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Week - 09
Lecture - 53
Multi-Criteria Decision-Making for Natural Resources Management: Part-03

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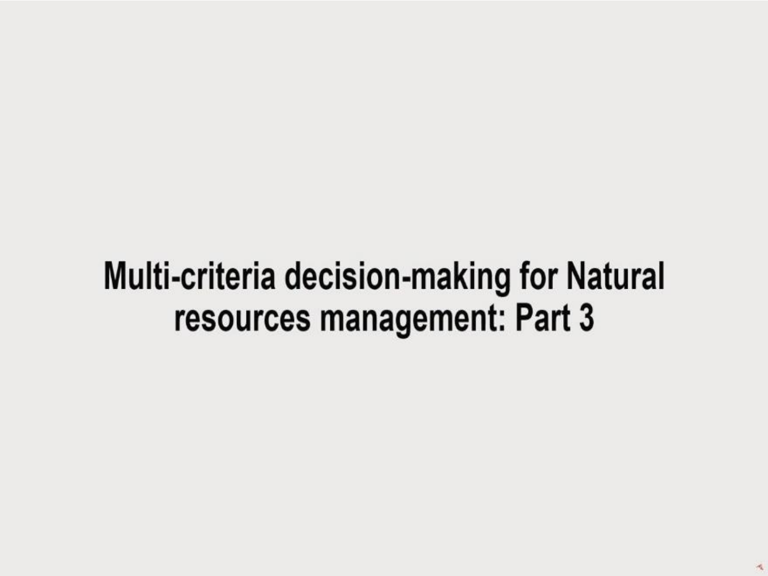

Dear participants, continuing our discussion on Multi-Criteria Decision Making for Natural Resource Management. In part 3, we will discuss about the techniques for order performance by similarity to ideal solutions.

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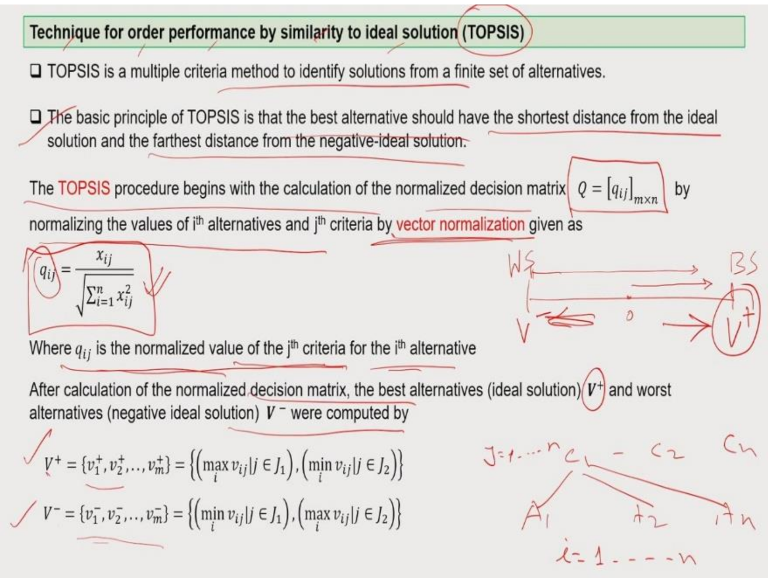

And this technique in brief it is known as TOPSIS method. If you recall at the introductory lecture of MCDA I have mentioned about couple of techniques of MCDA and TOPSIS is one of them. The full form of TOPSIS as you see here technique for order performance by similarity to ideal solution, little long name but TOPSIS is easy to remember. What is TOPSIS technique? TOPSIS technique is a multiple criteria method, which helps to identify solutions from a finite set of alternatives. Means, where your alternatives are known certain numbers say 6, 8, 10 finite numbers.

In TOPSIS the basic principle is that the best alternative should have the shortest distance from the ideal solution. Means, the alternatives which brings you very close to the ideal solutions and the farthest distance from the negative ideal solutions. So, suppose you have here the scale. This is your best solution and this is your worst solutions, you are here at the 0 line. So, TOPSIS allows you to reach as close as the absolute best solutions. So, you go closer means you go far away from your worst alternatives or choice. So, that is what is telling.

Now TOPSIS procedures, it is begins with the calculation of the normalized decision matrix. You remember, we discussed about various normalization process, log based, addition multiplication based. So, here TOPSIS procedure, it starts with the calculation of the normalized decision matrix that is your Q is equal to a m cross n matrix q

So, this normalized decision matrix by normalizing the values of the ith alternatives and jth criteria by vector normalization. How you do that, this is the process

qij equal to xij by summation of xij, i 1 to n.

qij equals to xij by square root of summation I equal to 1 to n xij square

So, where qij is the normalized value of the jth criteria and ith alternative. You remember, C1, C2, Cn criteria and then you have A1, A2, An.

So, this is your ith means i can be is equal to 1 to n and for criteria jth criteria this also can be j is equal to 1 to n. So, qij is the normalized value of the jth criteria for the highest alternative. After you do the calculation of the normalized decision matrix the best alternative or you can say the ideal solution which we call as V plus this is the ideal solution this is the worst solution. V plus and the worst alternatives negative solutions you compute by this and this equation, V plus equals to {v1 plus, v2 plus, up to vm plus} equals to {max i vij for j epsilon J1, min i vij for j epsilon J2}

where V plus means the positive best solution is a summation of v1 to vm.

And then you have for V minus. Similar way, you have

V minus equals to {v1 minus, v2 minus, up to vm minus} equals to {min i vij for j epsilon J1, max i vij for j epsilon J2}

All the negative value this side and this is the positive side. So, here on the basis of normalized decision matrix, you calculate the V plus and the V minus.

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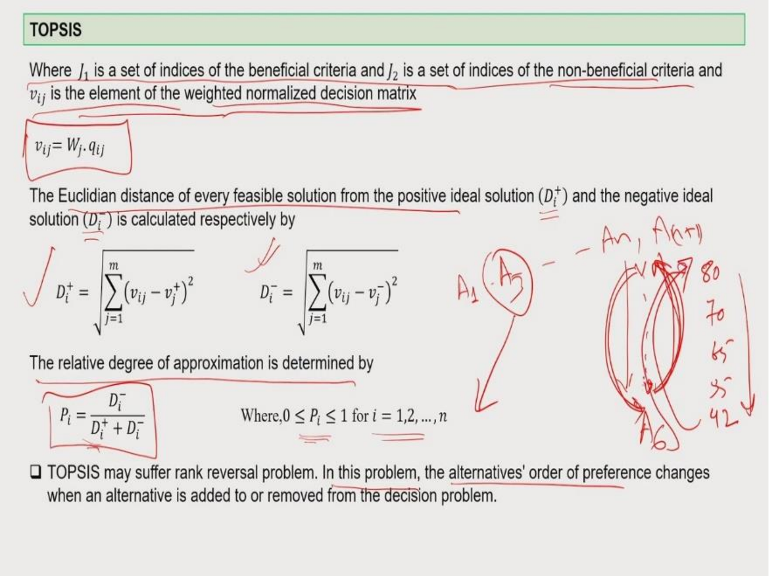

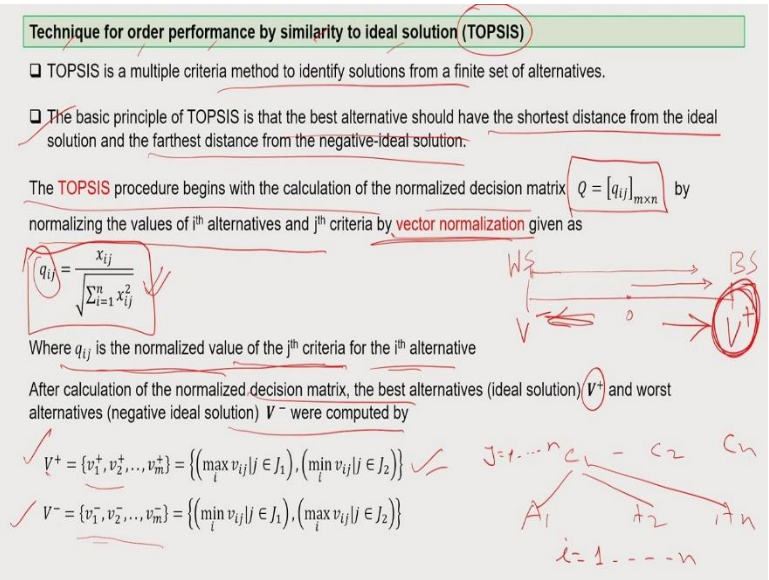

So, if your V plus value is closer to the best solutions, then it will be definitely much away from the worst solution. Both way, you can explain this. Where J1 is a set of indices of the beneficial criteria. Again you recall we discussed in previous lecture beneficial criteria and not beneficial criteria. J2 is a set of indices, which are non beneficial criteria and vij is the element of the weighted normalized decision matrix and you calculate vij in this way.

Now, the Euclidian distance of every feasible solution from the positive ideal solution means, this is your positive ideal solution, which is you can express as Di plus and the negative ideal solution Di minus. These two again is calculated by this formula and this formula. I am afraid you have to actually recall these formulas, when you actually calculate these in TOPSIS these kinds of matrices and then you try to find out the different solution options.

The relative degree of approximation in TOPSIS method is determined by this formula

Pi is equal to Di minus divided by Di plus plus Di minus

Pi is equal to Di minus means the negative or the negative ideal solution Di plus is the positive or good solutions. Where Pi values range between 1 and 0 and i is equal to 1 to n.

Now, TOPSIS sometime may suffer some rank reversal problem. You get to suppose a rank higher value to suppose lower value, but TOPSIS in case of TOPSIS sometime we suffer rank reversal. So, that means lower one comes up, upper one goes down. What happens is that in this problem the alternatives orders of preferences changes as I said these will go up and these may come down when an alternative is added to or removed from the decision problem, remember this.

In TOPSIS method your ranking may totally go up or down. This bottom one can go up and upper one can come bottommost. When it could happen? When you add another alternative or you remove it from the sets of alternatives during the decision-making process. Suppose you have in hand alternative A1 to An suddenly you decide that I will remove this A3 alternative or you decide that you want to add another alternative An plus 1. So, this kind of situation may happen that you are in order totally could change.

Suppose you on the basis of values suppose this is your 80 then 70, 65, 55 and suppose say 42. You range in like this way. Entire this value might change and you can see that, that your this alternative suppose A6 will go up there. The ranking could be totally reversal. So, that definitely would create a problem. Let us try to see with some simple example.

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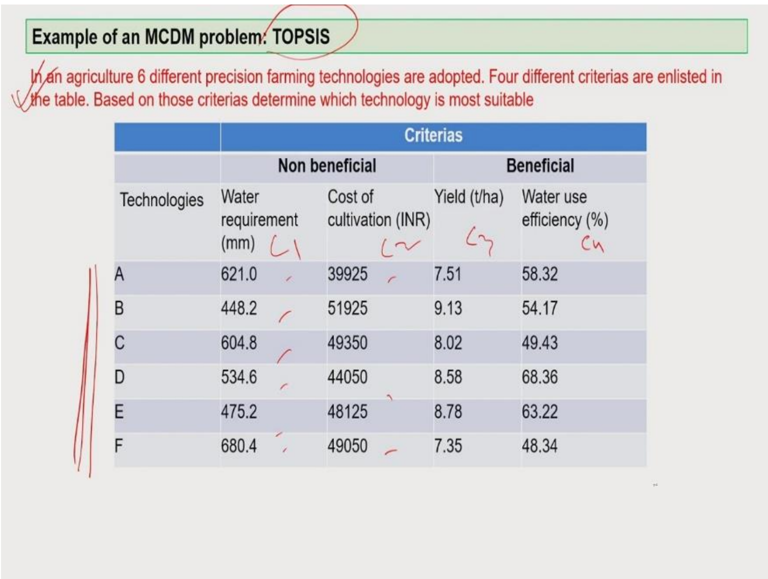

This might help us to understand this particular method TOPSIS. Again I will take the same kind of example. Suppose in an agriculture practice there are 6 different precision farming technologies are adopted, 4 different criteria C1 to C4 are enlisted in the below table as you see here C1, C2, C3 and C4. Based on those criterias, we have to now determine which technology is most suitable; this is your technology option, 6 technology options. Now in the criteria we have these values weightage assigned.

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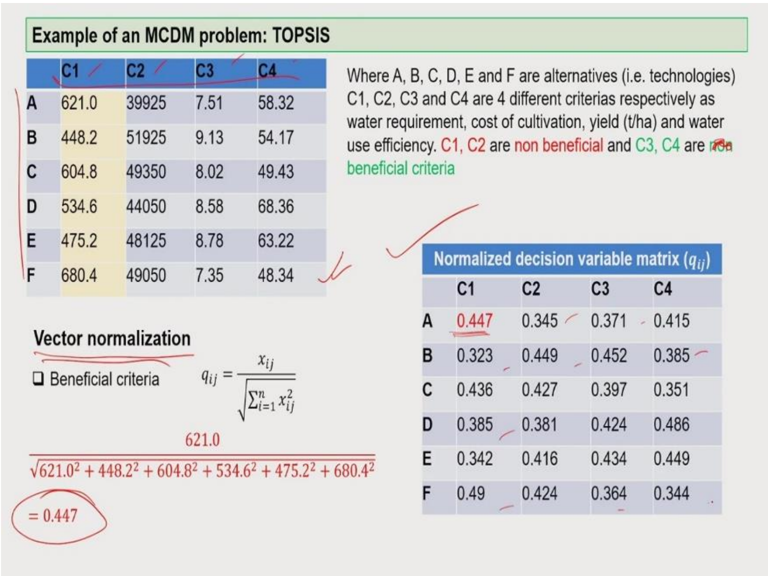

Now, let us see that how we actually complete the TOPSIS method. Here in this table as you see on the left side these are the technology options and then on the top you have the criterias. Now C1 and C2 are suppose non-beneficial criteria and C3 and C4 beneficial criteria. C1, C2 non-beneficial, C3, C4 suppose beneficial criteria. Then what happens? You go for normalized decision variable matrix same with that we discussed in the previous lecture, but here for TOPSIS we try vector normalizations.

So, after your vector normalization process, you get a value for suppose criteria C1 for alternative A, you get a value 0.447. You write it here. And then you calculate all these normalized values, put in this table, then let us go to the next.

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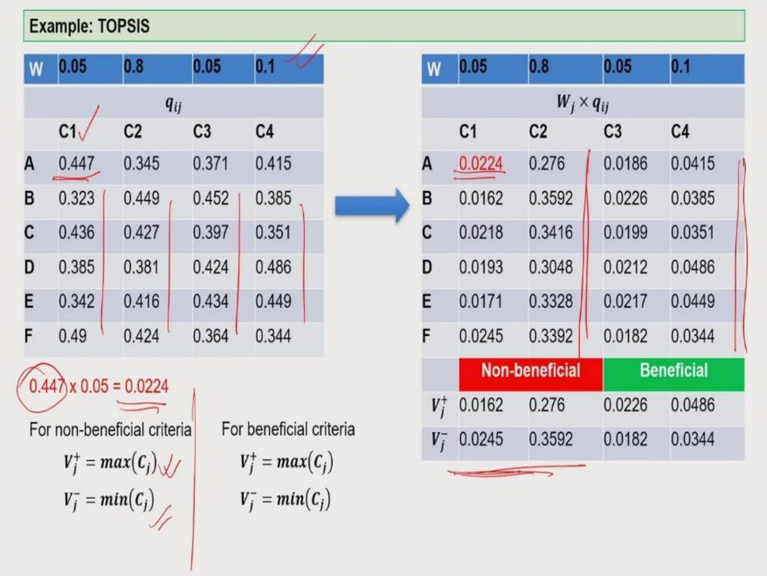

Now, here, we put you remember that criterias or values or weightage we put different weightage and we tried in previous example, if you recall, we have given same value once 0.25 and then different values for criteria. So, here let us see that for different values of criteria, how things work. Now, we have this normalized matrix value.

Now for C1, A, for A technology using criteria one what is your ultimate non beneficial criteria value. Now, you get 0.0224. This is your value, you put it here. Same way, you calculate for all the values for all criteria and alternative you get this is a non-beneficial set and here you get the beneficial set of values.

Now, we go for the for TOPSIS method, we need Vj plus and Vj minus means the best solutions and the worst solutions value to for both beneficial and non-beneficial we have to calculate. Now we have this value after we get Vj which is maximum Cj value and minimum Cj value. So, these two you have to put on the table and then you go next.

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Example: TOPSIS

W	0.05	0.8	0.05	0.1				
	$W_j \times q_{ij}$							
	C1	C2	C3	C4	D_i^+	D_i^-	P_i	Rank
A	0.0224	0.276	0.0186	0.0415	0.0102	0.0835	0.891	1
B	0.0162	0.3592	0.0226	0.0385	0.0838	0.0102	0.109	6
C	0.0218	0.3416	0.0199	0.0351	0.0673	0.0179	0.210	5
D	0.0193	0.3048	0.0212	0.0486	0.029	0.0565	0.661	2
E	0.0171	0.3328	0.0217	0.0449	0.0569	0.0296	0.342	3
F	0.0245	0.3392	0.0182	0.0344	0.0655	0.0835	0.234	4
	Non-beneficial		Beneficial					
V_j^+	0.0162	0.276	0.0226	0.0486				
V_j^-	0.0245	0.3592	0.0182	0.0344				

$$P_{i=1} = \frac{0.0835}{0.0102 + 0.0835} = 0.891$$

$$D_{i=1}^+ = \sqrt{(0.0224 - 0.0162)^2 + (0.276 - 0.276)^2 + (0.0186 - 0.0226)^2 + (0.0415 - 0.0486)^2} = 0.0102$$

$$D_{i=1}^- = \sqrt{(0.0224 - 0.0245)^2 + (0.276 - 0.3592)^2 + (0.0186 - 0.0182)^2 + (0.0415 - 0.0344)^2} = 0.0835$$

TOPSIS

Where J_1 is a set of indices of the beneficial criteria and J_2 is a set of indices of the non-beneficial criteria and v_{ij} is the element of the weighted normalized decision matrix

$$v_{ij} = W_j . q_{ij}$$

The Euclidian distance of every feasible solution from the positive ideal solution (D_i^+) and the negative ideal solution (D_i^-) is calculated respectively by

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^+)^2} \qquad D_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2}$$

The relative degree of approximation is determined by

$$P_i = \frac{D_i^-}{D_i^+ + D_i^-} \qquad \text{Where, } 0 \le P_i \le 1 \text{ for } i = 1, 2, \dots, n$$

- TOPSIS may suffer rank reversal problem. In this problem, the alternatives' order of preference changes when an alternative is added to or removed from the decision problem.

Example: TOPSIS

W	0.05	0.8	0.05	0.1
	q_{ij}			
	C1	C2	C3	C4
A	0.447	0.345	0.371	0.415
B	0.323	0.449	0.452	0.385
C	0.436	0.427	0.397	0.351
D	0.385	0.381	0.424	0.486
E	0.342	0.416	0.434	0.449
F	0.49	0.424	0.364	0.344

0.447 x 0.05 = 0.0224

For non-beneficial criteria

$V_j^+ = max(C_j)$

$V_j^- = min(C_j)$

For beneficial criteria

$V_j^+ = max(C_j)$

$V_j^- = min(C_j)$

W	0.05	0.8	0.05	0.1
	$W_j \times q_{ij}$			
	C1	C2	C3	C4
A	0.0224	0.276	0.0186	0.0415
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E	0.0171	0.3328	0.0217	0.0449
F	0.0245	0.3392	0.0182	0.0344
	Non-beneficial		Beneficial	
V_j^+	0.0162	0.276	0.0226	0.0486
V_j^-	0.0245	0.3592	0.0182	0.0344

Here what happens is that we now multiply the weightage for jth criteria for ith alternative. So, now we multiplied Wj with qij. Here you have these sets of values for non-beneficial and beneficial. Now, we will go for D, the ideal solution, the best positive ideal solution, negative ideal solution. Now positive ideal solution when i is 1, i alternative is 1 then you get this is the value. So, you calculate you remember we discussed about how to calculate the Di values same this is the one for Di plus and for negative ideal solution, this is the one.

So, now, let us come back again to this table. So, here you calculate and these values you are putting here under different criteria. Then you go for Di values, you get Di values for positive as well as negative. Now, finally, these values will be required to calculate your Pi value and Pi value finally, will give you the ranking. So, Pi value goes here, then on the basis of that finally you get a ranking.

Now, as I said that, suppose you have the alternatives here, 6 alternatives if you take away one alternatives or if you add one G then your entire this ranking will be exactly reversal. So, 4 will be here, 3, 2, 5, 6, 1. So, the reversal of the ranking will takes place. This is one issue with TOPSIS.

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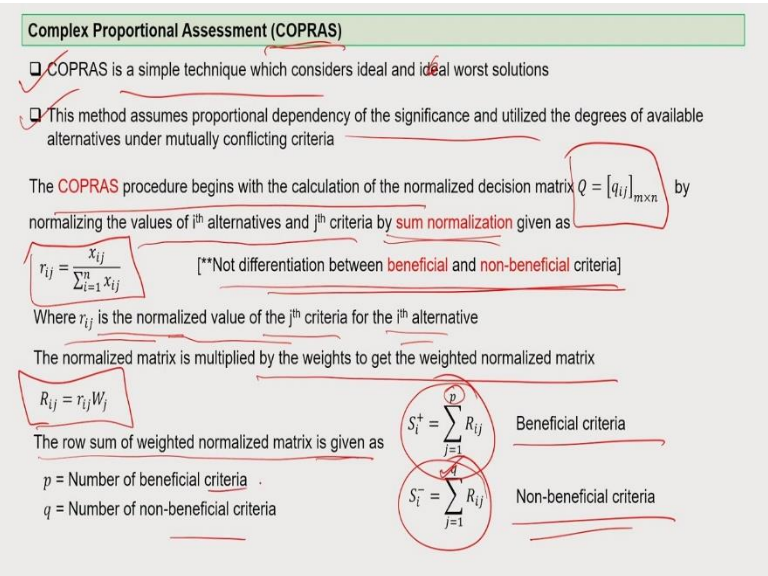

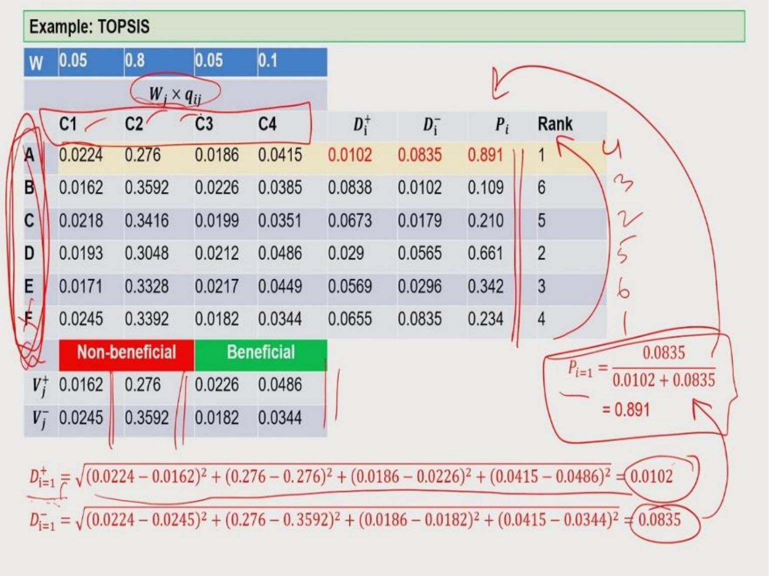

Example: TOPSIS

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	Non-beneficial		Beneficial					
V_j^+	0.0162	0.276	0.0226	0.0486				
V_j^-	0.0245	0.3592	0.0182	0.0344				

$$P_{i=1} = \frac{0.0835}{0.0102 + 0.0835} = 0.891$$

$$D_{i=1}^+ = \sqrt{(0.0224-0.0162)^2 + (0.276-0.276)^2 + (0.0186-0.0226)^2 + (0.0415-0.0486)^2} = 0.0102$$

$$D_{i=1}^- = \sqrt{(0.0224-0.0245)^2 + (0.276-0.3592)^2 + (0.0186-0.0182)^2 + (0.0415-0.0344)^2} = 0.0835$$

Now, another MCDA method which can be used at times is COPRAS. COPRAS stands for complex proportional assessment. COPRAS is a single technique which considers ideal and ideal worst solution that you can imagine. So, this method assumes proportional dependency of the significance and utilize the degrees of available alternatives. And those alternatives could be under mutually conflicting criteria means here suppose you will have a set of alternatives and these are the criteria. In case of COPRAS you could have that these criteria C1, C2 they could be mutually contrasting or mutually contradicting conflicting criteria.

So, this is COPRAS method is for more complicated decision-making process. So, the COPRAS procedure begins with the calculation of the normalized decision matrix same process like just previous one we discussed. You normalize the values again ith and jth alternatives and jth criteria by some normalization. So, you will do again rij which we carried out in the part one of this series of lectures. So, you will go for some normalization and here we are not going to have beneficial and non-beneficial criteria or differentiation in this process.

So, here again rij as you know is the normalized value for jth criteria and ith alternative. The normalized matrix is multiplied by the weights to get the weighted normalized matrix. This we have discussed earlier. Now, the row sum of weighted normalized matrix is given as Si plus and Si minus for beneficial and non-beneficial criteria. So, here p is the number of beneficial criteria and q is the number of non-beneficial criteria.

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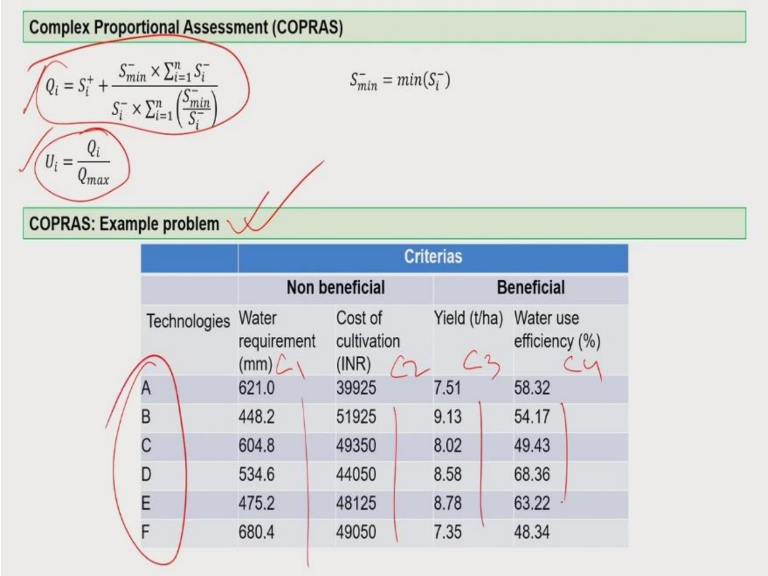

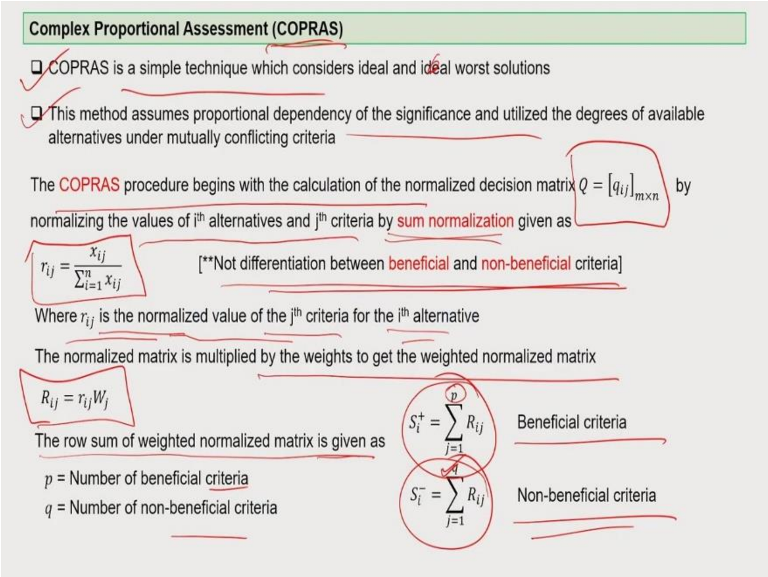

Now, again we will go for example, to understand it. So, in case of COPRAS, you can calculate the Qi and Ui through this equation.

Qi is equal to Si plus plus Smin minus multiplied by summation I equal to 1 to n Si minus divided by Si minus multiplied by summation 1 equal to 1 to n Smin minus by Si minus

Ui is equal to Qi by Qmax

First you go and calculate Qi and then you calculate Ui. This Qi and Ui will be required when we go for ranking through COPRAS method, let us see that how actually it can be carried out. So, in case of example, you have again C1 and C2 as non beneficial, C3 and C4 as beneficial criteria. You have 6 alternatives same example. Then you have set of values here.

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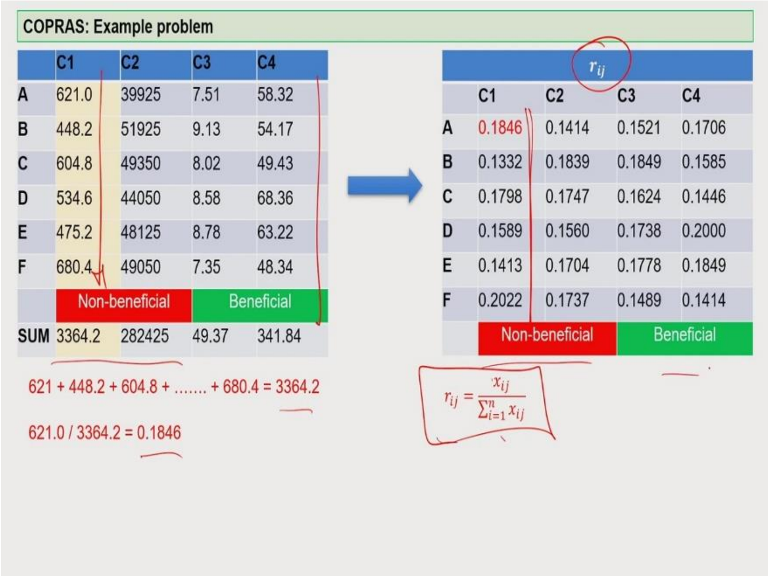

Then you try to calculate one by one step wise some normalizations which is much easy. You just add up these things non beneficial and you have beneficial. So, these are the values. Then you go for rij calculation. rij this is the formula xij by xij is equal to 1 to n. So, here you calculate for each criteria.

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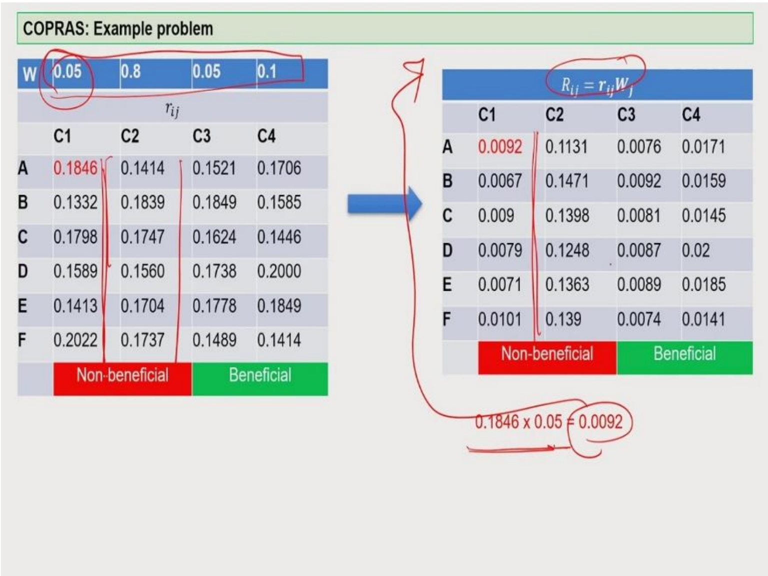

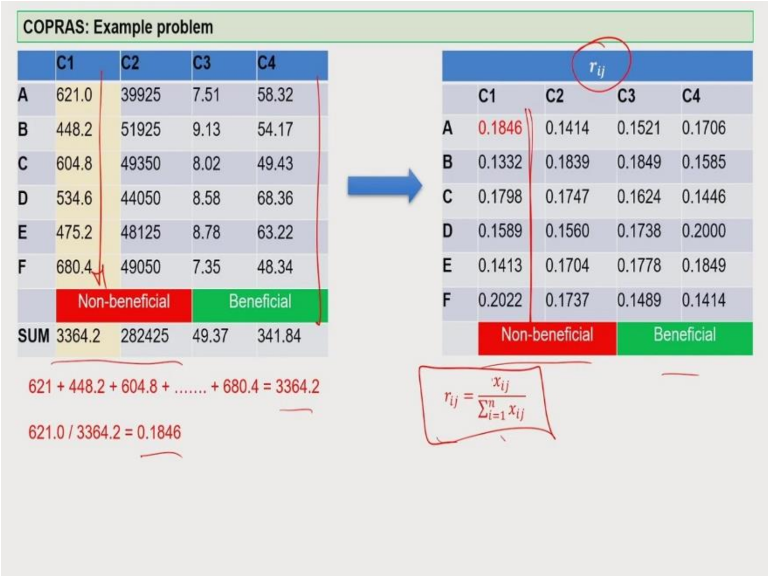

COPRAS: Example problem

	C1	C2	C3	C4
A	621.0	39925	7.51	58.32
B	448.2	51925	9.13	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	68.36
E	475.2	48125	8.78	63.22
F	680.4	49050	7.35	48.34
	Non-beneficial		Beneficial	
SUM	3364.2	282425	49.37	341.84

621 + 448.2 + 604.8 + ... + 680.4 = 3364.2

621.0 / 3364.2 = 0.1846

r_{ij}

	C1	C2	C3	C4
A	0.1846	0.1414	0.1521	0.1706
B	0.1332	0.1839	0.1849	0.1585
C	0.1798	0.1747	0.1624	0.1446
D	0.1589	0.1560	0.1738	0.2000
E	0.1413	0.1704	0.1778	0.1849
F	0.2022	0.1737	0.1489	0.1414
	Non-beneficial		Beneficial	

$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}}$

And then you go for next. You give the different values weightage for criteria. And then you calculate the criteria based alternative weightage. You put it here, then you go for once you get the rij so, then you go for your ranking. So, this is the value multiply with the criteria of value. You get the rij here. So, the rij value, you put it here in the table.

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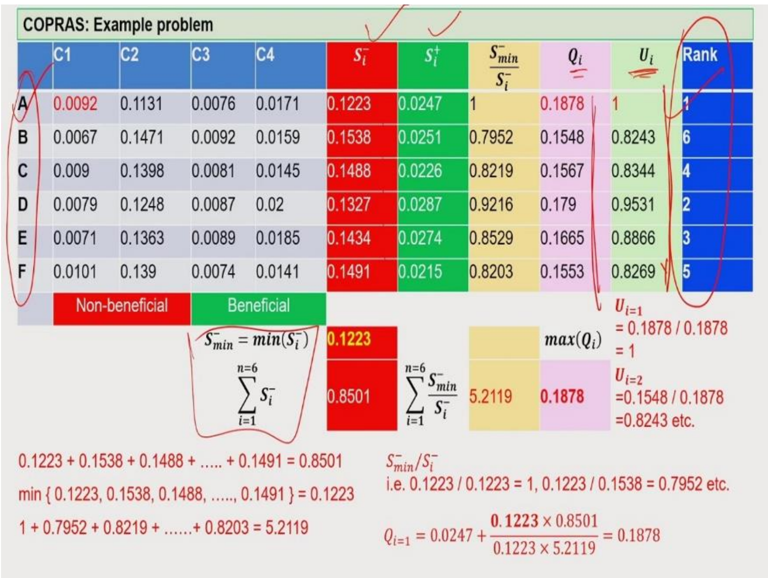


COPRAS: Example problem

	C1	C2	C3	C4	S_i^-	S_i^+	$\frac{S_{min}^-}{S_i^-}$	Q_i	U_i	Rank
A	0.0092	0.1131	0.0076	0.0171	0.1223	0.0247	1	0.1878	1	1
B	0.0067	0.1471	0.0092	0.0159	0.1538	0.0251	0.7952	0.1548	0.8243	6
C	0.009	0.1398	0.0081	0.0145	0.1488	0.0226	0.8219	0.1567	0.8344	4
D	0.0079	0.1248	0.0087	0.02	0.1327	0.0287	0.9216	0.179	0.9531	2
E	0.0071	0.1363	0.0089	0.0185	0.1434	0.0274	0.8529	0.1665	0.8866	3
F	0.0101	0.139	0.0074	0.0141	0.1491	0.0215	0.8203	0.1553	0.8269	5
	Non-beneficial		Beneficial							
				$S_{min}^- = min(S_i^-)$	0.1223			$max(Q_i)$		
				$\sum_{i=1}^{n=6} S_i^-$	0.8501	$\sum_{i=1}^{n=6} \frac{S_{min}^-}{S_i^-}$	5.2119	0.1878		

$U_{i=1}$ = 0.1878 / 0.1878 = 1

$U_{i=2}$ =0.1548 / 0.1878 =0.8243 etc.

0.1223 + 0.1538 + 0.1488 + ... + 0.1491 = 0.8501

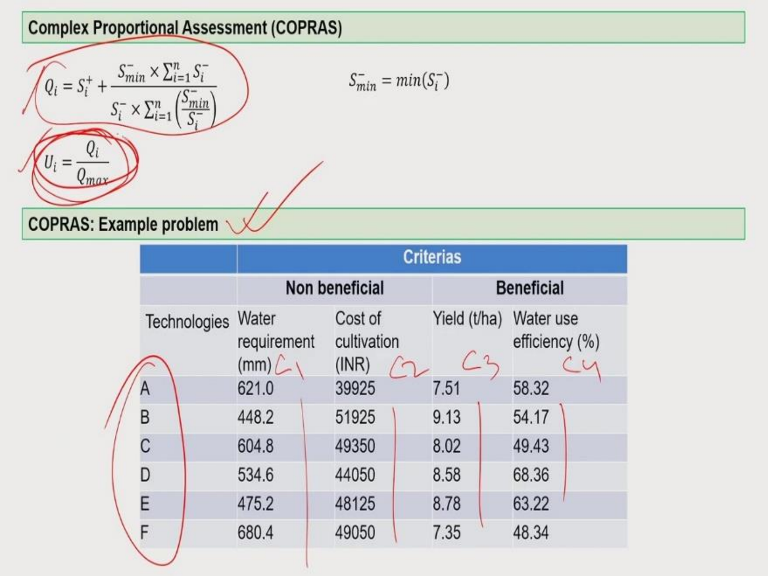
min { 0.1223, 0.1538, 0.1488, ..., 0.1491 } = 0.1223

1 + 0.7952 + 0.8219 + ...+ 0.8203 = 5.2119

S_{min}^-/S_i^-

i.e. 0.1223 / 0.1223 = 1, 0.1223 / 0.1538 = 0.7952 etc.

$Q_{i=1} = 0.0247 + \frac{0.1223 \times 0.8501}{0.1223 \times 5.2119} = 0.1878$



Once you get that, then your ranking process comes easier. So, here then you will go for Si plus and Si minus which we already discussed in the earlier example. And through this calculation, you will get certain value and then you go for Qi calculations and Ui calculation. So, once your Qi is there basically you already reach towards the ranking. Then from Qi you calculate the Ui, as you saw here, that this is the last step for COPRAS method. So, once Ui is with you, you get the ranking clearly here.

So, this is how step by step you calculate different index and then you finally reach towards the ranking process. Ultimate goal of everything is to come with the ranking, ranking of your alternative technologies. Means which one you are going to use here, you will going to use A, technology A that is the best solution, you have in hand among the 6 alternative technologies.

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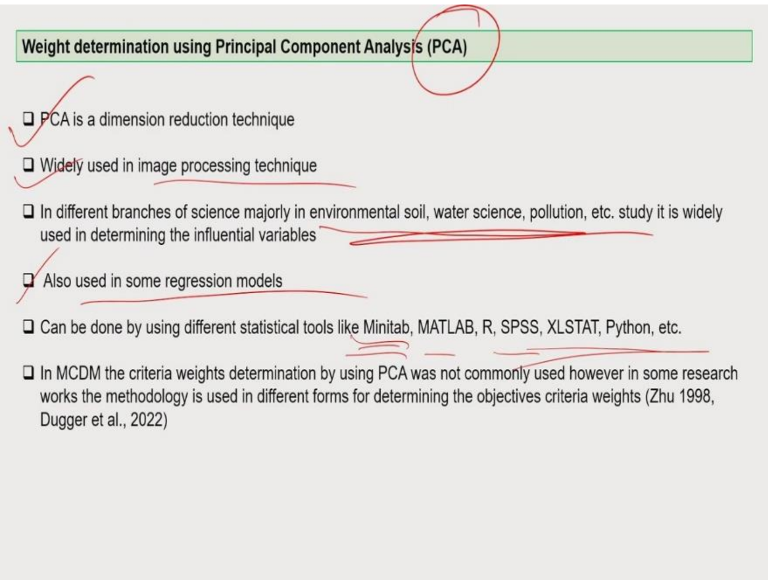

Next, weight determination using principal component analysis. Most of you have carried out during your research work PCA, famously known as PCA, Principal Component Analysis. PCA is a basically a dimension reduction technique and it is widely used also for image processing technique in different branches of science, engineering mainly in environmental soil and all natural resources related study heavily used this PCA to identify the sources. Source identification is another aspect that PCA helps us. Also it is used for regression model and it can be actually done using different statistical tool or programs Minitab, MATLAB, R, SPSS, many way.

In MCDA the criteria weights determination using PCA was not very common. However, in some cases, some people have used it in different forms to identify the weightage criteria as you by now might have understood that getting the weightage for your alternative and criteria is one of the key exercises within multiple criteria decision analysis.

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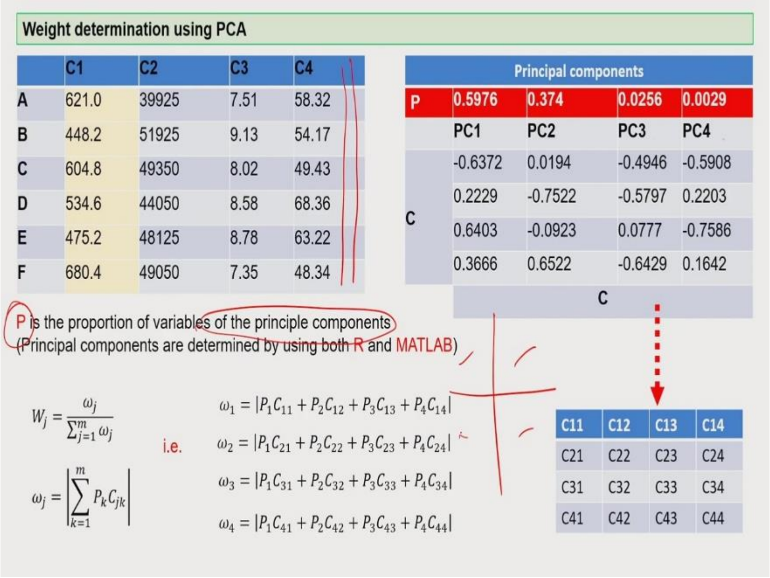

So, that is what actually it helps same example, if you see that how in case of PCA that you may try to do it. So, principal components here is the proportion of variables of the different principal component is P and it can be determined by various software as I mentioned. So, in case of PCA, you try to have PC1, PC2, PC3 and PC4 four area where actually you go with factor loading processes. So, there we actually consider certain Eigen values, on the basis of that, we consider whether our PCA exercise or PCA result output is acceptable or not.

So, again as I said that PCA is largely used for source identification and also for different other analysis relationships between different factors kind of regressions, but not much for multiple criteria decision analysis.

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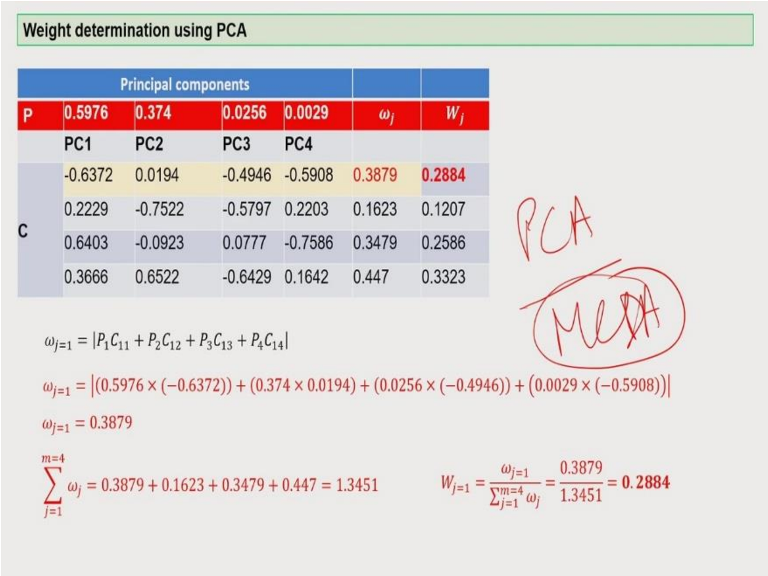

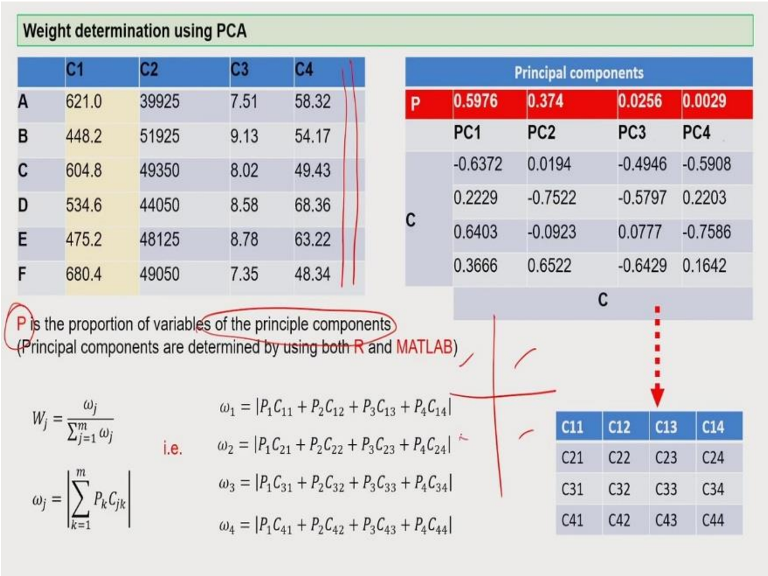

So, again, for PCA also, you have to identify or determine the weightage of your different P's and C's, like P's are the proportion of the variables and then you have the C's, which are actually your components. Then you have different way of analyzing, I am not going to talk in detail about PCA, as I said that this is very common tool many of you have been using it and also PCA is not a very preferred tool for MCDA, multiple criteria decision analysis, though there are few reports that people are using it.

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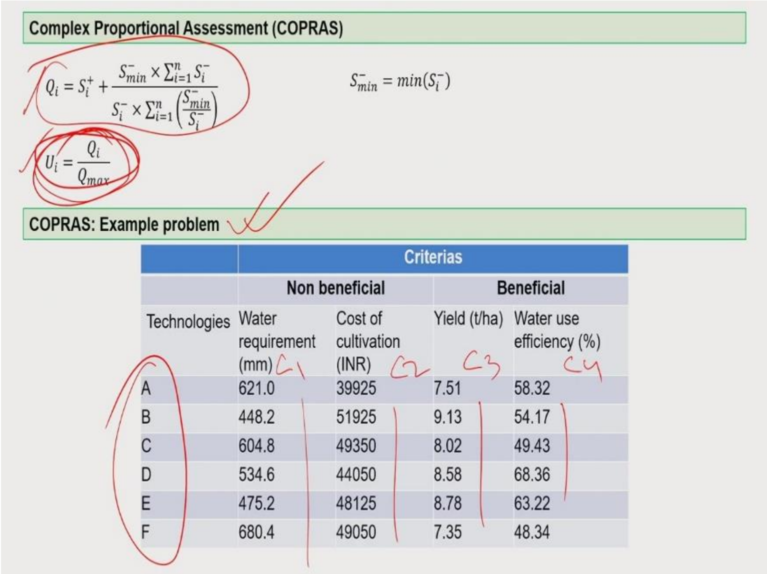

So, participants today, we actually end with this lecture, the entire MCDA technique and how in different process that we calculate different weightage and then how we normalize it for different methodology, different process. Some are a little complicated, some are very simple. As I said that analytical hierarchy process is one which many of you would be required to use when you work in the field of natural Resource Management.