) and heat flux q(x) in the slab.

Calculate the temperature at the surface x is equal to 0 and x is equal to L under the following conditions; L is 8 millimeter, k is 15 W/mºC, E_0 is 10×10^7 W/m^3, h is 5000 W/m^{2}ºC and T_e is 120ºC.

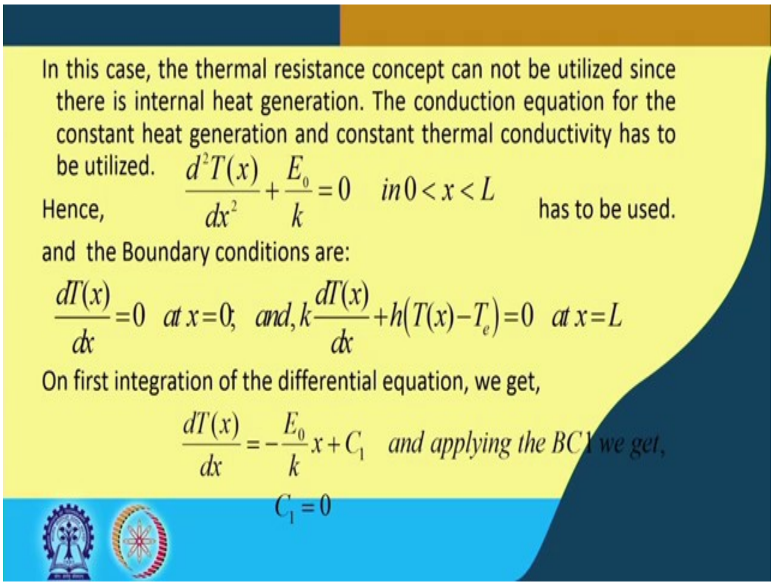
So, if we can solve this one also, but since here you see that internal energy generation is present. Since, internal energy generation is present we cannot use thermal resistance concept, 'right'. Had there been no internal resistance generation, we could definitely solve it with the thermal resistance concept, but it cannot be now, 'right'. So, we have to do analytical solution.

So, to understand you should read the problem number of times at least once or twice so, that you understand the problem mentally. The problem again is saying like this let us consider now a case where energy is generated at a constant rate of E_0 W/m^3. So, energy generation is present.

In a slab of thickness L having constant thermal conductivity k. The boundary surface now the boundary condition at x is equal to 0 is insulated; that means, you have some insulation that is that can be said equivalent to be under adiabatic condition 'right' no heat flow. And, that at x is equal to L dissipates heat so, other side is dissipating heat to the boundary by convection with a heat transfer coefficient h into fluid at a temperature T_e, 'right'. The derive expression for temperature distribution T(x) and heat flux q(x) in the slab and, also we have to calculate the values of T_{es} perhaps values of calculate the temperature at the surface x is equal to 0 and x is equal to L.

So, x is equal to 0 and L we have to find out the temperatures and we are given thermal conductivity heat transfer coefficient length of the slab that is thickness or L_0 to L. So, thickness of the slab and internal energy and environmental temperature everything are given. So, we can solve it, 'right'.

(Refer Slide Time: 05:17)



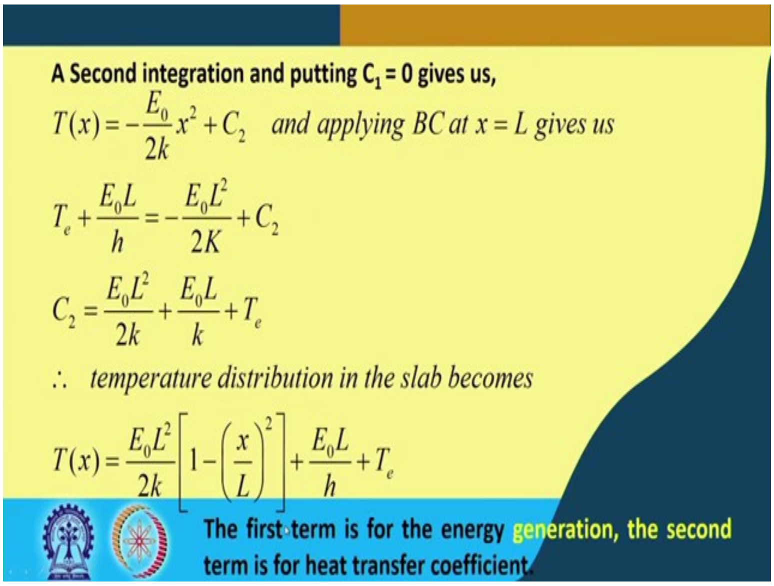
So, in this case the thermal resistance concept cannot be utilized, because there is internal energy generation. The condition or the conduction equation for the constant heat flux, or the constant not heat flux, for the constant heat generation, and constant thermal conductivity has to be utilized. And, now we start with that where we ended with you remember we said that the here it is steady state.

So, the moment it is steady state, so the right side becomes equal to 0. So, our equation becomes equal to $\frac{d^2T(x)}{dx^2}+\frac{E_0}{k}=0 \quad in\, 0<x<L$ and this is to be used for the solution. And, boundary condition given one side it is given you remember one side it is given that it is insulated. The moment the boundary condition is insulated we can say that $\frac{dT(x)}{dx}=0 \quad at\; x=0; \quad and, k\frac{dT(x)}{dx}+h\left(T(x)-T_e\right)=0 \quad at\; x=L$ So, conduction equal to convection that is the second boundary. First boundary was insulation and second boundary is conduction equivalent to convection.

So, our governing equation was if $\frac{d^2T(x)}{dx^2}+\frac{E_0}{k}=0 \quad in\, 0<x<L$ we integrate first integration give $\frac{dT(x)}{dx}=-\frac{E_0}{k}x+C_1$ and applying the BC $C_1=0$. And, now if we apply the first boundary condition that is dT(x)/d x is equal to 0 at x is equal to 0 we get C_1 is equal to 0.

Because, this x is 0 so, this is 0 and dT(x)/dx is 0 from here. So, we get C_1 is equal to 0.

(Refer Slide Time: 08:37)



So, on second integration we get C 1 is equal to 0, ‘right’. Then

$$T(x) = -\frac{E_0}{2k}x^2 + C_2 \quad \textit{and applying BC at } x = L \textit{ gives us}$$

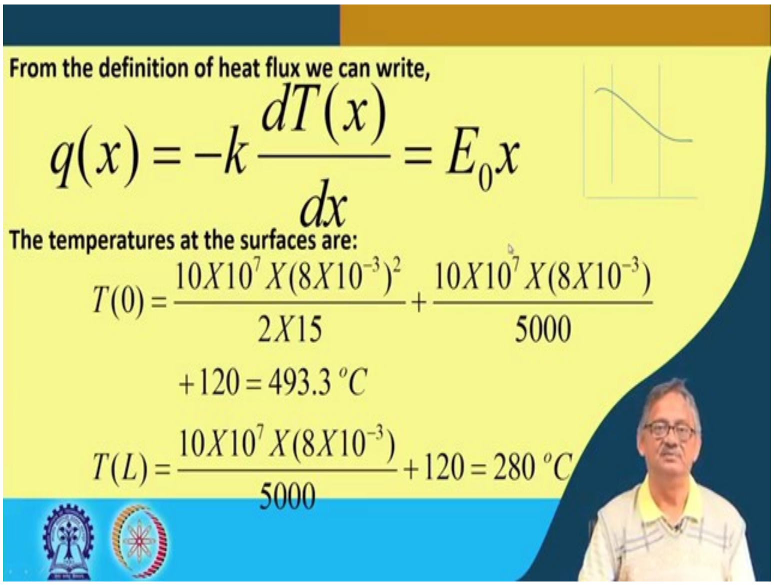
$$T_e + \frac{E_0 L}{h} = -\frac{E_0 L^2}{2K} + C_2$$

$$C_2 = \frac{E_0 L^2}{2k} + \frac{E_0 L}{k} + T_e$$

$$\therefore \quad \textit{temperature distribution in the slab becomes}$$

$$T(x) = \frac{E_0 L^2}{2k}\left[1 - \left(\frac{x}{L}\right)^2\right] + \frac{E_0 L}{h} + T_e$$

(Refer Slide Time: 11:47)



So, this is for energy generation and this is for the heat transfer coefficient. So, we can write that $q(x) = -k\frac{dT(x)}{dx} = E_0 x$. So, we have the solution we have found out the temperature distribution T(x) and also you have found out the q(x) at any x which is E_0x, 'right'. Now, we can solve easily and find out the temperatures at x equal to 0 and that x as to L

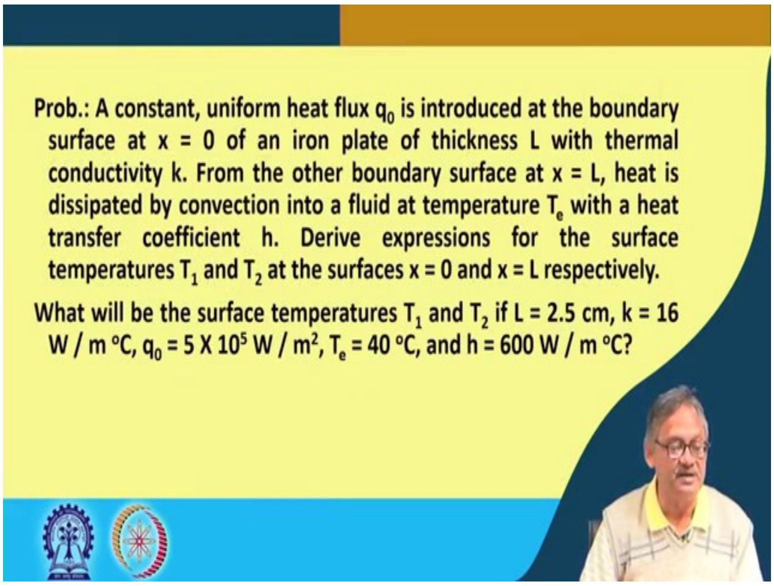
$$T(0) = \frac{10X10^7 X(8X10^{-3})^2}{2X15} + \frac{10X10^7 X(8X10^{-3})}{5000}$$
$$+120 = 493.3\,^oC$$
$$T(L) = \frac{10X10^7 X(8X10^{-3})}{5000} + 120 = 280\,^oC$$

Then, if we plot that the temperature distribution, then it comes like this you will see we can show that here one and here another, ok.

So, our temperature distribution that can be written that can be shown that this is equal to let us take this. So, here it was ok before that let us have the slab, this is the one part of this slab and this is the other part of this slab, 'right'. So, our boundaries are like that. So, the temperature then becomes equal to some temperature here, 'right' and then it became like this and then it became like that.

So, this is the temperature distribution which we are we have found out, 'right'. So, this is the solution which we have shown yeah this is the solution there this is 280 oh this was this was constant. So, this started from here itself, 'right' E_0, T_0 was 493 here and then it dropped to this 280 and outside was 120, 'right'. So, this way we can resolve the problems ok.

(Refer Slide Time: 15:35)



So, we let us take another problem a constant uniform heat flux q_0 is introduced at the boundary surface at x is equal to 0 of an iron plate of thickness L with thermal conductivity k. From the other boundary surface at x is equal to L, heat is dissipated by convection into a fluid at temperature T_e with the heat transfer coefficient h.

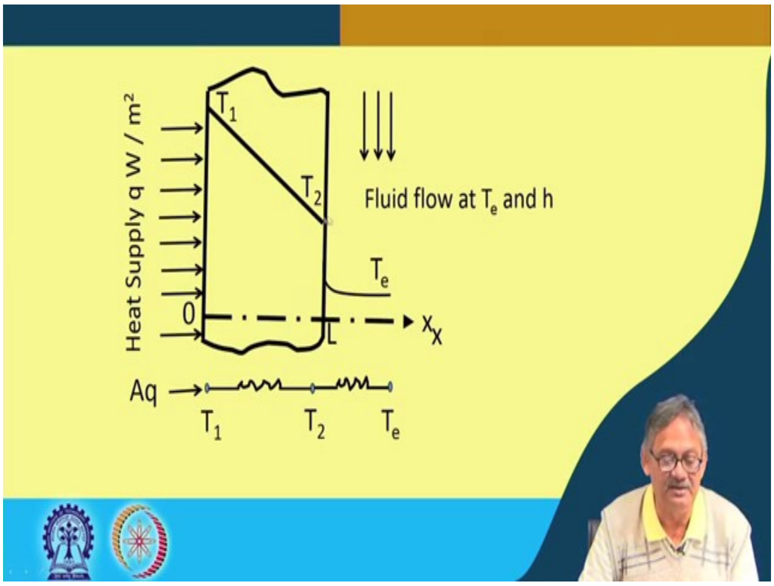
Derive expressions for the surface temperatures T_1 and T_2 at the surfaces x is equal to 0 and x is equal to L respectively. What will be the surface temperature T_1 and T_2 if L is 2.5 centimeter k is 16 W/m°C, $q_0 5\times10^5$ W/m², T_e 40 degree centigrade and h 600 W/m°C.

Say, here also you see that the since there is internal energy generation we cannot assume the thermal resistance concept, 'right'. So, again we need to solve it analytically. And again as you said to solve we have to read the problem and understand mentally and then follow it accordingly.

So, I am reading the problem or reading out the problem once again a constant uniform heat flux q 0 is introduced at the boundary surface at x equal to 0 of an iron plate of thickness L with thermal conductivity k. From the other boundary surface at x is equal to L heat is dissipated by convection into a fluid at temperature T_e with a heat transfer coefficient h. Derive the expressions for the surface temperatures T_1 and T_2 at the surfaces x is equal to L and x is equal to L_0 and x is equal to L respectively, 'right'.

So, this means that our problem is saying that you have a plate iron plate and one side is subjected to the boundary condition of second kind that is prescribed heat flux boundary condition. And the other side is boundary condition of the third kind that is prescribed convective boundary condition as well internal heat generation is there. So, you have to do first the analytical solution and if you then you have the temperature distribution equation, you can easily find out what is the two phases temperatures that is T_0 and T_L, 'right'.

(Refer Slide Time: 18:57)



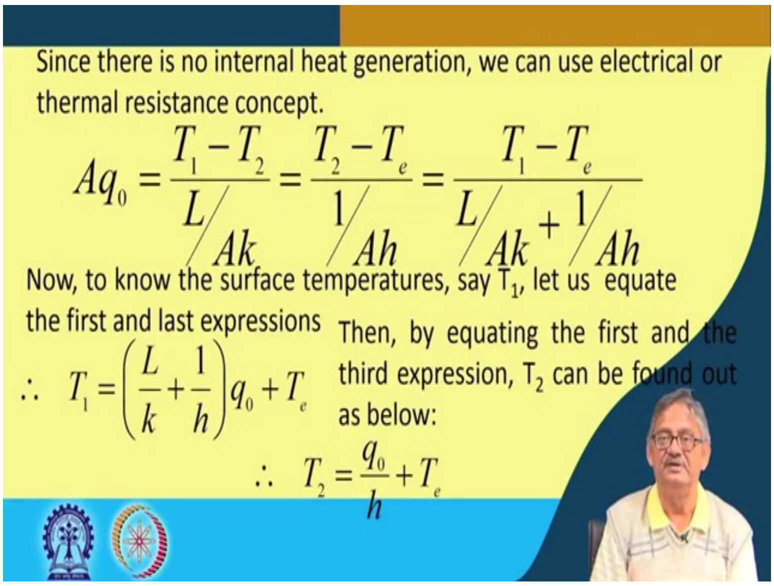
Here is that pictorially we can you can write it this is the iron slab and one side is heat supply by q we what so, much watt per meter square this is that, 'right'.

Internal heat one thing, internal heat generation that is why we have to read it, 'right'. So, here there is no internal generation is saying, but what did it saying that the flux. So, though looking at the q; I mean that the internal generation of heat was mentioned? no it is not mentioned here. So, here one boundary is heat flux, another boundary is convective boundary condition and the plate is there, 'right'.

So, since there is no internal generation of heat as well since it is a steady state though it is not said, but it is understood that there is a steady state. So, here we can use thermal resistance concept, 'right'.

So, that is what we are using W watt per meter square that is T_1 and T_2 and a boundary with convective boundary. So, with a fluid flow at T_e temperature h as the heat transfer coefficient 0 to L and it is in the X direction. So, A_q that is capital q quantity of heat is coming all, 'right', q is heat flux, A is the area. So, A_q, q quantity of capital q quantity of heat is coming and at the face this is T_1 and within that resistance of the material, it is coming to T_2 and then getting dissipated to T_e as the other side of the boundary. So, we have two resistances, one is internal and another is this and this is supplied by the heat flux.

(Refer Slide Time: 21:21)

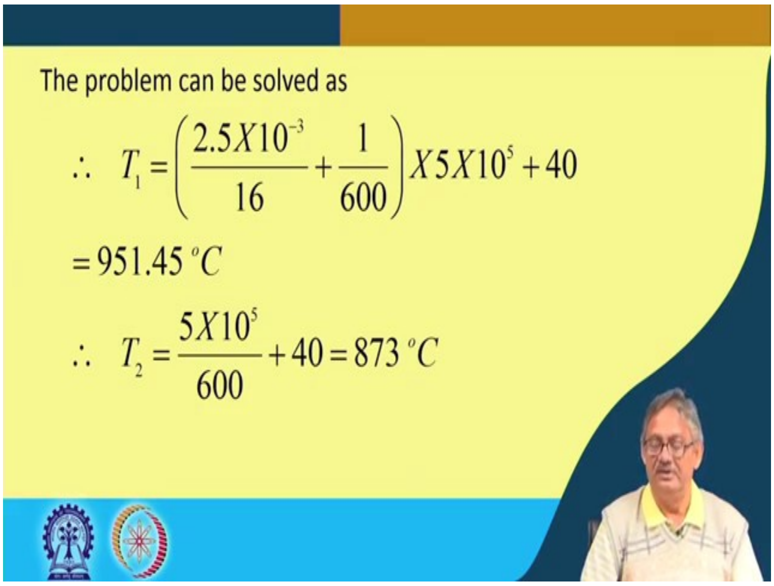


So, solution of heat can be that q is equal to capital Q that is

$$Aq_0 = \frac{T_1 - T_2}{L/Ak} = \frac{T_2 - T_e}{1/Ah} = \frac{T_1 - T_e}{L/Ak + 1/Ah}$$

Now, to know the surface temperature say T 1, let us equate the first and last expressions then we get $\therefore \quad T_1 = \left(\frac{L}{k} + \frac{1}{h}\right) q_0 + T_e$. And, then by equating the first and the third expression T_2 can be found out and this can be written as T_2 is equal to q_0 over h plus T_e, 'right'.

(Refer Slide Time: 22:33)



So, if we have already found out T_1 and T_2 we can easily now calculate the numerical values, because all others are given. So,

$$\therefore \quad T_1 = \left(\frac{2.5X10^{-3}}{16} + \frac{1}{600} \right) X5X10^5 + 40$$

$$= 951.45\ ^oC$$

$$\therefore \quad T_2 = \frac{5X10^5}{600} + 40 = 873\ ^oC$$

The material offers resistance of heat transfer. So, that you have to keep in mind that material is offering resistance and depending on how much may resistance it is offering the drop in temperature will be accordingly, 'right'. So, let us see whether any other problem we have or not; we can say that we observed all types of problems.

Once with the constant temperature boundary condition at left side, there is one side an another constant temperature boundary condition on the other side, we have made the analytical solution and we have also made the thermal resistance solution. So, the where internal energy was not present, internal energy generation was not present they are thermal energy also we have solved.

In another case we have also seen both the sides are convective, this side convective, this side also convective through boundaries and the material is getting conducted. So, we have solved that analytically in both the cases once when there was internal energy generation and in another case when there was no internal energy generation, ‘right’.

If internal energy generation is there we cannot solve it analytically, we cannot solve it by thermal resistance concept, but we have to solve it by analytical solution. So, we did it by analytical solution and then we also solved when internal energy was not there through the thermal resistance concept.

And, third one we have done when a constant heat flux was supplied that one boundary and the other boundary was by convective boundary condition and the in material is offering resistance. So, the temperature is dropping from one high one temperature to another. So, whether it is how high or low all will depend on what kind of values you are getting, what kind of values you are assuming for the different parameters like, heat transfer coefficient like, heat flux like, you know conductivity like, your thickness of the material or if there is internal energy generation.

So, depending on the values which you are choosing, your this thing will also be accordingly, ‘right’. So, with this let us conclude with thermal concept with one dimensional heat transfer coefficient. So, we thank you and thank you for the next class, ok, we will meet in the next class.

Thank you.