Urban Landuse and Transportation Planning
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Lecture - 36
Mode Choice Theory

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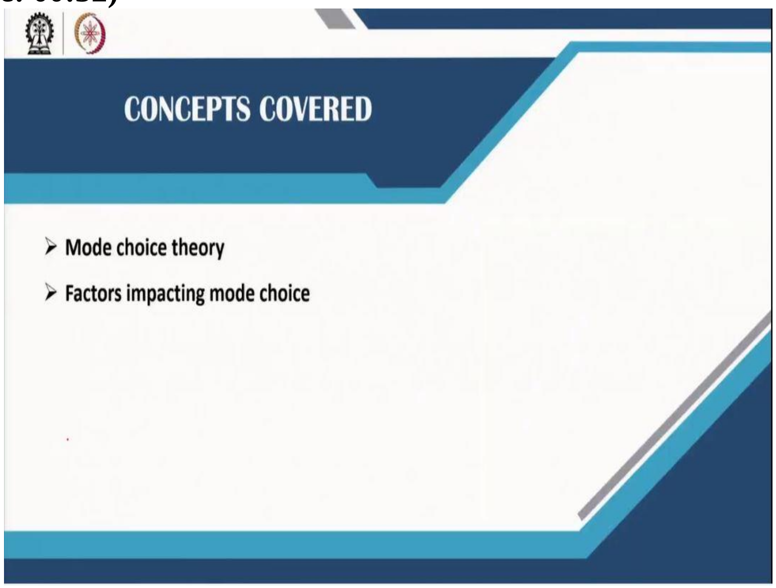

The concepts covered in this lecture are mode choice theory, and the factors impacting the mode choice.

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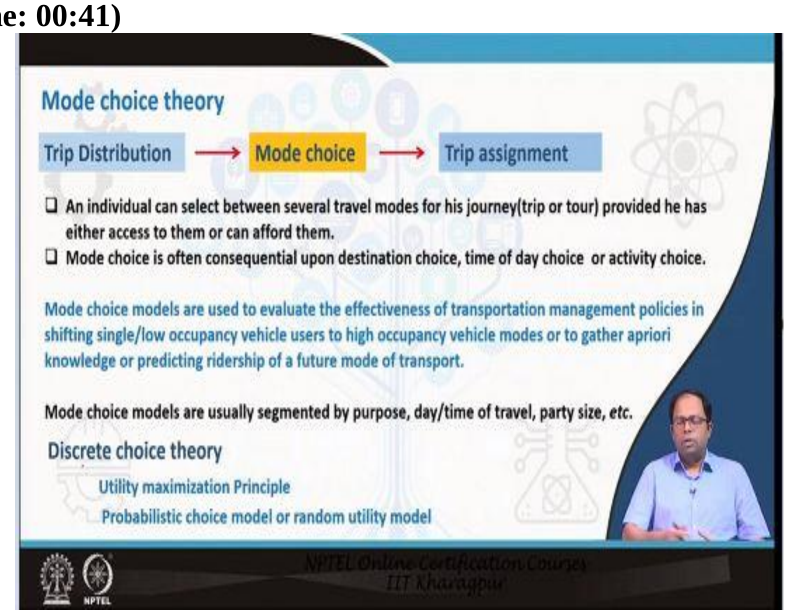

Mode choice theory

A conventional travel demand model follows a four-step modelling procedure, generally termed as a four-stage travel demand model. These stages, in sequence, are trip generation, trip distribution, mode choice, and trip assignment. So, mode choice is the third stage in the process which follows trip distribution and precedes trip assignment.

The trip distribution stage yields the number of trips made from zone ‘i’ to zone ‘j’, at a particular time of the day, or in a particular period of the time (peak or non-peak period). The next step, mode choice, determines the percentage of these trips made by each available mode of travel. Also, it analyses the individual’s behavior for selecting a particular mode of travel.

An individual can select between several modes of travel for his journey (trip or tour), provided that he has access to these modes or can afford to travel in these modes. For example, the modes available in a city are bus, shared taxis (Ola/Uber), and metro. An individual has to travel from location ‘A’ to location ‘B’ within this city. The modes available from location ‘A’ to location ‘B’ are bus and shared modes (Ola, Uber). The individual will only select Ola/Uber if he can afford to travel in these modes. Besides, he cannot select the metro because he does not have access to the metro line. So, affordability and accessibility play a role in mode choice.

Furthermore, mode choice is often consequential upon destination choice, time of the day choice, or activity choice. It means that even if an individual can afford to travel in a particular mode, or has access to use all the modes available in the city, the mode choice may depend on the destination choice, time of the day, or activity choice. For example, if a person is going shopping in a congested area, he may probably not use his car and may take the bus or metro. Similarly, a person may not take his car for a particular time of day because parking is not available in that location during that time of the day.

The mode choice models are developed to evaluate the effectiveness of transportation management policies that are primarily geared towards shifting single low occupancy vehicle users to high occupancy vehicle modes. For example, shifting from car to bus. The primary goal of model development is to understand the underlying factors why a person chooses a particular mode, and to identify those factors which can be changed to make an individual to shift to a different mode. These models are also developed to gather apriori knowledge, i.e. to understand the reasons why people choose certain modes, what attributes of decision maker, and the alternatives influence mode choice. In addition to this, the mode choice model can be used to predict ridership of future modes of transport. For example, a metro corridor is to be introduced in a city, then the likely effect of introducing this particular corridor needs to be evaluated. This is generally done by developing a mode choice model.

Mode choice models are usually segmented by trip purpose i.e. it can be for a work trip or recreational trip. Further segmentation can be based on the day or time of travel. For example, an individual may choose to ride a different mode depending on the day or time of travel. Additionally, it can be also segmented by party size. For example, five individuals are traveling together, then they may not be able to fit in a small car, so the mode choice will be different for them.

The mode choice models are formulated based on discrete choice theory. In discrete choice models, the utility maximization principle is the most widely used decision rule to select an alternative from a pool of available alternatives. This rule states that a person will choose an alternative if it gives him the highest utility among all the other alternatives. In order to calculate the probability of a particular mode being chosen by a particular individual, the utility maximization rule is converted into a format of a probabilistic choice model. These models take into consideration the modeler’s lack of information about the decision maker.

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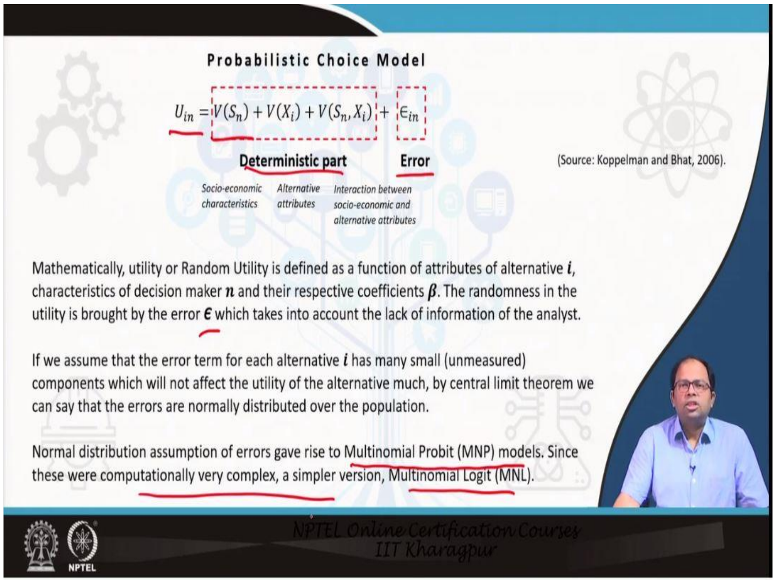

Probabilistic choice model

In the probabilistic choice model, the utility of a particular individual for a particular mode is a composite of two parts. One is the deterministic part and another part is the error term. This can be mathematically expressed as:

$$U_{in} = V_{in} + \varepsilon_{in}$$

$$U_{in} = V(S_n) + V(X_i) + V(S_n, X_i) + \varepsilon_{in}$$

Where U_{in} is the utility of alternative ‘i’ for individual ‘n’, V_{in} is the deterministic part of the utility, and ε_{in} is the error term. The deterministic part (V_{in}) is the function of characteristics of the decision maker ‘n’ ($V(S_n)$), attributes of alternative ‘i’ ($V(X_i)$), interaction terms between attribute of decision maker ‘n’ and attributes of alternative ‘i’ ($V(S_n, X_n)$), and their respective coefficients β. The error is the randomness in the utility, which takes into account the lack of information of the modeler/analyst.

The characteristics of the decision maker are income, gender, age, income group, and other socio-economic characteristics. The attributes of alternatives are characteristics of different modes of travel like travel time, in-vehicle time, etc. An example of interaction terms between socioeconomic characteristics and attributes of alternatives can be (gender=female) x travel time to work. These variables will be discussed in more detail in the subsequent section.

Based on the characteristics mentioned above, the deterministic part of the utility is a measurable quantity. On the contrary, the error term is an unmeasured value. So, the analyst assumes different distributions for the error term which results in different model forms such as logit, probit, and others. If it is assumed that the error for each alternative 'i' is very small, and it has got many small components which will not affect the utility of the alternative much, then, as per the central limit theorem, the errors will be normally distributed over the population.

The normal distribution assumption of errors give rise to Multinomial Probit (MNP) models. These models are computationally very complex and difficult to solve. Therefore, a simpler version, the Multinomial logit (MNL) model is used by assuming the error distribution to be a Gumbel distribution. As discussed in earlier lectures, Gumbel distribution approximates to normal distribution. So, MNL models are used in place of MNP models.

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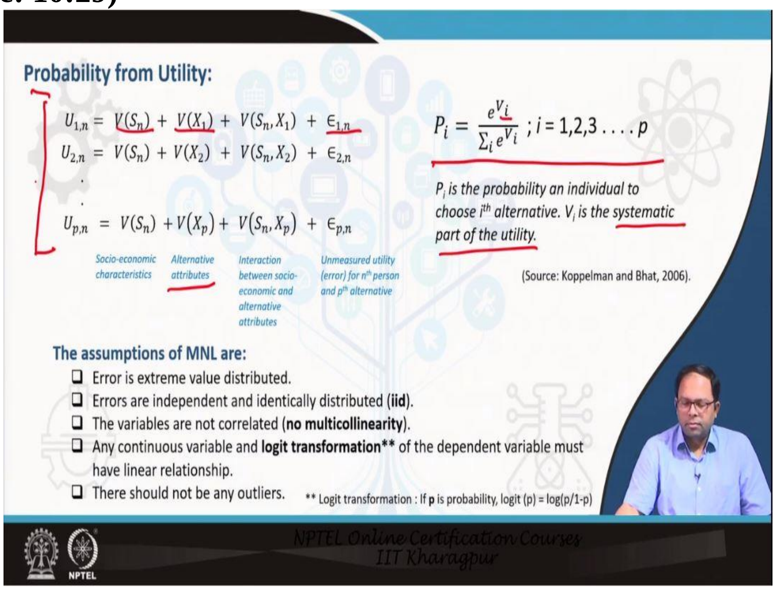

Suppose there are 'p' number of alternatives and 'n' number of individuals, then there will be p*n different utility equations. Each utility equation will correspond to one alternative. The given equations are for p number of alternatives, and each equation represent one alternative:

$$U_{1,n} = V(S_n) + V(X_1) + V(S_n, X_1) + \varepsilon_{1,n} \qquad \textit{for 1st alternative}$$

$$U_{2,n} = V(S_n) + V(X_2) + V(S_n, X_2) + \varepsilon_{2,n} \qquad \textit{for 2nd alternative}$$

$$U_{3,n} = V(S_n) + V(X_3) + V(S_n, X_3) + \varepsilon_{3,n} \qquad for\ 3rd\ alternative$$

$$U_{p,n} = V(S_n) + V(X_p) + V(S_n, X_p) + \varepsilon_{p,n} \qquad for\ pth\ alternative$$

It is important to remember that a person does not consider all the alternatives or does not have the option to consider all the alternatives. For example, a person only considers 3 alternatives out of p alternatives.

Based on these utility equations, the probability of an individual to choose 'ith' alternative can be expressed as:

$$P_i = \frac{e^{V_i}}{\sum_i e^{V_i}} \qquad i = 1, 2, 3 \ldots \ldots p$$

Here, Vi is the systematic part of the utility. The probability of choosing a particular mode of travel is equal to the exponent of the utility of that particular mode, divided by the summation of the utilities of all the modes (log sum), and it is taken as an exponent of the respective utility value.

There are few assumptions underlying a multinomial logit model. 1) Errors are extreme value distributed (Gumbel distribution), 2) errors should be independent and identically distributed, 3) the independent variables should be uncorrelated (or no multicollinearity), 4) any continuous variable and logic transformation of the dependent variable must have a linear relationship, and 5) there should not be any outliers. So, these are the different considerations which create the base for the development of a mode choice model (or MNL model).

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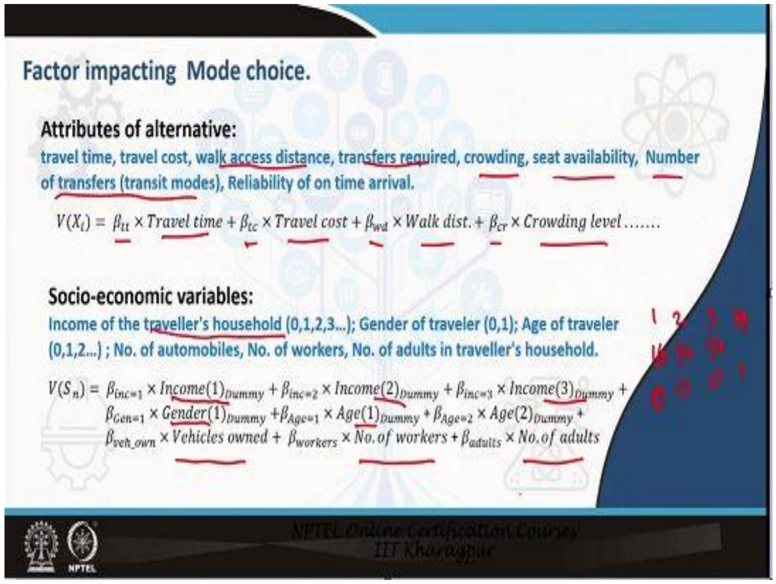

Factors impacting mode choice

There are broadly three categories of variables that influence the utility of a mode for a particular individual. These variables are attributes of alternatives, characteristics of the decision maker, and the interaction terms between attributes of alternative and decision maker.

The attributes of alternative are the different characteristics that are exclusive to a particular mode only. These characteristics affect the utility of each mode for all the individual alike. It includes travel time, travel cost, walk access distance to that particular mode (walking distance to bus stop), transfer required (transfer to another bus for the journey), crowding level, seat availability, and reliability of on time arrival. In addition to these, there can be other mode related variables as well.

Based on the above factors, the V (Xi) part of utility for a particular mode (e.g. bus) can be written as:

$$V(X_{bus}) = \beta_{tt} * travel\ time + \beta_{tc} * travel\ cost + \beta_{wd} * walking\ distance + \beta_{cl} * crowding\ level \ldots\ldots\ldots\ldots$$

Here, travel time, travel cost, walking distance, and the crowding level is for bus. Similarly, this portion of utility for other modes can be formulated. Also, the utility equation may vary from mode to mode. For example, crowding level is not considered in car. So, this portion of utility for car will not have crowding level as an influencing factor. βtt, βtc, βwd, and βcl are the beta coefficients for the given attributes. The beta coefficients signify the effect of a particular attribute on the utility of a mode. The value of these coefficients is estimated when the model is solved.

The different socio economic characteristics of the individuals also influence the mode choice. These variables include income of the travelers' household, gender of the traveler, age of the traveler, number of workers, number of automobiles, or number of adults in the household.

It is important to mention that the variables such as gender are not quantitative measures, since it is a nominal variable. So, for each category of nominal variable, there will be a separate utility equation. To avoid that, nominal variables are dummy coded to use them in a single regression equation.

If the variable has k levels or categories, then the dummy variables will be k-1. Because the kth category will be explained by the linear combination of k-1 dummy variables. For example, consider the income of household is a categorical (or nominal) variable with four categories, then there will be three dummy variables. Based on the household belonging to a particular income category or not, the value of the dummy variable will be 1 or 0 respectively. Suppose the household belongs to category 4, then the value of the dummy variable will be 0 for all three categories, and 1 for category 4. Even though the dummy variable for category 4 is not taken into account, one can easily interpret the same result based on the dummy value of the other three categories. So, k-1 dummy variables are included for k categories of an attribute.

The variables such as age of household, number of cars, number of workers, and number of adults in households can be categorical or continuous. In the present example, the age of the household head is considered as a categorical variable with two levels, and other variables as continuous variables. So, the V (Sn) part of the utility can be written as:

$$
\begin{aligned}
V(S_n) = {} & \beta_{inc=1} * Income\ (1)_{dummy} + \beta_{inc=2} * Income\ (2)_{dummy} + \beta_{inc=3} * Income\ (3)_{dummy} \\
& + \beta_{gen=1} * gender\ (1)_{dummy} + \beta_{age=1} * age\ (1)_{dummy} + \beta_{age=2} * age\ (12)_{dummy} \\
& + \beta_{veh\ own} * vehicle\ owned + \beta_{workers} * number\ of\ workers + \beta_{adults} \\
& * number\ of\ adults
\end{aligned}
$$

Here, β's are parameter coefficients for each attribute of the decision maker.

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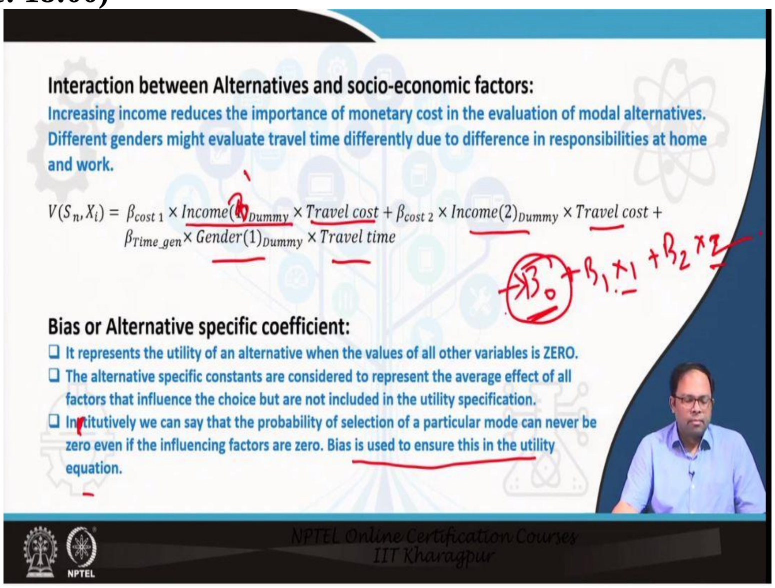

There are also interaction terms between alternatives and socio-economic factors that influence the mode choice. These variables describe the difference in the evaluation criteria of different individuals. For example, 1) increasing income reduces the importance of monetary cost in the evaluation of modal alternatives, and 2) different genders might evaluate travel time differently due to differences in responsibilities at home and work. It means that the factor such as income and travel cost, or gender and travel time are not independent. This interdependency is brought inside the utility equation through interaction terms. Based on the given interaction terms, the (V (Sn, Xi)) portion of utility can be written as:

$$
\begin{aligned}
V(S_n,X_i) = {} & \beta_{cost\ 1} * Income\ (1)_{dummy} * travel\ cost + \beta_{cost\ 2} * Income\ (2)_{dummy} * travel\ cost \\
& + \beta_{time,gen} * gender\ (1)_{dummy} * travel\ time
\end{aligned}
$$

Here, instead of four income categories, only two income categories are considered. It means that the travel costs for certain types of income groups like HMIG or HIG does not matter as they can choose any mode. But for income category like LIG and LMIG, travel cost plays a role as they can only afford certain modes.

In addition to V (Xi), V (Sn), V ((Xn, Sn)) in the utility equation of a mode/alternative, there is also a bias (β0) or the alternative specific constant. This bias term is assumed to be same for all the individual. It represents the utility of the alternative when the value of all other variables is 0. The alternative specific constants are considered to represent the average effect of all factors that influence the choice but are not included in the utility equation. Intuitively, it can be said that the probability of selection of a particular mode can never be 0, even if the influencing factors are 0. Hence, bias (β0) is used to ensure this in the utility equation.

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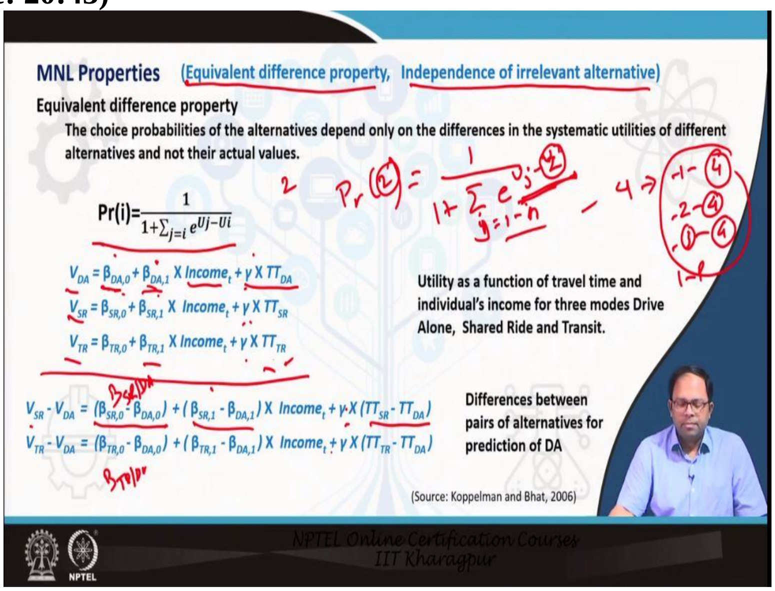

Multinomial logit model: Properties

The two most important properties of a multinomial logit model are equivalent difference property and the independence of irrelevant alternatives. These two properties are discussed in discrete choice theory. Here, both the properties are considered in the context of mode choice modelling.

Equivalent difference property states that the choice probability of an alternative depends only on the differences in the systemic utilities of different alternatives and not on their actual values. In other words, the difference between the utility is more important than the absolute utility value. Also, the addition, multiplication, or division of utility value by any scalar factor does not have any effect on the choice probability because the difference between the utility values of the two alternatives is important.

The probability of choosing an alternative 'i' can be written as:

$$\Pr(i) = \frac{1}{1 + \sum_{j \neq i} e^{Uj - Ui}} \qquad \nabla\, i \in J$$

Suppose, there are three modes of travel, which are drive alone (DA), shared ride (SR), and transit (TR). Then the probability of choosing DA, SR, and TR will be:

$$\Pr(DA) = \frac{1}{1 + e^{U_{SR} - U_{DA}} + e^{U_{TR} - U_{DA}}}$$

$$\Pr(SR) = \frac{1}{1 + e^{U_{DA} - U_{SR}} + e^{U_{TR} - U_{SR}}}$$

$$\Pr(TR) = \frac{1}{1 + e^{U_{DA} - U_{TR}} + e^{U_{SR} - U_{TR}}}$$

So, the probability of DA depends only on the difference of the utility between DA and SR, and DA and TR. Similarly, the probability of SR or TR depends on the difference between the utility value of that particular mode and other modes. This property of the multinomial logit model has an implication on the utilities of the alternatives. Let us consider that the systematic part of the utility is a function of income and travel time only, then the utility equation for DA, SR, and TR can be written as:

$$V_{DA} = \beta_{DA,0} + \beta_{DA,1} * income_t + \gamma * travel\ time_{DA}$$

$$V_{SR} = \beta_{SR,0} + \beta_{SR,1} * income_t + \gamma * travel\ time_{SR}$$

$$V_{TR} = \beta_{TR,0} + \beta_{TR,1} * income_t + \gamma * travel\ time_{TR}$$

where $\beta_{DA,0}$, $\beta_{SR,0}$, and $\beta_{TR,0}$ are bias or alternative specific constant, $\beta_{DA,1}$, $\beta_{SR,1}$, and $\beta_{TR,1}$ are coefficients for income variable, and γ is a coefficient for travel time. It is important to note that the γ coefficient is similar in all three utility equation because it is assumed that the experience of travel time in all the alternatives is evaluated similarly by every individual. Nonetheless, to determine the utility equation, these 7 parameters need to be estimated, but the estimation of these parameter coefficients is difficult. So, the equations are further reduced by taking the difference between the pair of alternatives based on equivalent difference property. For example, for the prediction of the probability of DA, the utility equation can be written in the following forms:

$$V_{SR} - V_{DA} = (\beta_{SR,0} - \beta_{DA,0}) + (\beta_{SR,1} - \beta_{DA,1}) * income_t + \gamma * (travel\ time_{SR} - travel\ time_{DA})$$

$$V_{TR} - V_{DA} = (\beta_{TR,0} - \beta_{DA,0}) + (\beta_{TR,1} - \beta_{DA,1}) * income_t + \gamma * (travel\ time_{TR} - travel\ time_{DA})$$

From the given equations, it can be inferred that the given utility equations are in reference to DA, i.e. utility of SR in reference to DA and utility of TR in reference to DA.

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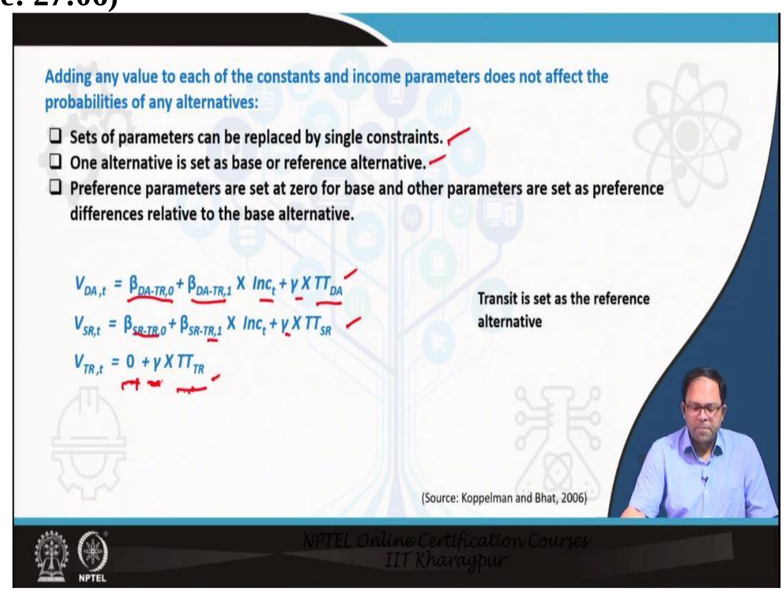

It is clear that adding any values to each of the constant and income parameters does not affect the probabilities of any alternative. So, to estimate the parameter coefficients, the sets of parameters are replaced by a single constraint. This constraint is to set considering one alternative as a base or reference alternative. Also, the preference parameters are set at zero and other parameters are set as preference differences relative to the base alternative.

In the present example, the transit (TR) alternative is set as the reference category. The preference or the bias ($\beta_{TR,0}$) is set at zero. Also, the other parameters are set as preference difference relative to transit alternatives. The utility equation can be written as:

$$V_{DA,t} = \beta_{DA-TR,0} + \beta_{DA-TR,1} * income_t + \gamma * travel\ time_{DA}$$

$$V_{SR,t} = \beta_{SR-TR,0} + \beta_{SR-TR,1} * income_t + \gamma * travel\ time_{SR}$$

$$V_{TR,t} = 0 + \gamma * travel\ time_{TR}$$

In here, the constant and the income coefficient are relative to the transit alternative, and the travel time is absolute travel time for DA, SR, and TR. Also, instead of the absolute travel time, the difference in travel time between DA and TR, and DA and SR can be taken.

So, in a multinomial logit model, the probability of one mode is determined in reference to another mode. The probability of the reference category can also be estimated. For example, there are 4 modes of travel. Then, the probability of modes 1, 2, and 3 can be estimated in reference to category 4 i.e., assuming category 4 as the reference category. Since the sum of all the probabilities should equate to 1, the probability of the reference category can be estimated from the probability of the other three alternatives.

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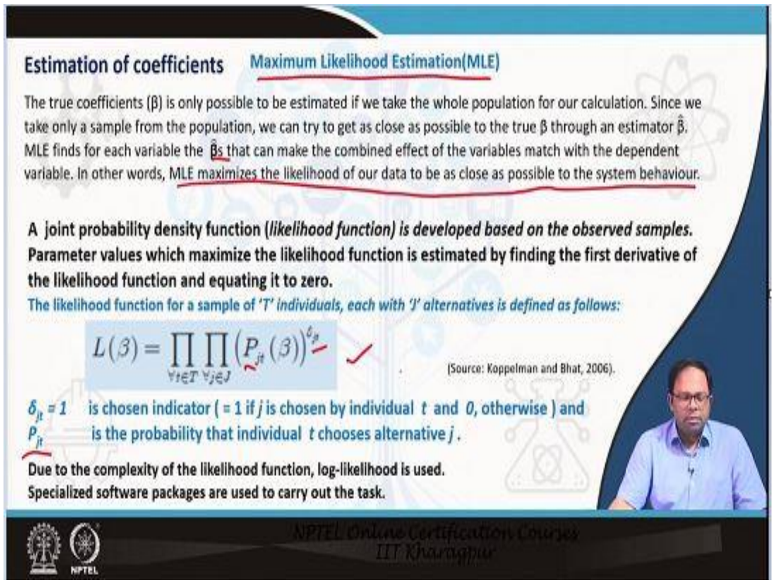

Estimation of coefficients

The true coefficient (β) is possible to be estimated if the calculations are done considering the whole population of the system (or study area). But for research studies, only a sample of the population is considered, because it is not possible to survey every individual/household. Therefore, an estimator of β (true population parameter) is calculated which is β^ (sample parameter). The estimation of β^ is accomplished by the Maximum Likelihood function (MLE). MLE finds the value of β^ for each variable that can make the combined effect of the variable match with the dependent variable. In other words, MLE maximizes the likelihood of data being as close as possible to system behavior (or population behavior).

In the Maximum Likelihood Estimation method, a joint probability density function (likelihood function) is developed based on the observed samples. The likelihood function for a sample of 'T' individual, each with 'J' alternatives is defined as follows:

$$L(\beta) = \prod_{\forall t \in T} \prod_{\forall j \in J} (P_{jt}(\beta))^{\delta_{jt}}$$

Where L(β) is the likelihood function, Pjt is the probability that the individual 't' chooses alternative 'j', and δjt is equal to 1 if alternative 'j' is chosen by the individual and 0 otherwise.

The values of the parameter which maximizes the likelihood function are estimated by taking the first derivative of the likelihood function and equating it to zero. Due to the complexity of the likelihood function, the log transformation of this function is used to estimate the parameter value. The log transformation of the likelihood function gives the same maximum as the likelihood function. The equation of the first derivative of the log-likelihood function is solved using different algorithms. So, software packages are used to carry out the task.

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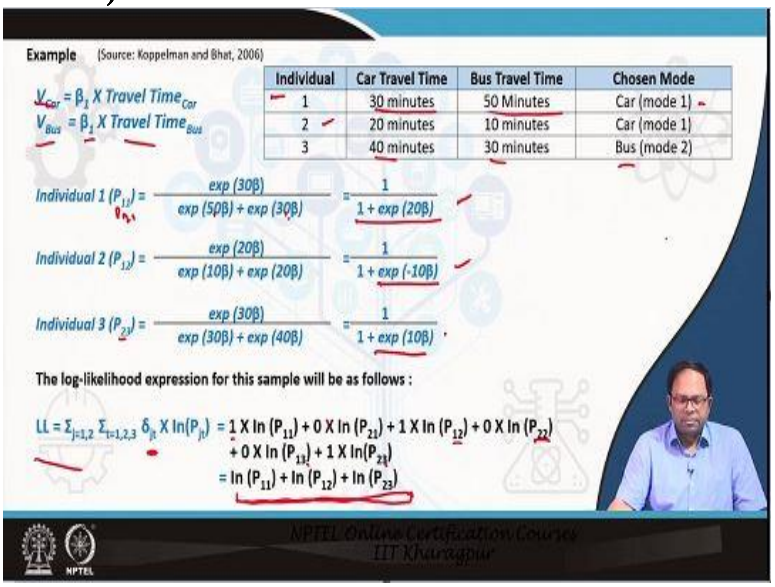

In order to understand the application of MLE, an example is considered where a binary logit model for choice between car (mode 1) and bus (mode 2) is to be estimated. The deterministic part of the utility for car and bus is given as follows:

$V_{car} = \beta_1 * Travel\ time_{car}$

$V_{bus} = \beta_1 * Travel\ time_{bus}$

Here, one parameter i.e. travel time is considered for simplicity only. Suppose the model is to be estimated based on the sample of three individuals only. The observed mode choice and the values travel time for each mode for all three individual is given in the table. For example, the observed mode choice for the first individual is a car, and the time taken by car and bus to reach the destination (B) from point of origin (A) is 30 minutes and 50 minutes respectively. For individual 2, the observed mode choice is a car, and the travel time by car and bus is 20 minutes and 10 minutes respectively.

The probability of the chosen mode for each individual can be written as:

$$Individual\,1\,(P_{car,1}) = \frac{e^{30\beta}}{e^{30\beta} + e^{50\beta}} = \frac{1}{1 + e^{20\beta}}$$

$$Individual\,2\,(P_{car,2}) = \frac{e^{20\beta}}{e^{20\beta} + e^{10\beta}} = \frac{1}{1 + e^{-10\beta}}$$

$$Individual\,3\,(P_{bus,3}) = \frac{e^{30\beta}}{e^{40\beta} + e^{30\beta}} = \frac{1}{1 + e^{10\beta}}$$

The probabilities of non-chosen modes are not calculated, because the value of δjt in likelihood function is 1 if alternative 'j' is chosen by the individual and 0 otherwise.

The log-likelihood function for the given sample (3 samples) can be written as:

$$LL = \sum_{j=car,bus} \sum_{t=1,2,3} \delta^{jt} * \ln(P_{jt})$$

$$LL = 1 * \ln(P_{car,1}) + 0 * \ln(P_{bus,1}) + 1 * \ln(P_{car,2}) + 0 * \ln(P_{bus,2}) + 0 * \ln(P_{car,3}) + 1 * \ln(P_{bus,3})$$

or

$$LL = \ln(P_{car,1}) + \ln(P_{car,2}) + \ln(P_{bus,3})$$

The last equation is the final form of the log-likelihood function for the present example.

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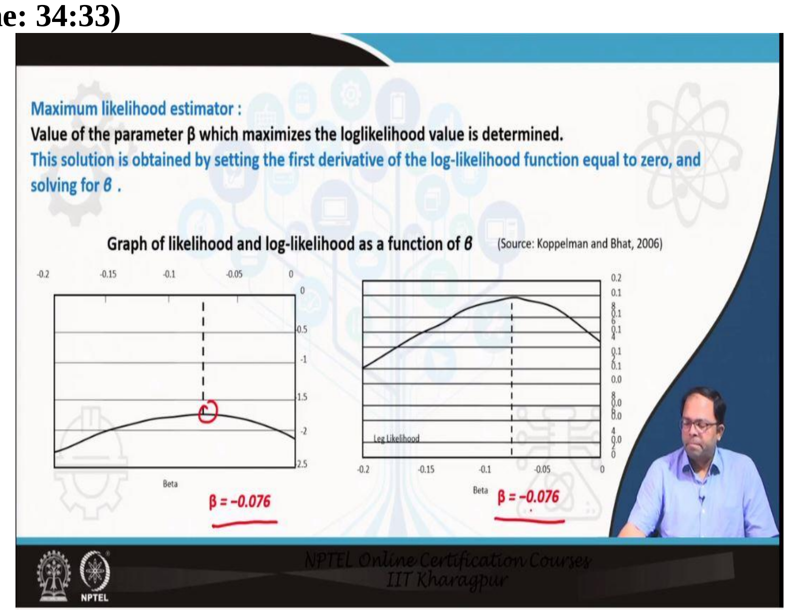

The solution for β values for the present example can be determined by taking the first derivative of the log-likelihood expression and equating it to zero. For the present example, the graph of likelihood and log-likelihood can be plotted for different values of β. The left graph represents the likelihood and the right graph is for log-likelihood. From the graph, it can be seen that at β=-0.076, the slope is zero. In other words, the value of likelihood and log-likelihood function is maximum at a point where β=-0.076. This solution is obtained by setting the first derivative of the log-likelihood function equal to 0.

In this case, the number of parameters to be estimated was one. But in practical problems related to mode choice, the number of parameters is more. So, the estimation of parameters becomes computationally significant. Therefore, softwares are generally utilized for the task.

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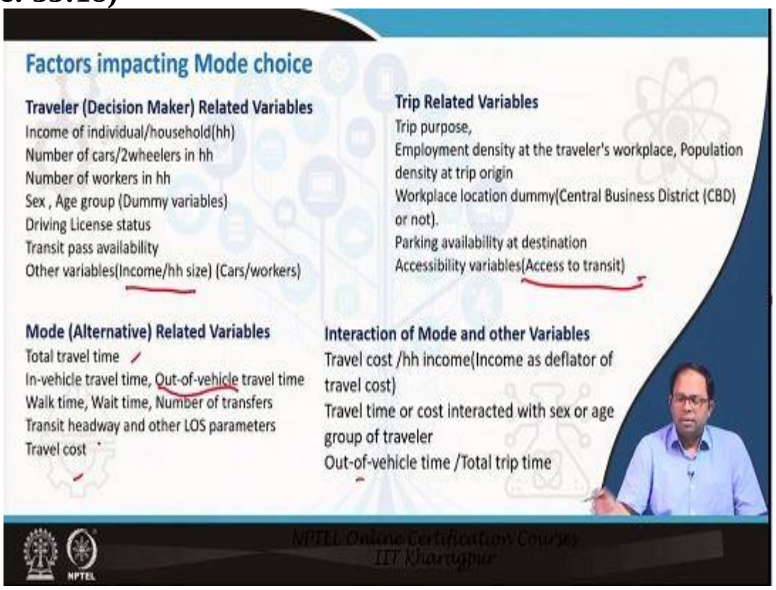

Factors impacting mode choice

The most commonly used factors in mode choice modelling are traveler (or decision maker) related variables, mode related variables, and interaction terms between mode and other variables, as discussed in earlier sections. In addition to these factors, there are trip related variables as well.

The traveler (or decision maker) related variables that influence mode choice and commonly used in mode choice studies are income of household, number of cars or two-wheeler, number of workers in the household, gender or age of household head, driving license, and transit pass availability. In addition to these, modified variables such as income per household size, or number of cars per number of workers are also used to understand the relative impact. For example, the income of a household per household size gives an impact of income on a particular person's decision making because the more the number of members in the household, the lesser will be the impact of income.

The trip related variables include trip purpose (work or recreational), employment density at the traveler's workplace, population density at trip origin, workplace location, parking availability at destination, and accessibility variables (access to transit).

The mode (alternative) related variables are total travel time, in-vehicle travel time, out-vehicle travel time, walk time, waiting time, number of transfers, transit headway, level of service related variables, and travel cost. These variables are either generic (or alternatives) or mode-specific. For example, travel cost is a generic variable, which means that it applies to all the alternatives or modes available. Whereas, transit headway is a mode- specific variable. Because it is specific to transit mode only.

The interaction terms between mode and other variables (traveler or mode related) include travel time/travel cost interacted with age or gender of traveler, out vehicle time per total trip time, or travel cost per household income. So, these are the different factors that influence mode choice and are commonly included in mode choice modelling.

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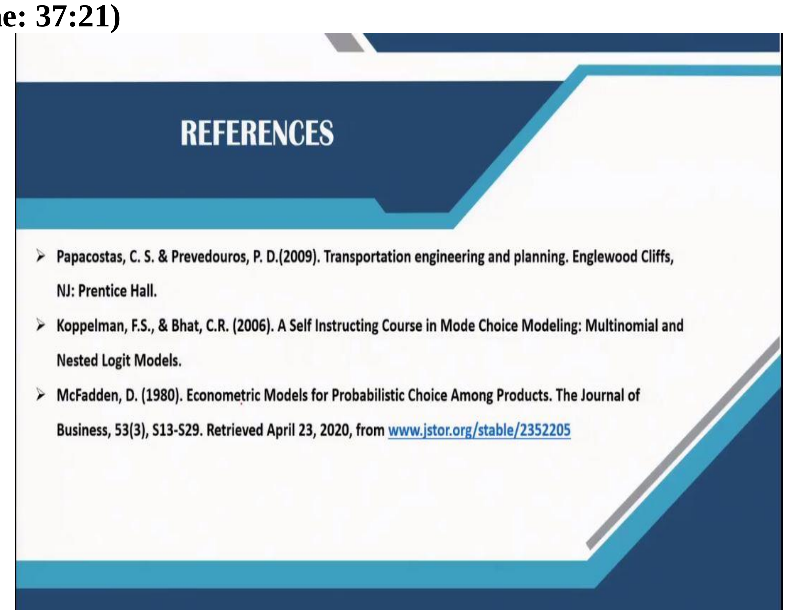

A list of references is given in the above slide.

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Conclusion

Mode choice models are used to evaluate transportation demand management policies and to evaluate the outcome of introducing new model alternatives. Both revealed and stated preference data are used to develop mode choice models. While random utility models are most popular, several new approaches are gradually being introduced to handle the increasing complexities and disaggregation levels. Also, the mode choice models are developed jointly with other choice models.