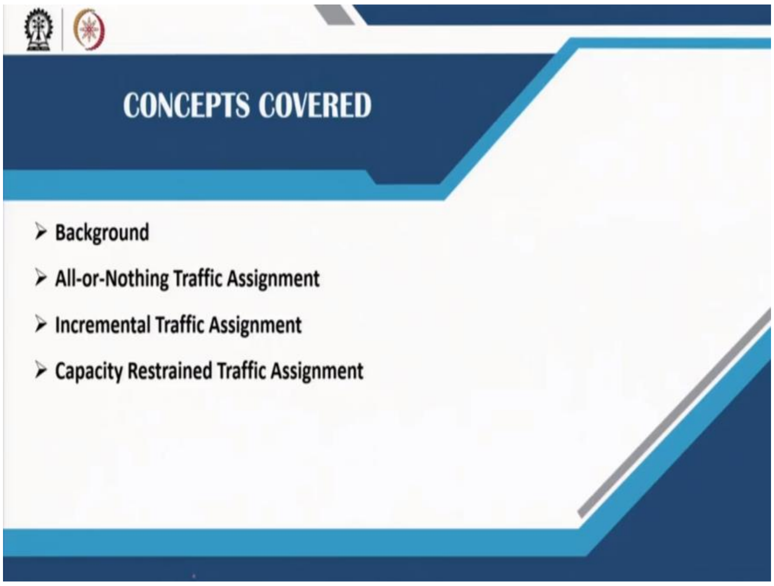
Urban Landuse and Transportation Planning

Prof. Debapratim Pandit
Department of architecture and regional planning
Indian Institute of Technology, Kharagpur

Lecture - 43
Link assignment 1

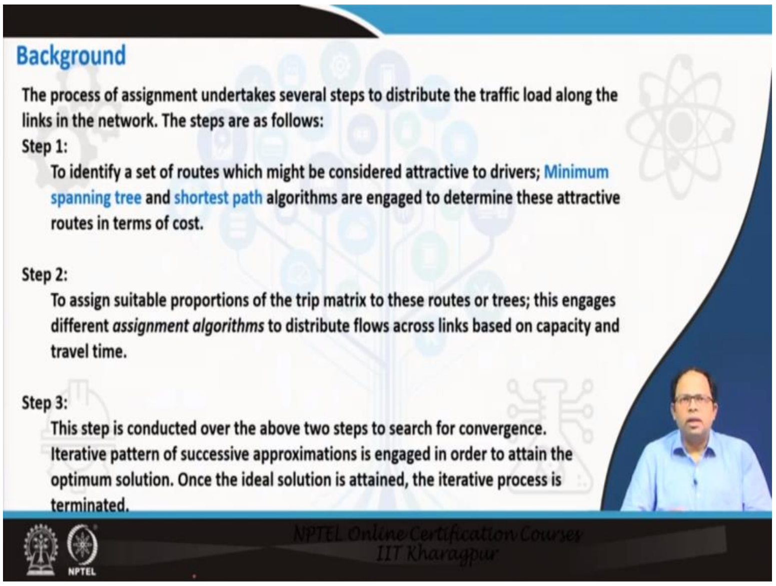
Welcome back in lecture 43. We will look into link assignment and link assignment module. The lecture is divided into two parts. Part 1 will be covered in this lecture.

(Refer Slide Time: 00:34)



First, we will discuss about the background of link assignment. Finally, different assignment techniques like all or nothing traffic assignment, incremental traffic assignment and capacity restrained traffic assignment will be discussed. Therefore, three traffic assignment techniques will be covered in this lecture. Other techniques will be covered in the next lecture.

(Refer Slide Time: 00:54)



Background

The process of assignment is conducted through several steps to distribute traffic load along the links in a network. The steps are discussed below:

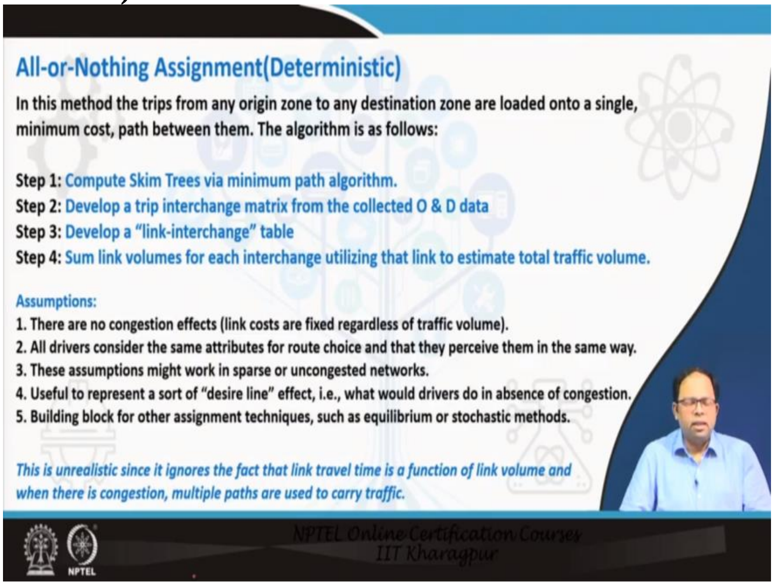
Step 1: The first step is to identify a set of routes which might be considered attractive to drivers. Minimum spanning tree and shortest path algorithms are engaged to determine these attractive routes in terms of cost. The techniques associated with route choice determination were discussed in the previous lecture.

Step 2: The suitable proportions of the trip matrix must be assigned to these routes or trees. This engages different assignment algorithms to distribute flows across links based on capacity and travel time. For example, if there is only one chosen route, then total amount of trips is assigned to that route. On the other hand, if there are multiple alternative routes, then there would be a certain proportion of the trips assigned to each of these routes. The objective is to understand the procedure behind the assignment of different flows to different links using the assignment algorithm.

Step 3: The final step is conducted to search for convergence. Iterative pattern of successive approximations is engaged in order to attain the optimum solution. Once the ideal solution is attained, the iterative process is terminated. After each iteration, there is an adjustment to the travel time along the link as well as the capacity. After each iteration, the algorithm tries to determine the optimum solution.

Therefore, the attractive routes are determined first using suitable algorithms. Then the flows are assigned to the links based on interzonal trip volumes using an appropriate assignment algorithm. The assignment process is continued until an optimum solution is found. The traffic assignment methods allocate the OD trips in a systematic manner to represent the features of the real network.

(Refer Slide Time: 03:30)



<u>All-or-Nothing Assignment</u>

The first assignment technique is deterministic and also known as all-or-nothing assignment. This is the simplest assignment technique. In this method, the trips from any origin zone to any destination zone are loaded onto a single minimum cost path between them. There are some assumptions related to this method. This method assumes that there is no congestion effect, i.e. the link costs are fixed regardless of traffic volume. According to this method, although the entire volume is assigned to one link, the travel time along that link will remain unchanged. Secondly, all drivers consider the same attributes for route choice and they perceive the attributes in the same way. This implies the perception does not vary across drivers. The shortest route remains the same irrespective of the drivers. Although these assumptions might be suitable for sparse or uncongested networks, i.e. in less demand areas, this algorithm cannot be utilized for a high capacity network like in case of central business districts. This assignment is helpful to represent a 'desire line' effect. This effect represents the drivers' 'desire' in absence of congestion, or in other words drivers' activities when there is no congestion. This technique also provides the building blocks for other assignment techniques such as equilibrium or stochastic methods which are the modification of this technique. The different steps of the algorithm are as follows:

Step 1: Compute Skim Trees via minimum path algorithm. The minimum paths must be determined for the connection between every origin zone *i* to every destination zone *j*.

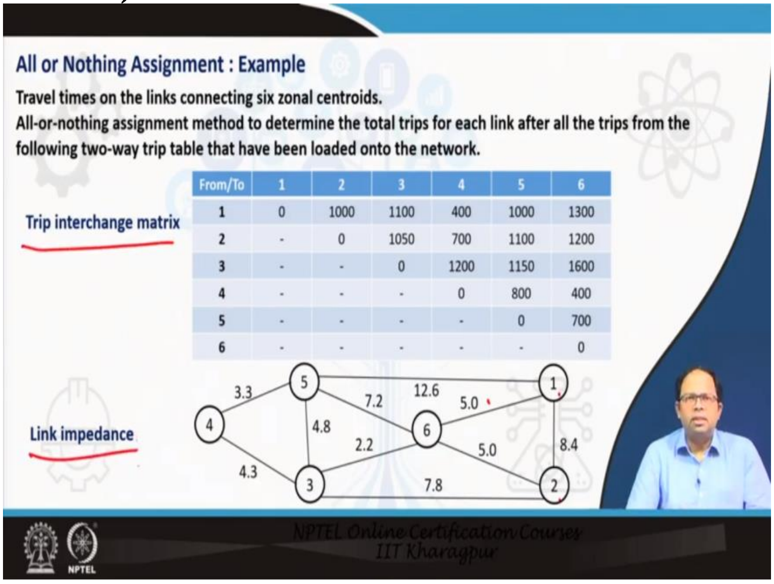
Step 2: Develop a trip interchange matrix from the collected O & D data. The data can also be taken as an input from the trip distribution model.

Step 3: Develop a “link-interchange” table

Step 4: Sum link volumes for each interchange utilizing that link to estimate total traffic volume.

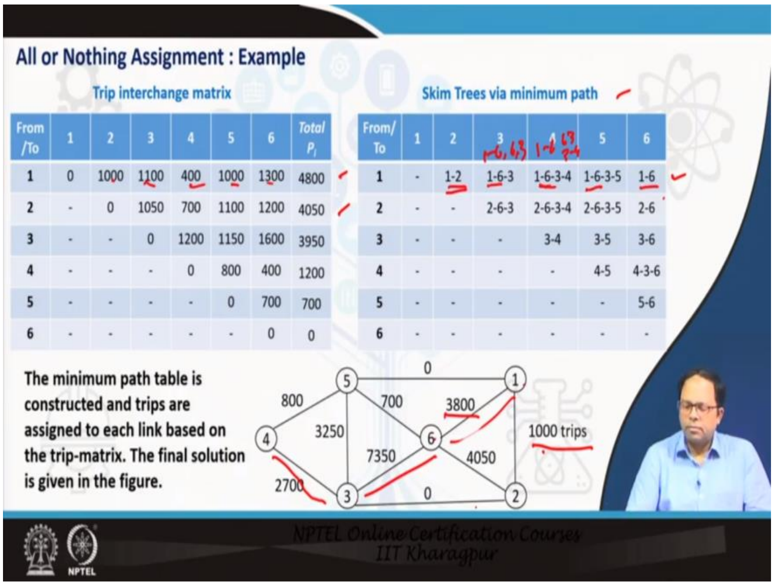
This technique is unrealistic since it ignores the fact that link travel time is a function of link volume and when there is congestion, multiple paths are used to carry traffic. The travel time increases with the increase in flow. If the travel time in one link rises, then another link automatically becomes more attractive. But in this technique, the travel time remains unchanged with the increase in flow which does not represent the characteristics of a real network.

(Refer Slide Time: 06:49)



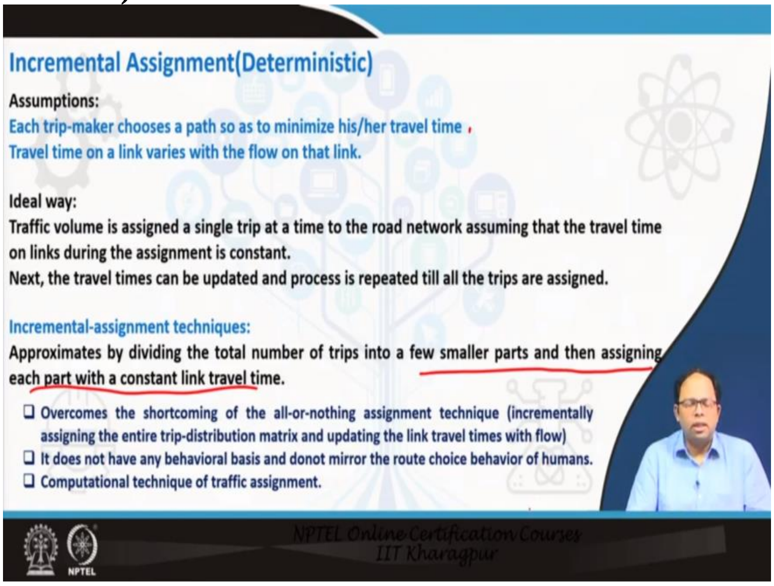
Let us explain the all-or-nothing assignment through an example. Consider the above network. In this example, there are six zonal centroids 1, 2, 3, 4, 5 and 6. The trip-interchange matrix has been provided. The link impedances or the travel times on the links are also given above each of the links in the given network. All the trips from the given two-way trip table are loaded onto the network using All-or-nothing assignment. Using this method, we will determine the total trips for each link of the given network after assignment.

(Refer Slide Time: 07:39)



In All-or-nothing assignment, the minimum path table is constructed, and trips are assigned to each link based on the trip matrix. The total trips produced at each of the nodes is calculated. The total trips produced at zone 1,2,3,4,5 and 6 are 4800, 4050, 3950, 1200, 700 and 0 respectively. The skim matrix is also prepared based on the impedance data given above. This matrix will give the shortest paths between any zone. These paths comprise of links. The flow must be assigned into each of these links. Let us consider the zone 1 first. There are three links connecting zone 1 i.e., 1-2, 1-6 and 1-5. For this network, 1-2 is same as 2-1. This assumption holds true for any link in the given network. Now from the skim matrix we need to identify the paths which include the above three links. None of the shortest path listed in the skim matrix include 1-5. So, link volume will not be assigned to this link. The link 1-2 is identified only between zone 1 and zone 2. The trip volume between zone and 2 will be assigned to this link i.e. 1000. The final link i.e. 1-6 is a part of four shortest paths. These shortest paths exist between zone 1 and zone 3,4,5 and 6. The trip volumes recorded between zone 1 and zone 3,4,5 and 6 must be assigned to link 1-6. The total flow in 1-6 will be 1100+400+1000+1300 i.e. 3800 trips. Similarly, the link 6-3 is included in eight shortest paths as identified from the skim matrix. The total flow in 6-3 will be the summation of the trip volumes observed in these eight cases. The final flow will be 1100+400+1000+1050+700+1100+1600+400 i.e. 7350 trips. Similarly, the traffic volume in each of these links in the given network is estimated. The final traffic flow in each of the links is shown in the figure above. Therefore, the different traffic volumes will be loaded into the links that are included in the shortest paths and eventually we will get the total amount of volume in each of these links. Once the final volume in all the links is obtained, we can also determine whether these volumes exceed the link capacity using v/c ratios. Based on the observations, some measures can be proposed to reduce the congestion in those links like expansion of right of way of certain links.

(Refer Slide Time: 11:32)

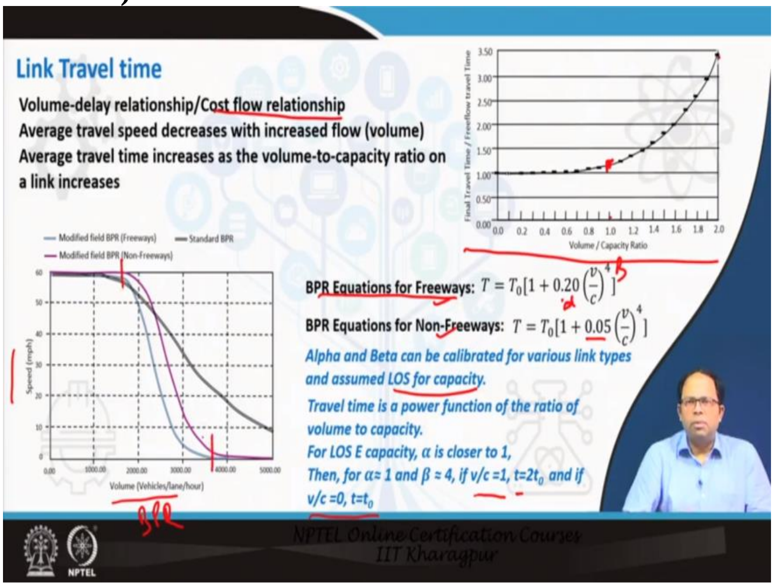


<u>Incremental Assignment</u>

Incremental assignment is another deterministic technique. This technique assumes that each trip maker chooses a path to minimize his or her travel time. This assumption is in line with the Wardrop's first principle as discussed earlier. The second assumption of this is that the travel time on a link varies with the flow on that link. The ideal way to solve this technique is to assign the traffic volume one trip at a time on the road network assuming that the travel time on links during that assignment process remains constant. In the next step, the travel time can be updated considering the current traffic volume in that link and then the next trip is assigned. In the previous assignment technique, the travel time on a link remain unchanged irrespective of increase in traffic flow.

If this assignment is conducted in the ideal way, then one trip is added to a link at a time and the travel time is updated in that link. As the flow in that link increases, the travel time will also increase. This may change the shortest path. If the shortest path changes, then the entire trip volume will be loaded onto another route or link. Ideally, this can help us to determine the assignment of traffic volume across different paths. But assigning one trip at a time will increase the time to execute the process. Instead, we utilize the incremental assignment technique to divide the total amount of trips into smaller parts. The proportion of the trips so generated after the division is assigned with a constant link travel time. The link travel time is updated, and the process is repeated till all the trips are assigned. This assignment overcomes the shortcomings of all-or-nothing assignment technique by incrementally assigning the entire trip-distribution matrix while updating the link travel times with flow. Therefore, the link travel times can be updated after certain time intervals. However, this does not have any behavioral basis and do not mirror the route choice behavior of humans. Although this technique updates the travel time along a link after assigning flow, the entire traffic volume is assigned to a shortest path since there is no route choice option. So, the change in route choice options based on human behavior cannot be evaluated based on this technique. This is a computational technique of traffic assignment. Although this technique does not have behavioral basis, it is a technique which assigns the traffic to different paths instead of just a single path as observed in all-or-nothing assignment.

(Refer Slide Time: 15:17)



The cost-flow relationship which was discussed in the previous lecture can be utilized to explain the volume-delay relationship. This relationship explains the delay along a link with the change in traffic volume in that link. The delay is explained through speed and the traffic volume is same as flow. The average travel speed decreases with the increased flow or volume. As the speed decreases, the travel time along the link increases since the speed is inversely proportional to travel time. Besides, the average travel time increases with the increase in volume to capacity ratio on a link. If the flow/volume along a link is nil, the travel time along a link is equal to the free flow travel time. If the volume is equal to the link capacity i.e. the v/c ration is 1, the travel time increases linearly since the power function is nullified with a v/c ratio of 1. But, when v/c ratio increases, i.e. v/c is greater than 1, the power function must be considered. This increases the travel time along that link exponentially. The graph shown above represents the same. In the bureau public roads (BPR) function, the travel time of a link is estimated based on link volume, link capacity and the free flow time/the previous travel time of the link. The BPR equation for freeways is given by,

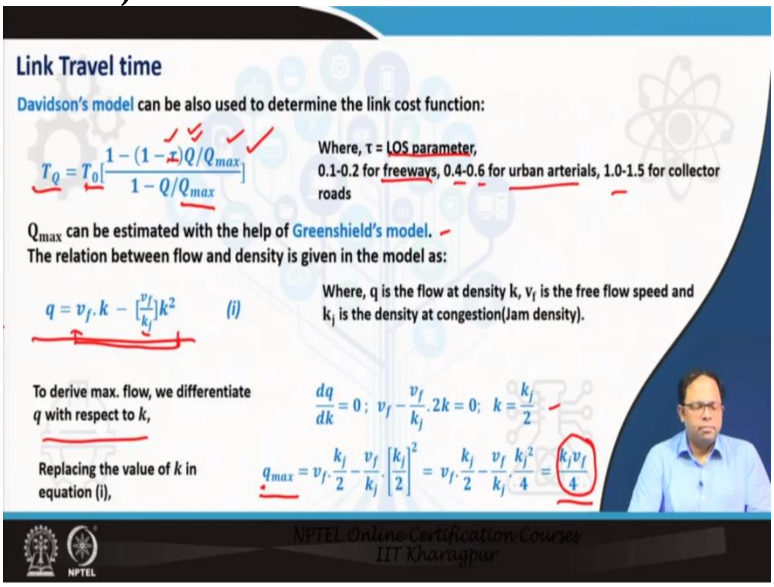
$$T = T_0[1 + 0.20\left(\frac{v}{c}\right)^4]$$

On the other hand, the BPR equation for non-freeways is given by,

$$T = T_0[1 + 0.05\left(\frac{v}{c}\right)^4]$$

In the previous lecture, alpha value was considered to be 0.15. The alpha and beta parameters of the link cost function have been modified for freeways and non-freeways. Therefore, alpha and beta values can be calibrated for various link types and an assumed Level of Service (LOS) for capacity. As mentioned earlier, travel time is a power function of the ratio of volume to capacity. For LOS E capacity where the link is designed for higher speed as well as capacity, the alpha so calibrated is closer to 1. The beta value is calibrated to be 4. If v/c = 1 with an alpha value equal to 1 and beta value equal to 4, then, then the travel time becomes 2 times the free flow travel time along a link. But if v/c = 0, then travel time is equal to the free flow travel time. The above plot represents curves for standard BPR and modified BPR curves for freeways and non-freeways. In all the curves, the speed is plotted against volume. When the volume increases, the speed will gradually come down. So, initially the speed remains same with a drastic reduction in the middle. Finally, the speed becomes more or less stable. The type of curve represented in the graph shown above is also known as S curve, which is found in many cumulative distributions. The characteristics of S-curves are that the initial and the tail parts of the curve is mostly parallel to the x-axis, while drastic change is observed in the middle.

(Refer Slide Time: 18:55)



In addition to the BPR equation, the Davidson's model can be used as well to determine the link-cost function. The davidson's model can be expressed as,

$$T_Q = T_0\left[\frac{1-(1-\tau)Q/Q_{max}}{1-Q/Q_{max}}\right]$$

where, T_Q is the final travel time, T_0 is the free flow time, Q is flow, Q_{max} is the maximum flow on the link, τ is called the LOS parameter which takes the value of 0.1-0.2 for freeways, 0.4-0.6 for urban arterials, 1-1.5 for collector roads. The values for τ are empirically estimated values based on wherever this model was calibrated. So, Q_{max} can be estimated with the help of another model also known as the Greenshield's model. The relation between flow and density is given in the model as,

$$q = v_f.k \; - \; \left[\frac{v_f}{k_j}\right]k^2$$

Here, q is the flow at density k and v_f is the free flow speed and k_j is the jam density or the congestion density. To estimate q_{max}, we need to determine the value at which q becomes maximum. In order to determine the maximum value for q, we can take a derivative of the function given above and equate it with 0. To derive the maximum flow, we differentiate q with respect to k. The following results are obtained,

$$\frac{dq}{dk} = 0\,; \;\; v_f - \frac{v_f}{k_j}.2k = 0; \;\; k = \frac{k_j}{2}$$

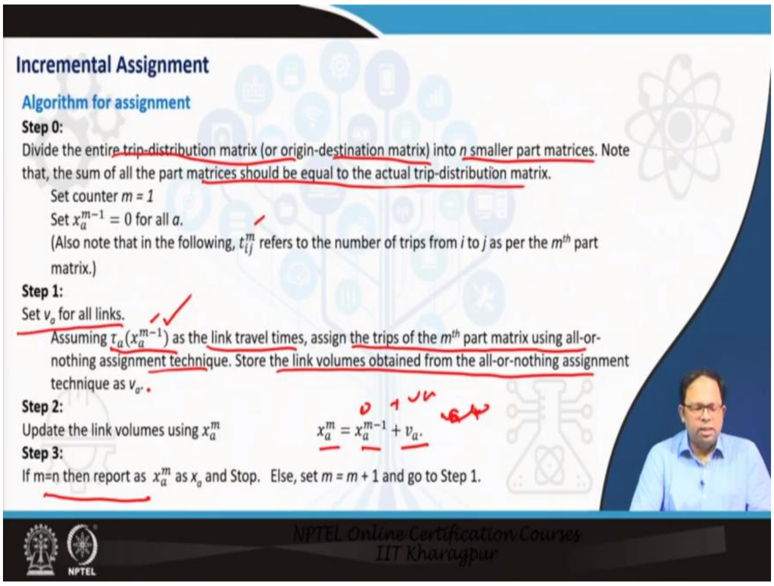
Now we replace the value of k in the equation given for Greenshield's model. The value of q_{max} is,

$$q_{max} = v_f.\frac{k_j}{2} - \frac{v_f}{k_j}.\left[\frac{k_j}{2}\right]^2 = v_f.\frac{k_j}{2} - \frac{v_f}{k_j}.\frac{{k_j}^2}{4} = \frac{k_j v_f}{4}$$

where, k_j is the jam density and v_f is the free flow speed.

Therefore, Q_{max} will provide us with the link capacity of a corridor. This will also give us the flow/capacity ratio. τ can be determined based on the type of corridor. Thus, we can determine the increment in the travel time or the impedance along that corridor. This is another alternative equation to determine the link travel time.

(Refer Slide Time: 22:14)



The incremental assignment is conducted using the following steps:

Step 0:

Divide the entire trip-distribution matrix (or origin-destination matrix) into *n* smaller part matrices. Note that, the sum of all the part matrices should be equal to the actual trip-distribution matrix.

Set counter $m = 1$

Set $x_a^{m-1} = 0$ for all a.

(Also note that in the following, t_{ij}^m refers to the number of trips from *i* to *j* as per the m^{th} part matrix.)

Step 1: Set v_a for all links.

Assuming $\tau_a(x_a^{m-1})$ as the link travel times, assign the trips of the m^{th} part matrix using all-or- nothing assignment technique. Store the link volumes obtained from the all-or-nothing assignment technique as v_a.

Step 2:

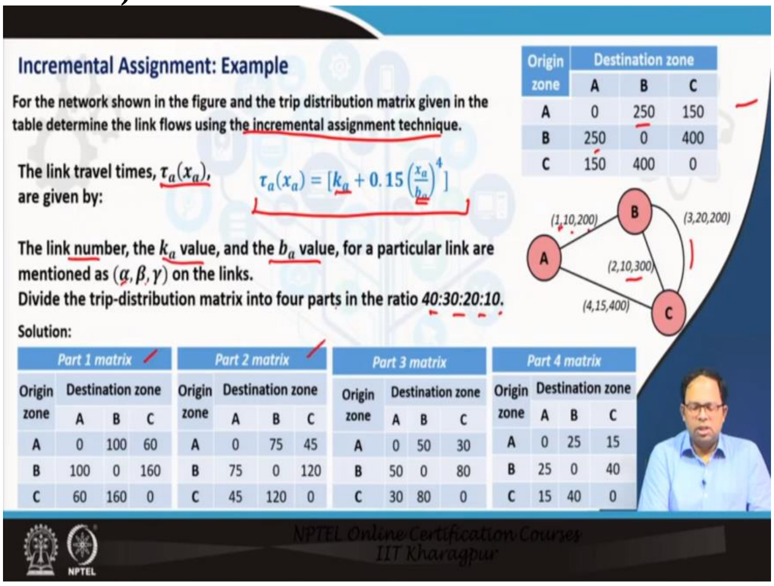
Update the link volumes using x_a^m , $where\ x_a^m = x_a^{m-1} + v_a$.

Step 3:

If m=n then report as x_a^m as x_a and Stop. Else, set $m = m + 1$ and go to Step 1.

In other words, the trip distribution matrix is divided into *n* smaller proportions. This division can be done in equal or different proportions. All the link volumes are set to zero for the first iteration. Here, v_a is nothing but the link volume for a given link obtained from all-or-nothing assignment. The trips from the m^{th} part of the trip distribution matrix are assigned to the link with shortest travel time based on all-or-nothing assignment. The link volumes are updated based on the volumes obtained in current iteration. The process is repeated till the iteration number is equal to the number of proportions of trip-distribution matrix. So, the flow will be assigned, and the travel time will be updated in each iteration based on the given link-cost function. The travel time on a given link will determine whether the flow will be assigned to that link in the next iteration.

(Refer Slide Time: 25:16)

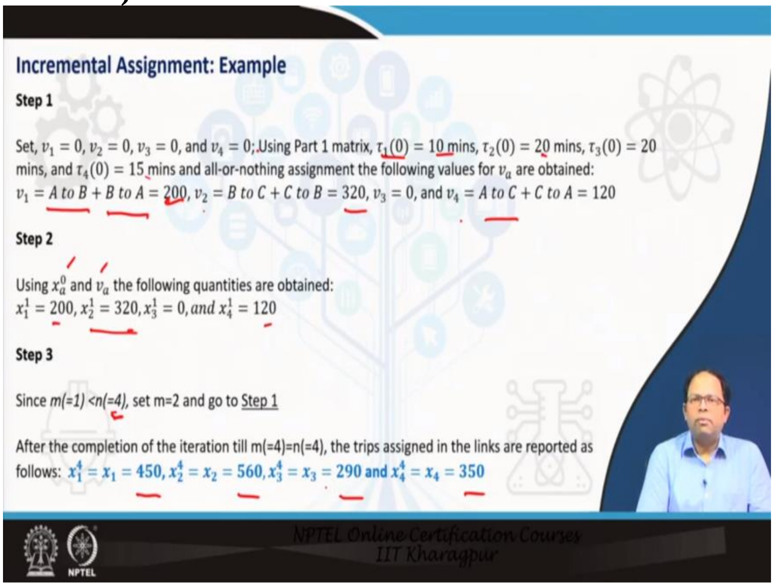


Let us explain the incremental assignment with an example. We need to determine the link flows for the given network using the incremental assignment technique with the help of trip distribution matrix shown above. The matrix must be divided in the ratio of 40:30:20:10. The link travel times, τ_a (x_a) are given by the following equation,

$$\tau_a(x_a) = [k_a + 0.15 \left(\frac{x_a}{b_a}\right)^4]$$

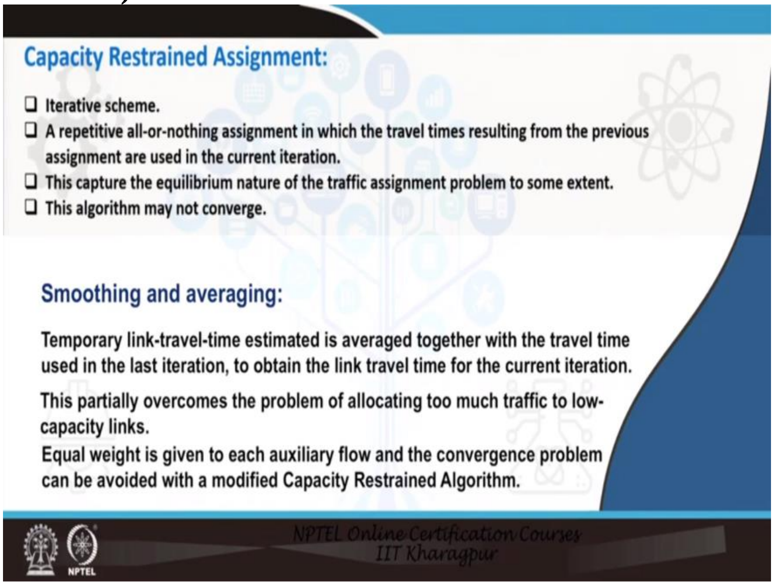
The link number, the k_a value, and the b_a value, for a particular link are mentioned as (α,β,γ) on the links where k_a is the initial travel time and b_a is the link capacity. For example, if (α,β,γ) for link is given as 1, 10 and 200 respectively, then the link number is 1, 10 is the travel impedance and 200 is the link capacity. Note that, the link-time function is different from the Bureau of Public Roads (BPR) equation. If the link-time function is not specified, BPR equation can be used. There are three nodes and four links in the given network. There is one link between A and B and A and C, whereas there are two links between B and C. If an individual wants to travel from A to B, the travel impedance is 10. If a person wants to travel from B to A, then either he can take the direct route or he can go via the node C. In the first instance, the time taken by the person will be 10 whereas for the second case the travel time will be 10+15 i.e. 25. So, the former route is the shortest route between B to A. But eventually if we keep on loading values on the shortest route, we may find that another route has become faster. The trip volume will then be assigned to the new shortest route. Before the assignment is initiated, the trip distribution matrix is divided into four parts based on the proportions mentioned above.

(Refer Slide Time: 27:48)



As per the algorithm discussed earlier, the initial flows in all the links are set to 0. The initial travel impedances for each of the links are obtained from the network diagram which are, $\tau_1(0)$ =10 mins, $\tau_2(0)$ =20 mins, $\tau_3(0)$ =20 mins, and $\tau_4(0)$ =15 mins. The flow values for each of the links are estimated based on all-or-nothing assignment. The values thus obtained are as follows, v_1=$A\ to\ B$+$B\ to\ A$=200, v_2=$B\ to\ C$+$C\ to\ B$=320, v_3=0, and v_4=$A\ to\ C$+$C\ to\ A$=120. Now, the flow in link 3, v_3 is zero because link 3 has higher travel time than link 2. So, the trips between nodes B and C will get assigned to the route with shorter travel time as per all-or-nothing assignment. The final flow on the links will be obtained based on the summation of flow in previous iteration and flow assigned in current iteration which are represented as x_{a-1} and v_a respectively. Based on the initial flow and the assigned flow, the final flow on the links after first iteration are 200, 320, 0 and 120. The travel time on each of these links are updated based on the link-cost function provided above. Since the iteration step is less than number of proportions in which the matrix has been divided, the process must be continued. So, we must update the travel time values based on the volumes obtained in each iteration and repeat the steps till the termination criteria is satisfied. So, after completion of iteration till m = 4, the total link volumes in link 1, link 2, link 3 and link 4 are 450, 560, 290 and 350 respectively. It can be observed that although link 3 has higher travel time initially, trips has been assigned to that link unlike all-or-nothing assignment where no volume would have been assigned to link 3. But in incremental assignment, the trips eventually get assigned because the travel time of the alternative route gradually becomes higher. So, we can observe traffic flows in link 3 as well.

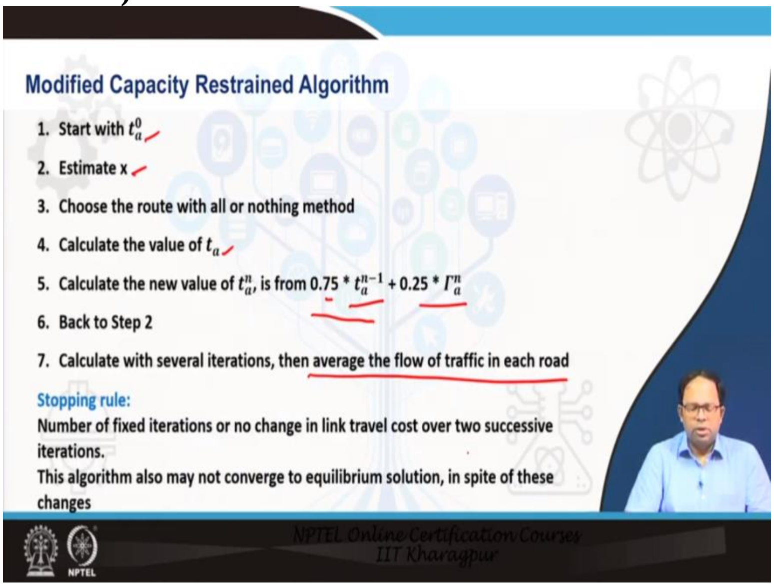
(Refer Slide Time: 30:35)



Capacity Restrained Assignment

This assignment technique is an approximation of the equilibrium method which will be discussed in the next lecture. The technique is Based on Wardrop's first principle where the equilibrium is gradually established when everybody chooses the shortest path. It is an iterative scheme and a repetitive all-or-nothing assignment in which all travel times resulting from the previous assignment are used in the current iteration. This assignment also captures the equilibrium nature of the traffic assignment problem to some extent. The algorithm may not converge to optimum solution unlike all-or-nothing assignment. The entire traffic volume is not divided into smaller proportions, rather the entire traffic volume is assigned to a single link using all-or-nothing assignment and the travel time for that link is obtained. The shortest path is determined in each iteration based on the updated travel time. Once the shortest path is known, the assignment can be conducted, and the process can be repeated. So, there may be no convergence. So, some other strategies must be adopted to terminate this algorithm or to obtain the result. These technical strategies are smoothing and averaging. The temporary link travel time estimated is averaged together with the travel time used in the last iteration to obtain the link travel time for the current iteration. Therefore, the travel time estimated from the link cost function is not considered as the link travel time. Instead, a composite measure is considered to average travel time obtained in earlier iterations. The travel times increases when flow is assigned to a link. If such composite measures are not adopted, the flow will continue to shift between two links and a solution can never be obtained. Therefore, such weighted average can converge to a solution. Such smoothing and averaging can partially overcome a problem of allocating too much traffic to low capacity links. If the entire traffic volume is assigned to a single link as observed in all-or-nothing assignment, there is a possibility of assigning too much traffic in that link. Equal weight is given to each auxiliary flow and the convergence problem can be avoided with a modified Capacity restrained algorithm.

(Refer Slide Time: 34:00)

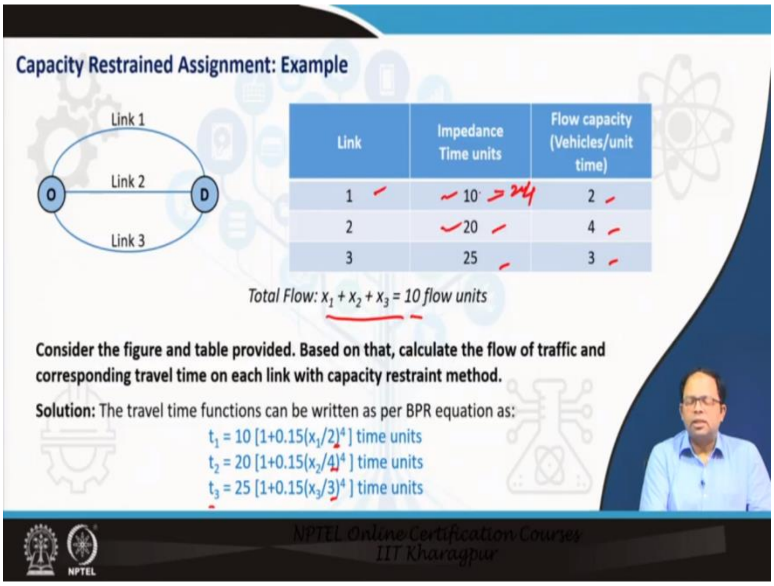


The algorithm is initiated with a free flow travel time, t_a^0 and the flow x on each link is estimated. The route is chosen using all-or-nothing assignment. The updated value of t_a is calculated. The link travel time, i.e., t_a^n is calculate using the following equation,

$$t_a^n = 0.75 * t_a^{(n-1)} + 0.25 * \Gamma_a^n$$

The process is repeated with several iterations and the average of both flow and time obtained from all the iterations is calculated for each link. Therefore, the travel time is smoothened, and final flow is obtained based on the average value. The stopping rule for the algorithm is that either the number of iterations is fixed or there is no change in link travel cost over two successive iterations. Besides, there is a possibility that this algorithm may not converge to equilibrium solution in spite of these changes.

(Refer Slide Time: 35:24)



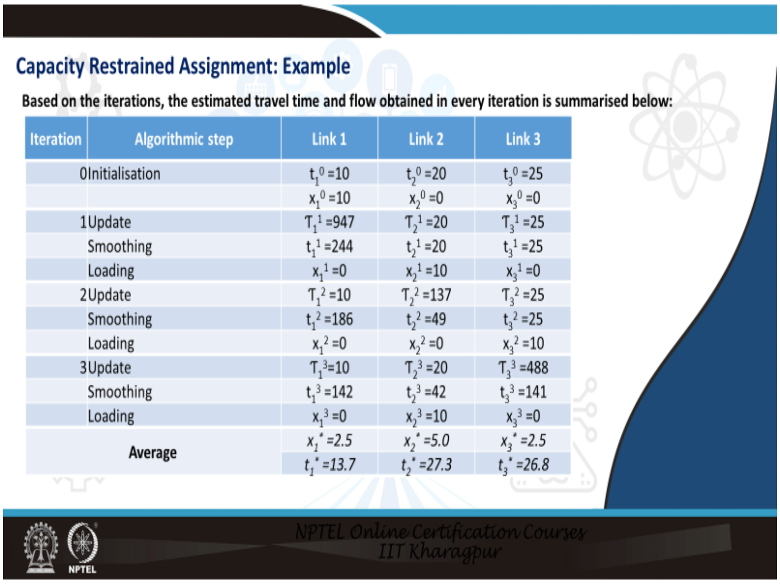
Let us consider the above network to explain the capacity restrained algorithm. In the given example, there are three links between O and D, i.e., link 1, link 2, and link 3. The free flow impedance of these three links are provided as 10, 20 and 25 and the link capacity is given as 2, 4 and 3 vehicles per unit time. Now, the total flow between O and D is 10 units. Based on the network diagram and the table provided, the BPR equation can be utilized to determine the travel time function which are t_1, t_2 and t_3. The travel time functions so determined for link 1, link 2 and link 3 are as follows:

$$t_1 = 10\ [1+0.15(x_1/2)^4\ ] \text{ time units}$$

$$t_2 = 20\ [1+0.15(x_2/4)^4\ ] \text{ time units}$$

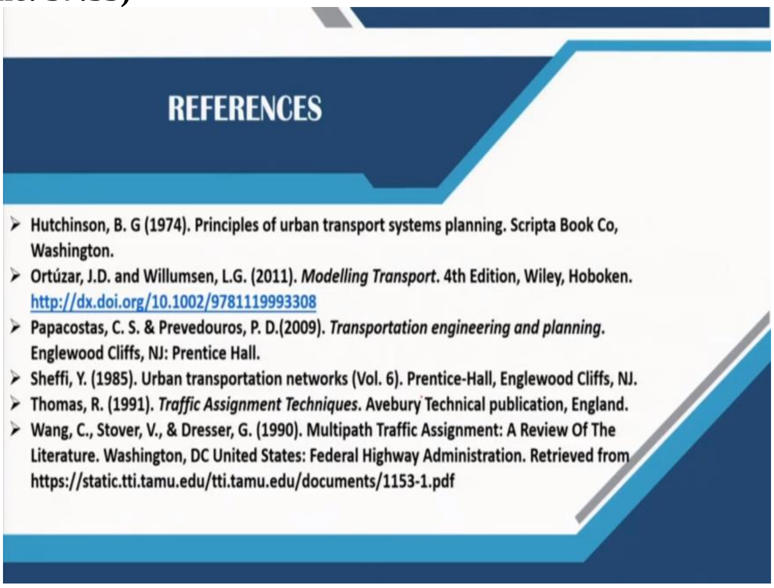
$$t_3 = 25\ [1+0.15(x_3/3)^4\ ] \text{ time units}$$

(Refer Slide Time: 36:41)

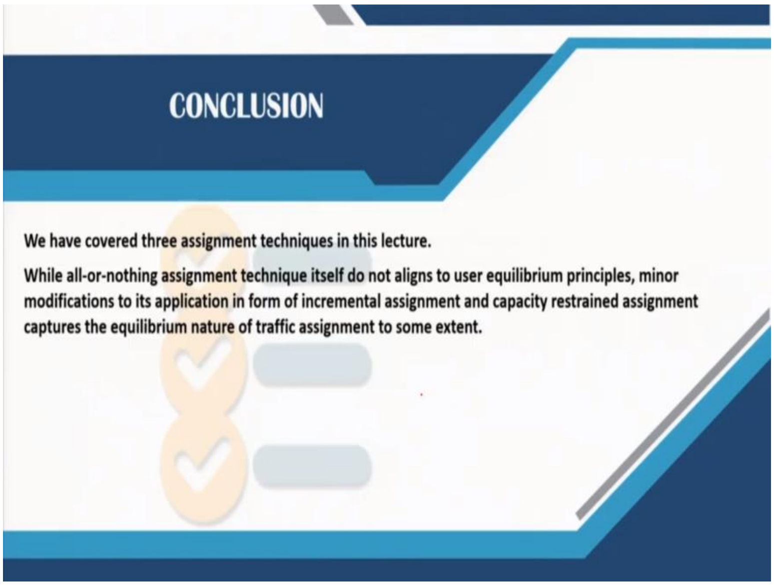


Let us initialize the algorithm with the free flow travel time for each link. Based on the free flow time, the link 1 is the shortest route. So, the entire flow is assigned to link 1 based on all-or-nothing assignment. Based on the assigned flow, the travel time in each link is estimated based on the function defined earlier. The updated travel times are T_1^1 =947, T_2^1 =20, and T_3^1 =25 for link 1, link 2 and link 3 respectively. The link travel time for each link is obtained through the average smoothing process. While link travel time for link 1 becomes 244 using the formula, the link travel time for link 2 and 3 remains the same as the free flow travel time. Based on the updated link travel times, the flow is assigned to link number 2 since it is the shortest route in the current iteration. The similar process is repeated as above. The updated link travel time now becomes 186, 49, and 25 for link 1, link 2 and link 3 respectively. The same is repeated and the assignment is terminated after three iterations. The final flow in each link is determined by averaging the link flows in each link observed in each iteration. The final flow on link 1, link 2 and link 3 are 2.5, 5 and 2.5 respectively and the corresponding travel times are 13.7, 27.3 and 26.8. Based on the capacity restrained assignment, the link volume is assigned to each of the links between a pair of origin and destination.

(Refer Slide Time: 37:55)



(Refer Slide Time: 38:00)



Conclusion

Different techniques can be utilized to assign the traffic volume to the links in the network. In this lecture, we have covered three assignment techniques. While all-or-nothing assignment technique itself do not align to user equilibrium principles, minor modification to its application in form of incremental assignment and capacity restraint assignment captures the equilibrium nature of traffic assignment to some extent.

Thank you!