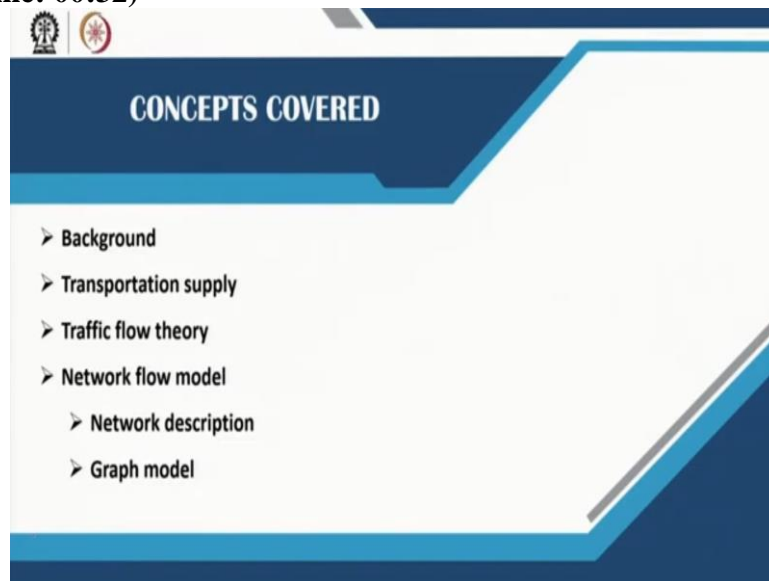


Urban Landuse and Transportation Planning
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Lecture - 41
Introduction to Trip Assignment

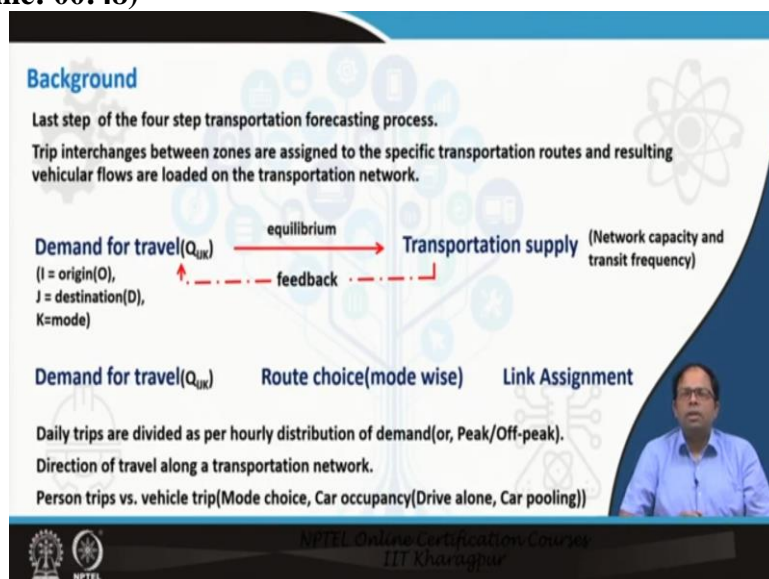
In module 9 we will cover trip assignment and in lecture 41 we will introduce the concepts behind trip assignment.

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In order to understand trip assignment, it is important to study the background. The concepts of transportation supply, the traffic flow theory, the network flow model are also discussed to understand the network and to represent it using a graph model.

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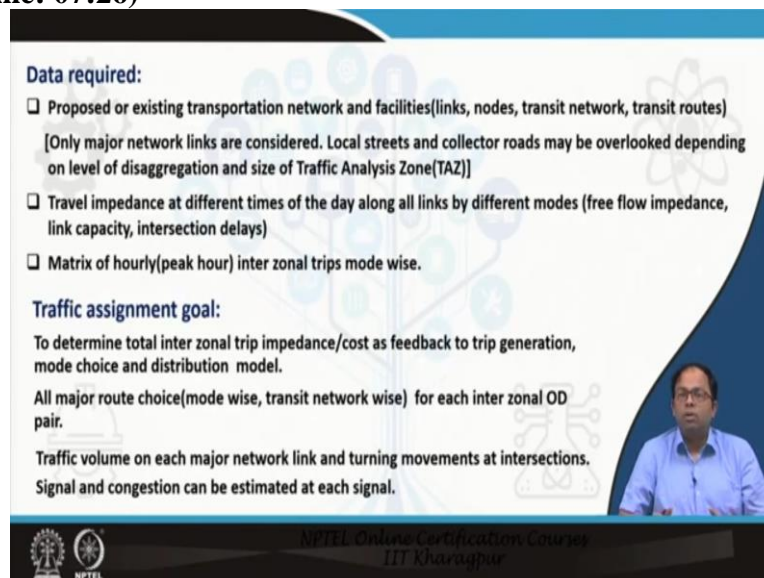
Background

The traffic assignment is the last step of the four-step transportation forecasting process. In traffic assignment, the trip interchanges between the zones are assigned to specific transportation routes and resulting vehicular flows are eventually loaded onto the transportation network. For example, whenever a person wants to go from zone i to zone j he will try to identify the route he must adopt. Such adoption will be based on either the shortest route or the shortest cost or the shortest length or a combination of both. So, there could be different ways a person chooses a route. Besides, the person must be aware of the existence of different routes available between origin and destination while choosing a specific route. Once the route is chosen, the corresponding interchange volume would be assigned to the route. A route is composed of many road segments which results in vehicular flows in each of these segments. Besides, these segments can be part of multiple shortest paths. Such overlap must be taken into consideration while summing up the vehicular flows in each segment. In this way one segment can get loaded from different directions. Finally, the vehicular flows are loaded onto the transportation network which needs to be assorted. For example, if the demand for travel is given as Q_{ij} and the mode k is known, the origin and the destination zone of a person as well as his travel mode can be identified. The total vehicular flow resulting from the person's travel must be loaded into the network. The demand is determined from such vehicular flows. The transportation infrastructure provided to cater to this demand comprise of the transportation supply. The total amount of transportation supply is determined by two components, transit frequency and network capacity. For example, if we are looking into transit network, the capacity of transit will depend on the transit frequency, which implies that the number of buses coming within an hour determines the number of passengers that could be transferred along a route. The demand varies with supply and vice versa. So, a feedback loop is maintained between the two. The equilibrium is established when the demand matches with the total amount of supply. Besides, an individual must choose routes in such a way that equilibrium is attained. For example, if there are two routes going from A to B, we may find that the inter zonal volume is loaded in such a way that the travel time is almost same. In this case, both the routes have attained equilibrium.

The following three components are crucial to carry out the traffic assignment, the demand for travel, the route choice for every mode, and the link assignment. Once we know the route along which a person is going to travel or the vehicle in which the person will be travelling,

we need to assign it to a link. While assigning the trips, the first process includes the estimation of daily trips either based on per hour distribution of demand or based on peak and off-peak hour demand distribution. For example, we estimated Q_{ij} values for the inter zonal trips for a day, but for assignment, we must determine Q_{ij} values either for each hour or for specific periods (peak/off-peak) based on certain distribution. For instance, if 40% of the total load is evenly distributed during off-peak hour, trip interchange for off-peak hour will be $0.4Q_{ij}$. On the other hand, the rest 60% is concentrated for a specific time periods of the day making the trip interchange for peak hour as $0.6Q_{ij}$. The direction of travel along a transportation network is also important because each road segment has two directions. The direction of vehicle movement can vary spatio-temporally which can affect the traffic assignment. Finally, the vehicle trips are required to assign onto a link. Therefore, the person trips, i.e., Q_{ij} must be converted into vehicle trips. Vehicle trips depend on mode choice as well as car occupancy. If a person goes by transit, we know the origin and destination stops of the individual and we will assign the person to that transit node. We need to further determine whether he would be able to go along that route based on the transit capacity. If he is using a two-wheeler or a car, then we must identify whether he will be travelling alone or in group. Accordingly, we can estimate the number of cars resulting from the person trips.

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Data required:

- Proposed or existing transportation network and facilities(links, nodes, transit network, transit routes)
[Only major network links are considered. Local streets and collector roads may be overlooked depending on level of disaggregation and size of Traffic Analysis Zone(TAZ)]
- Travel impedance at different times of the day along all links by different modes (free flow impedance, link capacity, intersection delays)
- Matrix of hourly(peak hour) inter zonal trips mode wise.

Traffic assignment goal:

To determine total inter zonal trip impedance/cost as feedback to trip generation, mode choice and distribution model.

All major route choice(mode wise, transit network wise) for each inter zonal OD pair.

Traffic volume on each major network link and turning movements at intersections.
Signal and congestion can be estimated at each signal.

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Data Required for traffic assignment

Three different types of data are required to carry out the traffic assignment process. Data on the proposed or existing transportation network and facilities such as the links, the nodes, the transit network, the transit routes etc. must be collected. Only the major links are considered since the local street or the collector street can result into a huge amount of data and the route

choice options can increase exponentially. For example, if we are choosing road between two points and consider the local/collector streets along with the arterial and sub-arterial roads, we will end up with several route choice options which can generate multiple shortest paths. Therefore, we might be required to assign loads to these routes equally. But, in general, the local paths are not chosen in the middle of the journey of an individual. A person may start from a zone. At the initial part of the journey, he may use some local streets. But then he takes a major corridor (arterial/sub-arterial) to reach his destination where again he might take local streets. In between, individuals prefer to stay in the major corridors rather than local/collector streets. To carry out the assignment process, we assume that an individual chooses major corridors to complete his trip. If people have a detailed knowledge about the network, they can take local streets to avoid congestions in special scenarios. But usually, we are only concerned with the major network links. The local streets and collector road can be overlooked depending on the disaggregation level and the size of the traffic analysis zone. The next important data set required for traffic assignment is the travel impedance. The travel impedance at different times of the day along all the links must be considered for each mode. An impedance could either be travel time or travel cost or a combination of both. Let us assume for the time being, that the impedance is travel time. So, we need to identify the travel time for each mode for each of the link in the network. The travel time for each link depends on the free flow impedance, link capacity, and the intersection delays. The travel time is a result of the link capacity and the free flow impedance. The free flow time of a link is the travel time of the link when there is no congestion. To determine shortest path based on travel time, these data sets are crucial. Finally, we require the mode-wise matrix of inter zonal trips (hourly or peak hour) for traffic assignment. Based on the above information, the traffic must be loaded onto the network.

Traffic assignment goals

1. The first goal of traffic assignment is to determine the total inter zonal trip impedance/cost as feedback to trip generation, mode choice and distribution model. The trip impedance determined through assignment is considered as an input into the trip distribution model. Similarly, not only trip distribution, the amount of travel time also plays a role in determining accessibility and accordingly it may affect trip generation as well as the mode choice models. In previous lectures, we have discussed that cost experienced by the people are derived from the network time schemes which are

generated from the trip assignment stage. So, this is the final stage as well as the most important stage in the four-stage transportation planning process.

2. The next goal is to identify all major route choice both mode wise as well as transit network wise for all for each inter zonal OD pair. For example, for each origin i to destination j , there could be some route choices which can vary based on the mode. If the mode chosen is transit, then it could also vary based on the transit network. If the person is using a car, the individual will have different route options than the one using a motorbike. But in order to reduce the complexity of the computational process we must restrict the total number of options for route choice. We can consider 2 or 3 or maximum 4 routes depending on our available resources. If number of options increases, the choices can increase exponentially.
3. Finally, traffic assignment process can help us to study the traffic volume on each major network link and turning movement at intersections. Once we know the route choices of the individuals, we can assign the traffic volume on each of these major network links. This will also determine the turning movement at intersections. The number of vehicles in each direction (straight/left/right) at each intersection will help us to determine the signal times. Once the signal times are known, the number of arriving vehicles can also be estimated at each signal. If the number of arriving vehicles at each signal is known, we can estimate the changes in traffic signal pattern, congestion level and the queue lengths with the increase in traffic flow. We can determine the changing impedance of the link accordingly. This implies that the link impedance changes dynamically, and people might change their decision to choose a route instantaneously. This implies that at certain intervals people update their route choices as well. Change in route choice can influence the link assignment as well. The signals can therefore have an influence on the travel impedance as well as the route choice. So, we need to have a detailed understanding of these traffic signals.

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Transportation Supply

Transportation supply can be defined as a system of models which, estimate the flows resulting from users' demands on one hand and determine the technical and physical aspects of transportation supply infrastructure.

- 1. Traffic Flow theory**
Traffic flow theory are used to analyse and simulate the performances of the main supply elements.
- 2. Network Flow Models**
Network Flow models are used to represent the topological and functional structure of the system.

With increase in traffic flow, speed reduces.
Shortest path before trip assignment will change after trip assignment.

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Transportation Supply

Transportation supply can be defined as a system of models which, estimate the flows resulting from users' demands on one hand and determine the technical and physical aspects of transportation supply infrastructure. The transportation supply comprises of following two concepts:

- 1. Traffic Flow Theory** – this theory is utilized to analyze and simulate the performance of the main supply elements. The change in performance of the system along with the physical characteristics of the transportation is investigated with the variations in demand.
- 2. Network Flow Models** - The structure of the transportation network is taken into consideration for such models. These models are used to represent the topological and functional structure of the system and its impacts on route choices, total impedances and so on.

There are two important concepts which must be considered to carry out the traffic assignment process. Firstly, with the increase in traffic flow, the speed of the vehicle reduces. Secondly, the shortest path before trip assignment will change after trip assignment. So, the assignment is an iterative process. The equilibrium is said to be attained after single/multiple iterations, when no change in the link impedance is observed. The corresponding traffic volume is considered as the link assignment.

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Traffic Flow Theory
Models in Traffic Flow Theory
 The models in traffic flow theory simulate the interactions among the vehicles which are using a given transportation facility or service.

Such Models can be classified in two main groups:

1) Models for running links
 These models simulate the interactions among users moving along the same transportation facility.
 Important variables –
Length of road segment, vehicle speed, headway, number of vehicles crossing a point at a given time, spacing between vehicles, etc.

2) Models for queuing links
 These models simulate the interactions among users waiting for receiving a service at a specific location.
 Important variables –
Number of users in the queue, vehicle arrival time, headway between arrival times, service time of the vehicle, waiting time, etc.

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Traffic Flow Theory

The models in traffic flow theory simulate the interactions among the vehicles which are using a given transportation facility or service. Such models can be classified into two groups i.e., *model for running links* and *model for queuing links*.

Model for queuing links- Queuing links is mostly related with transit. This model simulates the interactions among users waiting for receiving a service at a specific location. For example, number of users in queue, vehicle arrival time, headway between arrival times, service time of the vehicles, waiting time, etc. For example, if number of people waiting at the bus stop and the headway between two buses are known, we can determine the number of people who will be waiting in queue. We can further estimate the number of people who will be able to board based on the arrival time of passengers. This will give us the number of people remaining in the queue which will be useful in determining the waiting time of each passenger. Based on these variables, we can determine the load on our transit network and the transit capacity required to support the load. These models can be utilized to investigate the level of service in the transit network, which, in turn, can influence the number of people travelling through transit.

Model for running links- We are primarily concerned with the model for running links because we assume that the transport authorities usually design the transport infrastructure based on the demand. For example, a transit authority should design the service as per the number of people arriving at each bus stop or a railway station. For the road network, this process is more dynamic. The model for running links simulate the interaction among users moving along the same transportation facility and the parameters of concern are length of the road segment, vehicle speed, headway between different vehicles, number of vehicles

crossing a point at a given point of time, spacing between vehicles etc. This model considers flow, the concentration, the total speed or travel time in a particular corridor. Speed or travel time can be considered as the impedance discussed before. Based on different loading pattern in a corridor, the impedance will also change which can influence the traffic assignment.

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Traffic Flow Theory
Speed-Flow-Concentration

Fundamental equation describing traffic stream
 $q=uk$ Where,
 u = Mean Speed, k =concentration/density, q =flow.

- These three variables vary simultaneously.
- u - k relationship is monotonically decreasing.
- q - k and q - u relationships are backward bending.
- Maximum flow (q_{max}) happens at intermediate speed u_m and concentration k_m .
- u_f = free flow speed

The traffic flow is inversely proportional to the speed. The reduction rate of speed decreases as the flow reaches the link capacity.

In traffic assignment, generally the speed-flow relationship is handled in terms of travel time per unit distance versus flow leading to a cost-flow relationship

Flow curves

The three primary components of traffic flow theory are speed, flow and concentration. The relationship between these three components is given by the following fundamental equation, also known as speed-flow-concentration equation,

$$q = u * k$$

where u is mean speed, k is concentration or density and q is flow. These three components vary simultaneously. Such variations can be found in the graphs shown above. The flow and speed are plotted against concentration. The speed is also plotted against flow. For speed-concentration relationship, the speed reduces with the increase in concentration. When the concentration is 0, the speed u_f is maximum. The maximum speed is also known as free flow speed. As the concentration increases, the speed reduces to zero after certain point implying the function to be monotonically decreasing. On the other hand, q - k and q - u relationships are backward bending. The flow q increases gradually with the increase in concentration. Once maximum flow, q_{max} is reached, the flow gradually decreases with the increase in concentration. In case of flow-speed relationship, the speed u gradually decreases with the increase in flow. When the maximum flow (the total vehicles thriving along the link per hour is equal to the capacity of the link), q_{max} is reached, the rate of reduction of speed gradually decreases. The maximum flow q_{max} is observed at intermediate speed u_m and at concentration k_m . So, from the relationships discussed above we can conclude that the traffic flow is

inversely proportional to speed and the reduction rate of speed decreases as the flow reaches the link capacity. Generally, in traffic assignment the speed-flow relationship is handled in terms of travel time per unit distance versus flow, also known as a cost-flow relationship since travel time is also considered as a cost incurred by the people travelling in the network.

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Traffic Flow Theory

Cost-Flow relationship

With increase in flow towards capacity average stream speed decreases from free flow speed(u_f) to speed at maximum flow(u_m) thus also increasing the impedance. The relation between link flow and link impedance is called Link Cost Function.

As per equation given by BPR, link impedance/travel time at different flow (vehicles/hour) of a link is given as:

$$T = T_0 \left[1 + \alpha \left(\frac{v}{c} \right)^\beta \right]$$

Where,

- T = congested link travel time; T_0 = original link (freeflow) travel time;
- v = assigned traffic volume; c = the link capacity
- α and β are the model parameters (typical values, $\alpha = 0.15$ and $\beta = 4.0$)

Since, the minimum path changes with trip assignment, a number of iterations are performed till convergence. (Capacity restrained methods)

Bureau of public Roads link capacity function.

As discussed earlier, with the increase in flow towards capacity, average stream speed decreases from free flow speed(u_f) to speed at maximum flow(u_m) thereby increasing the impedance. With the decrease in speed, the link cost gradually increases with the link flow. This relation between link flow and link impedance is called the link cost function. The link impedance is plotted against link flow in the graph shown above. Before the link flow equals the link capacity, there is a gradual increase in the link impedance. Beyond the maximum flow, the link impedance increases exponentially. The link impedance/travel time at different flow(vehicles/hour) of a link can be represented through the general equation given by Bureau of Public Roads (BPR),

$$T = T_0 \left[1 + \alpha \left(\frac{v}{c} \right)^\beta \right]$$

Where, T is equal to congested link travel time, T_0 is the original link travel time or free flow time, v is the assigned traffic volume and c is the link capacity. If the assigned volume is lower than the capacity, v/c will be less than 1. Therefore, the increment rate of link impedance will be less. On the other hand, higher v/c will lead exponential increase in link impedance. The calibration coefficients for the above equation is α and β . These are the model parameters and the typical values commonly used are, $\alpha = 0.15$ and $\beta = 4.0$. If we substitute the values of α and β in the given equation, we obtain the congested link travel time

as $1.15T_0$ when traffic flow (vehicles/hour) is equal to the link capacity. In the graph shown above, it is represented as $1.15W$ where W is denoted as link impedance. So, with the above values of the model parameters the travel time at maximum flow will be $1.15W$. Since the shortest path changes with trip assignment, the numbers of iterations are performed till convergence. The criteria for convergence may vary based on different algorithms.

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Network Flow Model
Network Description

The network comprises of a system of links and nodes.

Links: A link is a segment between two intersections in a network.

Nodes: A node can either be described as a zone centroid or the intersection point of two segments within a network.

In analysis, the road network is abstracted as a coded network to facilitate processing using computers and usually does not include the local and minor streets.

Different link characteristics like link travel time and distance, level of service, capacity, facilities and geometry are coded.

The diagram shows a network with three nodes: Node A (green), Node B (orange), and Node C (yellow). Node C is labeled as a 'Zone'. Link A connects Node A and Node C. Link B connects Node A and Node B. Link C connects Node C and Node B. The slide also features the NPTEL logo and a video inset of a presenter.

Network Flow Model

The description of the network can be carried out through network flow models. The network comprises of a system of links and nodes. As discussed earlier, a *link* is a segment between two intersections in a network. The *node* can either be described as a zone centroid or the intersection point of two segments within a network. The zone centroid is the point from where the traffic volume for a traffic analysis zone (TAZ) is loaded into the network. The centroid is referred to as the centroid of a link to which the traffic is assigned. The road network is abstracted as a coded network for the purpose of analysis. The term ‘coded network’ is used since each of the links and the nodes in the network are denoted with certain attributes or values. A matrix is prepared for those nodes and links which can be processed using computers. Manual handling of large network can give rise to computational complexities. Matrix calculations are computationally efficient. Besides, the local and minor streets are also excluded from the matrix to reduce that overall complexity of the network and to enable fast computation. Moreover, different link characteristics like link travel time distance, level of service, capacity, facilities and geometry are also coded and aligned with the network which are used for further computation.

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Network Flow Model
Network description

Links are chosen in such a way that physical and functional characteristics can be assumed to be *homogeneous* for the whole link (e.g. the same average speed).

Nodes can correspond to points with *different space and/or time coordinates* in which the events occur.

The different entry or exit times in a road segment, an intersection, a station, may be associated to a single node, representing all the entry/exit events.

Centroid nodes, represent the beginning and/or the end of individual trips.

A path, k , is a sequence of consecutive links connecting an initial node (path origin) and a final node (path destination).

Only paths connecting centroid nodes are considered in transportation graphs.

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The links are chosen in such a way that physical and functional characteristics can be assumed to be homogeneous for the whole link. A corridor with multiple intersections can be considered as a single link if too many connections from other centroid zones are absent. In this regard, this corridor will include multiple signals. Generally, a single link is considered either between consecutive signals or consecutive intersections. The nodes can correspond to points with different space and/or time coordinates in which events occur. The significance and characteristics of the nodes may vary spatio-temporally. The direct entry or exit times in a road segment, an intersection, a station may be associated to a single node, representing all the entry and exit events. In other words, a single node can serve multiple purposes. The centroid nodes represent the beginning and/or end of individual trips. As discussed earlier, the traffic can be loaded into the middle of a road segment. But in some cases, it can also be loaded into a transit station or an intersection. Based on this background, let us now understand the concept of a path. A path, k is a sequence of consecutive links connecting an initial node (origin) and a final node (destination). So, a path is a combination of multiple links which connects the origin and the destination. It is noteworthy that only paths connecting centroid nodes are considered in transportation graphs. This is because origin and destinations are centroid nodes and all other intermediate intersections or nodes are either secondary or tertiary nodes.

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Network description: Trip Matrix

An estimated trip matrix must represent the three preliminary estimates required for network loading.

- 1) First being the difference between *interzonal person-trips* and *interzonal vehicle-trips*. The matrices may be available in terms of person trips which should be converted into vehicle trips.
- 2) The second is related to the difference between *daily trips* (i.e., the estimate of the 24-hr. demand) versus the *diurnal distribution* of this demand.
- 3) The third is concerned with the *direction of travel* of the trips to be assigned on the transportation network.

Legend:
 → Unidirectional route
 Bidirectional route

C is a city.
 Different zones in the city are A,B,D,E,F,I and J.
 Each zone is connected to another through routes.

Let us determine the trip-matrix for zone A.

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The trip matrix is a crucial component required for network description. An estimated trip matrix must represent the three preliminary estimates required for network loading. The first thing we need to understand is the difference between interzonal vehicle trips and person trips. As discussed earlier, vehicle trips must be considered to develop the matrices instead of person trips. If person trips are provided, it must be converted into vehicle trips before developing the matrix. The second estimate is related to the difference between daily trips and diurnal distribution of the trips. We need to consider the diurnal distribution of trips instead of the overall estimate of 24-hour demand since traffic will vary with the time of the day. The interzonal trips must be subdivided either based on hour or on period (peak/off-peak). Finally, we also need to know the direction of travel of trips to be assigned to a transportation network. The direction of travel will help us to determine the signal timings and congestion at intersections as discussed previously. Let us imagine that C is a city. The image of the city with its network is represented above. The city has different zones, A, B, D, E, F and I. Each zone is connected to another through routes which are directed routes as represented above. While there are unidirectional routes represented through an arrow in a single direction, bi-directional routes (flow is in both directions) represented through dotted lines also exist between some zones. In order to carry out assignment, trip matrix must be determined for each zone. For example, let us determine the trip-matrix for zone A.

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Network description: Trip Matrix

If the survey data is available to us, the trip matrix for zone A can be chalked out as:

Origin Zone	Destination Zone	Activity	Time of Day						
			6 to 9	9 to 11	11 to 14	14 to 17	17 to 20	20 to 22	22+
A	B	Work	44	46	7	1	1	0	0
		Edu	59	12	17	5	7	1	0
		Other	5	7	20	14	54	0	0
	D	Work	36	53	9	1	1	0	0
		Edu	35	22	20	10	12	2	0
		Other	3	6	22	14	53	2	0
	E	Work	32	58	8	1	1	0	0
		Edu	41	33	15	3	7	0	0
		Other	6	17	2	13	62	0	0
	F	Work	50	33	13	2	1	0	0
		Edu	48	6	28	8	10	0	0
		Other	2	10	18	13	54	2	0
	I	Work	51	45	4	0	0	0	0
		Edu	48	27	11	5	9	0	0
		Other	0	7	17	17	57	2	0
	J	Work	42	47	8	1	0	0	0
		Edu	46	20	18	6	9	0	0
		Other	4	9	15	14	56	1	0

Similar matrix need to be prepared for other zones.

Row wise addition will give us the number of daily trips for each trip purpose.

This matrix consists of the person trips which needs to be translated to vehicle trips prior to assignment.

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In the trip matrix for zone A, A will be treated as the origin zone while B, D, E, F, I, J will be treated as the destination zones. This is true for any trip matrix for any zone. If the survey data is available, the data can be divided among the zones. The first segregation can be based on the trip purpose as work, education and so on. Once the data based on the trip purposes is available, then the segregation must be based on the different times of the day like 6 to 9, 9 to 11, 11 to 14 and so on. The matrix so prepared will comprise of person trips conducted during different time periods for every origin-destination pair for each trip purpose. Once the matrix is prepared, the next step is mode wise categorization of trip purpose. This will help us to determine the number of people travelling in each mode for each trip purpose. Similar matrix needs to be prepared for other zones. The above matrix is only prepared for zone A. Row wise addition will give us the number of daily trips for each trip purpose. For instance, work trips from zone A to zone B for the entire day will be 99 (44+46+7+1+1) trips. Moreover, the matrix consists of person trips which must to be converted to vehicle trips prior to assignment. So, once the mode choice is known, mode-wise trips can be determined. It will help us to estimate the total number of vehicles i.e. the vehicle trips that will be loaded onto that route segment.

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Network description: Coded network

The network layout can be transformed to a coded network using the GIS technology.

(Papacostas & Prevedouras, 2009)

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As discussed earlier, the actual network must be transformed to a coded network for carrying out multiple analyses. The image above shows a map of a network which comprises of different roads within it. The network can also be divided into different nodes. The centroid nodes in the network must be identified based on the concentration of activities. In the above map, five centroid nodes are taken into consideration. For example, node 1 is connected to node 5 with four different links and three intermediate nodes. The links are either unidirectional or bidirectional which implies the link volumes must be identified separately for each direction. For example, the link volume from centroid node 1 to intermediate node 6 is different from 6 to 1. Moreover, the free flow travel times along each link can be different. These impedances are also specified in the coded network. For example, from node 6 to 7, the travel impedance is 8, whereas from 7 to 6, it is 7. The entire network must be coded in the similar manner mentioned above. The route network diagram so prepared has been shown above. Such transformation can be conducted using the GIS technology.

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Network Flow Model

Graph Models

The coded network, is represented through graph models.

A graph is an ordered pair of sets, represented by:

$$G = (N, L)$$

where, N , the set of elements known as nodes or vertices and links or arcs, i.e., $L \subseteq NXN$, a set of pairs of nodes belonging to N

Graphs used in transportation services are oriented, i.e. the links have directions and the nodes are ordered pairs.

$G \equiv (N, L)$
 $N \equiv \{(1,2,3,4)\}$
 $L \equiv \{(1,2), (1,3), (2,3), (2,4), (3,4)\}$

(Cascetta, E., 2013).

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In graph models the network could be represented in graphs and the coded network could be represented through these graph models. A graph is an ordered pair of sets represented by,

$$G = (N, L)$$

Where, N is the set of elements known as nodes or vertices, and L is a set of links or arcs where L is a subset of $N \times N$, which is a set of pair of nodes belonging to N . The graphs used in transportation network are oriented i.e. the links have directions and the nodes are ordered pairs. In the graph represented above 1, 2, 3, 4 are nodes N and the links L are 1-2, 1-3, 2-3 2-4 and, 3-4. While there are five different links in the above graph, the number of paths are six since a path is a combination of links between origin and destination node as mentioned earlier.

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Network Flow Model

Link-Path Incidence Matrix

A binary matrix called the **link-path incidence matrix**, Δ , can represent the relationship between links and paths.

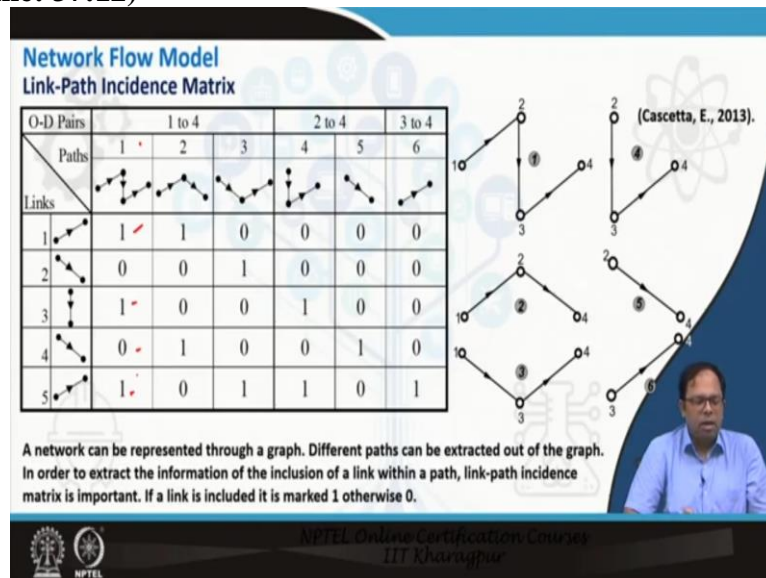
It has a number of rows equal to the number of links, n_l , and a number of columns equal to the number of paths, n_p .

Row of the link-path incidence matrix corresponding to the generic link l identifies all the paths including that link (columns k for which $\delta_{lk} = 1$). Same procedure is adopted for the columns as well i.e. rows l for which $\delta_{lk} = 1$.

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Once we determine the links and paths from the graph, we can also create a link-path incidence matrix. It is a binary matrix, where Δ can represent the relationship between links and paths. The relationship can identify the presence of a link within a path. If $\Delta = 1$, a link is included in the path. $\Delta = 0$ represents that path does not include the identified link. To create the matrix, first it is required to identify the O-D pairs from the given graph. In a link path incidence matrix, the total number of rows are equivalent to the total number of links, n_L and the number of columns is equal to the total number of paths, n_P identified from the graph. Therefore, the paths are listed on each column whereas the links are listed on each row. The corresponding Δ values for each link-path is determined to identify the links associated with different paths. The row of the link-path incidence matrix corresponding to generic link L identifies all the paths which include that link (columns k for which $\delta_{lk} = 1$). Same procedure is adopted for columns as well (rows l for which $\delta_{lk} = 1$).

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The link path incidence matrix has been determined for the graph shown above. The O-D pairs identified for the above graph are 1 to 4, 2 to 4 and 3 to 4. The number of paths between O-D pairs may vary for different pairs. In the given graph, there are three paths between O-D pair 1 to 4. For 2 to 4, one path can be 2-3-4 or we can directly reach 4 through the path 2-4. Similarly, we can determine the path for O-D pair 3 to 4. Therefore, the number of paths for 1 to 4 are three, paths for 2 to 4 are two and a single path for 3 to 4. The rows represent the links and for the given graph, there are five links. Now the Δ values are estimated for each cell depending on the presence of a link within a path. In the given graph, path 1 includes link 1. So, the corresponding δ is 1. Similarly, link 2 is not included in path 1 and the corresponding δ is mentioned as 0. A similar procedure is adopted for other paths and links

and corresponding δ values are determined to create the matrix. To extract information regarding the inclusion of a link within a path, the link-path incidence matrix is important.

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Network Flow Model
Cost Function
 As discussed earlier, each path k is associated with a path cost, which can have two different attributes:

1. Additive path costs – These costs are dependent on the link flows for congested networks. Such costs can vary spatio-temporally.
2. Non-additive path costs – These costs are specific/constant costs which are specific to a path or an OD pair.

The total path cost for a network depends on the link-path incidence matrix and the costs associated with the links. The function can be defined as follows:

$$g = \Delta^T \cdot c + g^{NA}$$

Where,
 g is the vector of the total path cost for a network
 Δ^T is the transpose/inverse of link-path incidence matrix
 c represents link cost vector
 g^{NA} is the vector of non additive path costs

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Once we identify the link combinations of the paths, we need to determine the cost of each path. To identify the shortest path between each O-D pair, it is important to determine the cost of each path. Each path, k is associated with a path cost, which can have two different attributes, the additive path cost and the non-additive path cost. Additive path cost depends on link flows for congested networks. This cost increases with the congestion in the link. Therefore, this cost varies spatio-temporally i.e., congestion will differ across locations and time. In other words, costs are associated with the link flow. The non-additive path costs are specific/constant costs for each link which are specific to a path or an OD pair. This is a fixed cost that will be incurred while traveling along a route. For example, there is a fixed bus fare along a route, but the travel time in bus along that route can vary. In this case, a generalized cost function must include the fixed cost i.e. the fare as well as the additive cost i.e. the travel time. The total path cost for a network depends on the link-path incidence matrix and the costs associated with the links. The function can be defined as follows,

$$g = \Delta^T \cdot c + g^{NA}$$

where g is the vector of total path cost for a network, Δ^T is the inverse/transpose of the link-path incidence matrix, c represents the link cost vector which is the additive path cost and g^{NA} is the vector of non-additive path cost. The above function has been explained through the following example.

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Network Flow Model

Cost Function

Let us look at the following example:

If we take the link-incidence matrix discussed previously, let us assume that the cost of the links are as follows:
 Link 1 = 2 units; Link 2 = 1 unit; Link 3 = 3 units; Link 4 = 2 units; Link 5 = 1 unit
 What will be the cost of each path in the network?

Given,

$$\Delta = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

Putting the values in the equation discussed in our previous slide we get,

$$g = \Delta^T \cdot c + g^{NA}$$

Therefore, the solution is as follows:

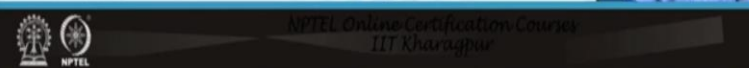
Therefore,

$$\Delta^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad g^{NA} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(Considering Non-additive costs to be null)}$$

$$g = \begin{bmatrix} 6 \\ 4 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

Path 1 = 6 units;
 Path 2 = 4 units;
 Path 3 = 2 units;
 Path 4 = 4 units;
 Path 5 = 2 units;
 Path 6 = 1 unit.

(Cascetta, E., 2013).



We use the graph mentioned earlier. The link-path incidence matrix discussed previously is considered and let us assume that the costs of the links are as follows: Link 1 = 2 units, link 2 = 1 unit, link 3 = 3 units, link 4 = 2 units and link 5 = 1 unit. We assume that there are no fixed costs, g^{NA} for the paths in the given graph. A null matrix is created to represent the non-additive path cost vector where all g^{NA} values are 0. The link-incidence matrix i.e. Δ is already available from previous discussion. The inverse of the link-path incidence matrix is given by Δ^T . The additive cost matrix, c is determined through the costs associated with each link given above. Then Δ^T is multiplied with c based on matrix multiplication in the following manner,

$$(1*2) + (0*1) + (1*3) + (0*2) + (1*1) = (2+3+1) = 6$$

With the non-additive cost vector being a null matrix, the path cost vector is generated based on the equation discussed earlier. The resultant matrix, g represents the costs associated with each path in the network. So, the cost of path 1 is 6 units, path 2 is 4 units, path 3 is 2 units and path 4 is 4 units and so on.

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CONCLUSION

Traffic assignment is conducted to derive and estimate different patterns of vehicle movements along the transportation network.

Aggregate estimates of the network such as total vehicular flows, total distance covered by the vehicle, total system travel time etc. can be computed using the trip assignment model.

Travel cost between different zones can be estimated which helps in estimating or refining other steps of the four step travel demand forecasting model.

Trip assignment can also help in determining supply side deficiencies or the impact of provision of transportation infrastructure and amenities.

Conclusion

The final stage of four-stage transportation model, traffic assignment is conducted to derive and estimate different patterns or vehicle movements along the transportation network. Besides, the aggregate estimates of the network such as total vehicular flows, total distance covered by the vehicle, total system travel time etc. can be computed using the trip assignment models. Travel cost between different zones can also be estimated which helps in estimating or refining other steps of the four-step travel demand forecasting model. Finally, trip assignment can also help in determining supply side deficiencies or the impact of provision of transportation infrastructure and amenities. If the volume exceeds the link capacity after assigning traffic onto a link, the excess volume can be accommodated by increasing the capacity of that link.

Thank you!