**Urban Land use and Transportation Planning**
**Prof. Debapratim Pandit**
**Agricultural and Regional Planning Department**
**Indian Institute of Technology-Kharagpur**

**Lecture-35**
**Trip distribution**

This lecture deals with trip distribution.

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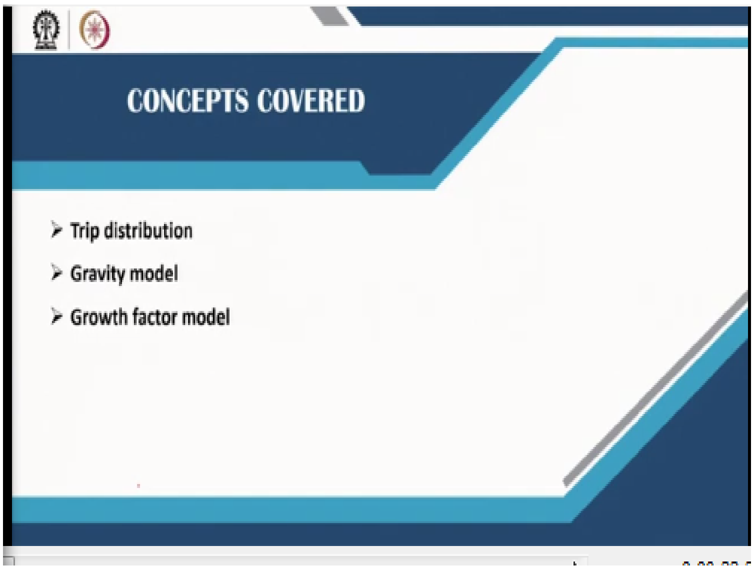


The different concepts that would be covered are the basics of trip distribution; gravity model; and growth factor model.

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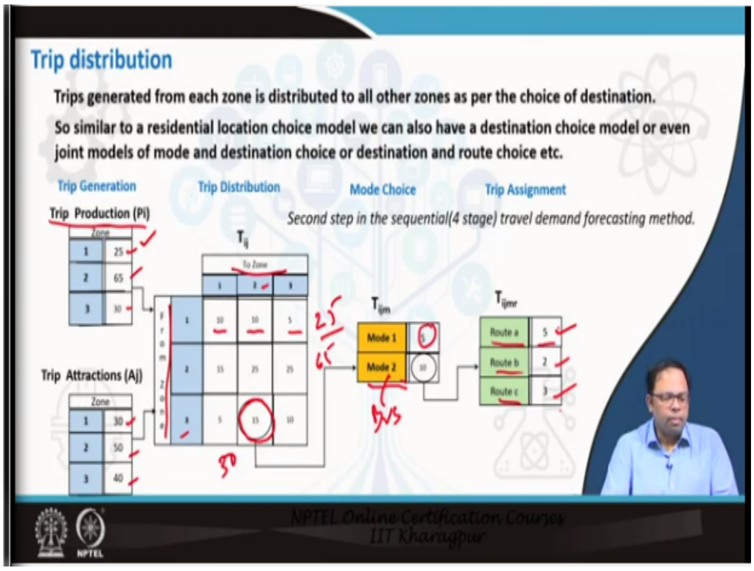
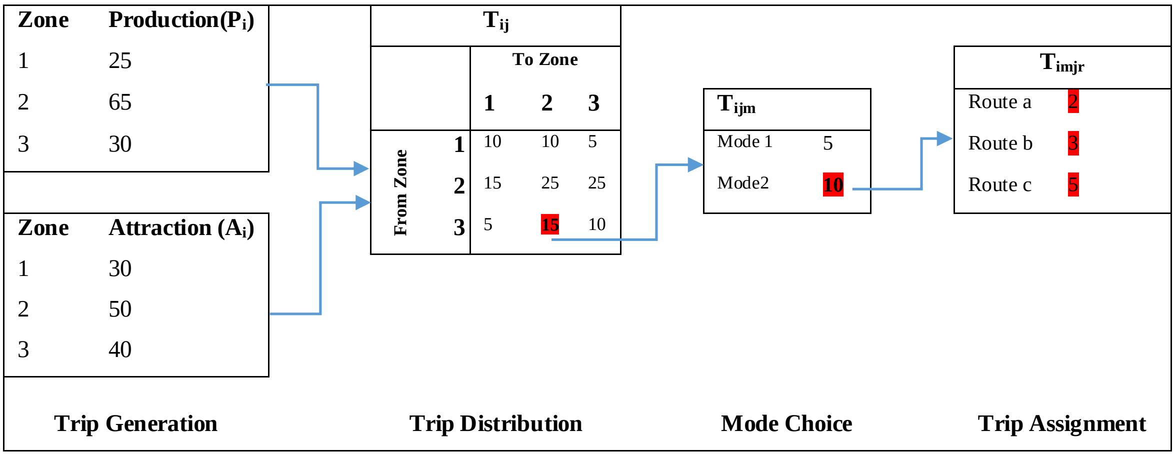



**<u>Trip distribution:</u>**

In sequential four stage model for travel demand forecasting, the stage after trip generation is trip distribution. After estimation of the number of trip produced and attracted for each zone in the urban area, the number of trips starting from each zone and ending at different zones needs to be estimated. This estimation of flow of trips between different zones is called trip distribution which involves distributing the trips generated at each zone to other zones based on the choice of destination.

There are several methods which can be used to determine destination choice. Similar to that of residential location choice model, there can be destination choice model or even joint models of mode and destination choice or even destination and route choice. Apart from this there can be other forms of complexities. For example, if an urban area has got 200 zones, and trips from each zone to all the other zones are to be mapped, there will be 40000 values of $\mathbf{T_{ij}}$ (trips originating from $\mathbf{i^{th}}$ zone and ending at $\mathbf{j^{th}}$ zone). If '**n**' different purposes of trips are considered, then the number of $\mathbf{T_{ij}}$ values become $\mathbf{nT_{ij}}$. This can be further disaggregated into trips made by various socio-economic groups, different kinds of households, and so on.

Trip distribution covered in this lecture is concerned with only simple models that can be adopted for the whole city. The aim is to be able to forecast the future trip distribution using the base year data. Alternative methods are also discussed to estimate the number of trips which are generated in the surrounding areas and ending in the city area. The following example shows the number of trips produced and attracted for three zones, namely 1,2, and 3.




Among the total 120 trips produced by the three zones, zone 1,2, and 3 produces 25, 65, and 30 trips respectively. These trips are attracted to zones 1,2, and 3 as 30, 50, and 40 trips respectively. Trip distribution stage gives the matrix that gives the detailed information about the number of trips originating from and arriving to each zone from every other zones, and maybe within itself. In the trip distribution matrix, the rows represent the origins, and the columns represent the destinations. Each value in the matrix is represented as $\mathbf{T_{ij}}$, which means the number of trips originating from zone **i** and destined to zone **j**. For example, among the total 25 trips generated from zone 1, 10 are destined to zone 1 itself, 10 are destined to zone 2, and 5 are destined to zone 3. Similar observation can be made for the other zones. Adding all the $\mathbf{T_{ij}}$ values row wise must be equal to trip productions ($\mathbf{P_i}$), and a column wise sum must be equal to the trip attractions ($\mathbf{A_j}$), where **i** and **j** represents the zones.

The trip distribution matrix leads to the third stage i.e. determining mode choice for each trip, in the four stage sequential travel demand forecasting process. For example, a flow which is starting from zone 3 and ending at zone 2 has 15 trips. In mode choice modelling it is determined how these 15 trips would be distributed among different modes. In this case, mode 1 is used for 5 trips and mode 2 is used for 10 trips.

After mode choice, the last step of the four-stage model is trip assignment, where the trips by various modes are assigned to routes in the network. Let us assume mode 1 to be a **transit** mode i.e., **bus**, and mode 2 as **personal car**. Considering the case of bus, the 10 trips needs to be assigned a particular route among the many routes available to move from zone 3 to zone 2. Let us assume there are three routes; **route a**, **route b**, and **route c**. After mode choice, 2 bus trips are assigned to route a; 3 bus trips to route b; and 5 bus trips to route c. Similar assignment can be carried out for the personal car trips. There can be many factors behind people choosing a particular route over other, and these are likely to change overtime throughout the day. So this is the schematic flow of four-stage model of travel demand management.

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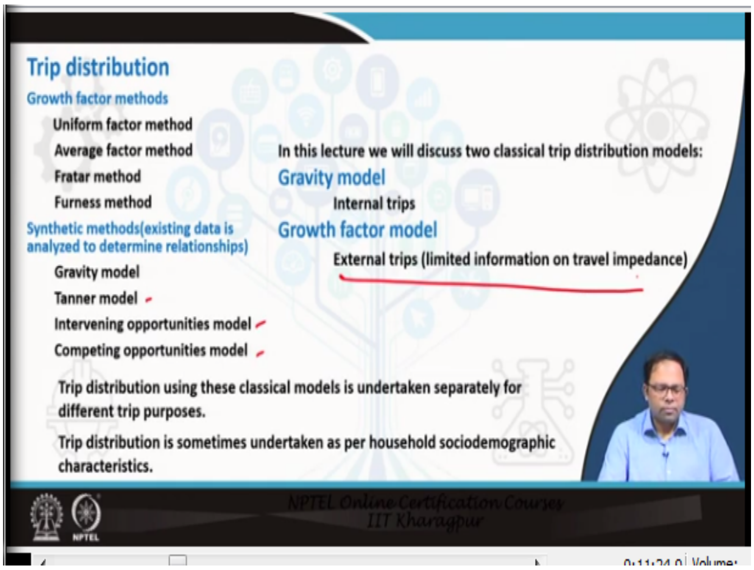


**Methods of trip distribution:**

There are different methods to distribute trips in an urban area, once the total number of trips produced and total number of trips attracted for each zone in an urban area or TAZ are determined. The methods can be broadly classified into two groups, **growth factor** methods and **synthetic** methods. In growth factor methods, existing number of trips travelling between zone i to zone j is multiplied with some factor to get the projected number of trips at some time in future. There are different growth factor methods like uniform growth factor method, average factor method, Fratar method, Furness method etc. Synthetic methods on the other hand, analyses the exiting data on $\mathbf{T_{ij}}$ values and production and attraction data, and establishes some relationships among these based on some principles. These relationships are used to develop trip distributions for future. Some of the popular models in synthetic approach are gravity model which is the most famous one; Tanner model; intervening opportunities model; competing opportunities model etc. In this section, gravity model and growth factor model is discussed in detail. Trip distribution is performed for different types of trips like; work trips, business trips, school trips, etc. Since trips based on purpose depend on the landuse pattern of an urban area, estimating distribution for all these trips in the same way i.e. using the same equations may give erroneous results.

Gravity model is the most common model used to determine trip distribution for internal trips. On the other hand, growth factor model is particularly used for external trips i.e. trips coming from the surrounding area into the particular urban area under consideration. Since external trips arrive from surrounding areas, reliable information on the travel times, distances travelled, or any other form of impedance is usually not available. Population and predicted growth of such adjoining areas may be available though. Based on these information, the total number of trips produced from the surrounding zones in future can be estimated which can be further used to derive the number of trips from these areas that may enter the urban area under consideration.

Trip distribution is also sometimes undertaken as per household and socio-demographic characteristics. This is done because, in the framework of four-stage modelling, the stage following trip distribution is mode choice. In mode choice, data showing different types of socio economic groups and different economic categories are required. People belonging to different groups, tend to have a bias towards choosing mode or travelling. This also needs to be reflected in the mode choice model too. So, often the trip distribution data also requires to preserves the socio economic characteristics of the observations. For example, if trip distribution is done based on trip purpose only, it is difficult to say how many people travelling from zone **i** to zone **j** belongs to a particular group. Hence segregating the data based on socio-economic data prior to performing trip distribution facilitates the inclusion of socio-economic linkages in mode choice model.

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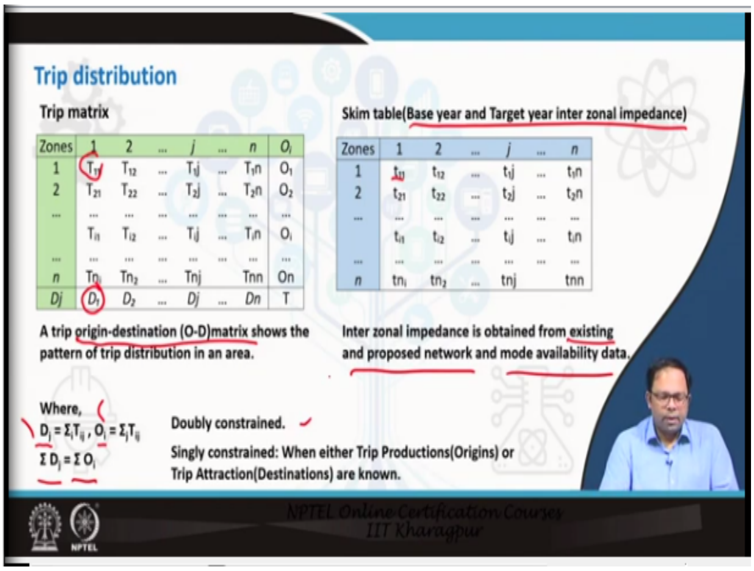


There are two tables that are used in trip distribution; Origin-destination matrix, and skim table. Origin-destination matrix or OD matrix, as it is popularly called, is a matrix representing the number of trips originating from and destined to a pair of zones. OD matrix has the zones 1,2,3… n mentioned in rows and columns. Each entry in the matrix is indicated by $\mathbf{T_{ij}}$, where **i** represents the origin zone and **j** represents the destination zone, and **T** represents the number of trips. Adding all the $\mathbf{T_{ij}}$ values row wise, or $\sum_j T_{ij}$ represents $O_i$ or the total number of trip generated by $\mathbf{i^{th}}$ zone. Similarly, adding all the $\mathbf{T_{ij}}$ values column wise, or $\sum_i T_{ij}$, represents $D_j$ or the total number of trip destined at $\mathbf{j^{th}}$ zone.

Skim table is the base year and target year inter zonal impedances. Impedance can be represented by travel time, travel cost or any other form of impedance one might face while travelling from one zone to other. Usually this information is obtained from base year data and is collected from surveys done in the urban area under consideration. Inter-zonal impedances are estimated based on the existing and proposed networks and mode availability data. Usually the impedances are kept the same for target year. But there may be cases where new modes are introduced in the network of the urban area, which might alter the inter-zonal impedances. In such cases, the skim table might be required to be estimated again. When predicting trip distribution for future, the target year skim table must be used, whereas while developing the model, or calibrating the model, base year skim table is used.

As already mentioned, the total number of trips attracted to a particular zone '**j**' is summation of all trips originating in different zones which are attracted to zone '**j**'. Similarly, all trips originating in zone '**i**' is a summation of all trips originating in that zone and ending in all the other zones '**j**'. Also, the summation of $\mathbf{D_j}$ is equal to summation of $\mathbf{O_i}$ which is the basic constraint that is used in the OD matrix. Since the matrix is constrained from two sides i.e. from the destination side and also from the origin side, it is called a doubly constrained model. There might be cases where only either of total productions (origins), and total trip attractions (destinations) are known for base year as well as for target year. In such cases, the model is called a singly constrained model.

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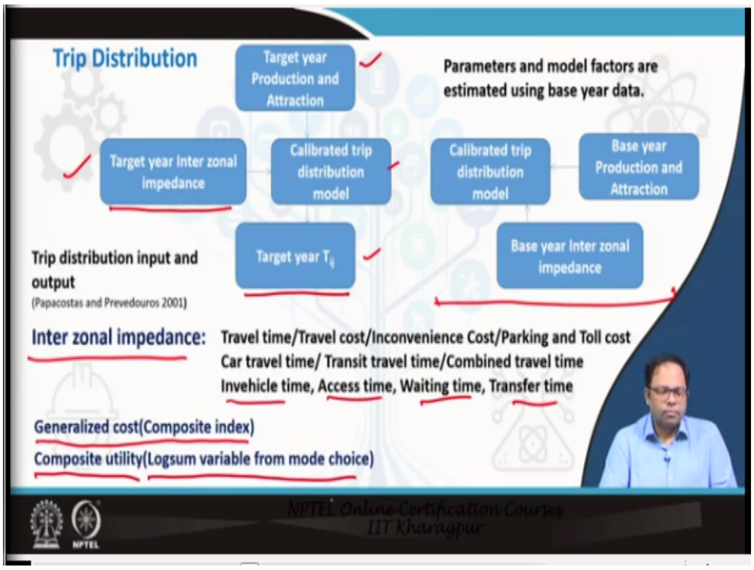
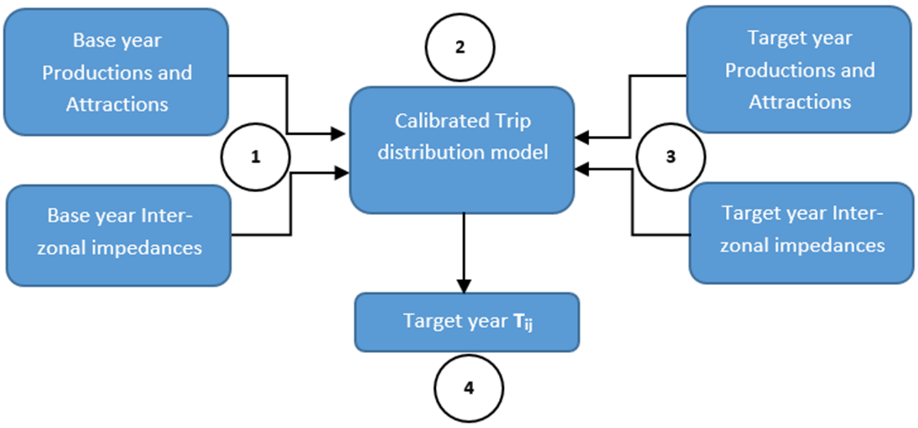



**Components of trip distribution:**




**Components of Trip Distribution**

While developing a trip distribution model the first objective is to calibrate the model. Calibrated trip distribution model is a set of equations, developed based on base year production and attraction values and base year inter zonal impedance. Following the fulfillment of this objective, the second objective is to use the target year productions and attractions, and target year inter-zonal impedances to finally get the $\mathbf{T_{ij}}$ values for target year. The figure shown above shows each component involved in the process of obtaining the $\mathbf{T_{ij}}$ values for target year.

Any kind of cost or impedance could be included in the inter zonal impedance functions that we use. These impedances can be travel time; travel cost; inconvenience cost; parking and toll costs, car travel time; transit travel time; combined travel time of different modes; in vehicle travel time; access time; waiting time; transfer time; etc. A generalized cost, which is a composite index which may include all the different cost parameters and gives one value of impedance for each inter-zonal flow. In other words, a generalized cost model gives a composite impedance value for each pair of zones **i** and **j**, where **i** represents origin zone and **j** represents destination zone.

Apart from generalized cost, the use of a composite utility, based on the available modes between a pair of zones is also found in composing the skim table. **Logsum** variable is used expressing the composite utility. It is the log of the sum of exponential of utilities of all the modes available between a pair of zones. The use of logsum can also be seen in discrete choice model, where there are several alternatives to choose from, the probability to choose an alternative is expressed as the ratio of exponential of the utility of the alternative, to the sum of exponential of all other alternatives.

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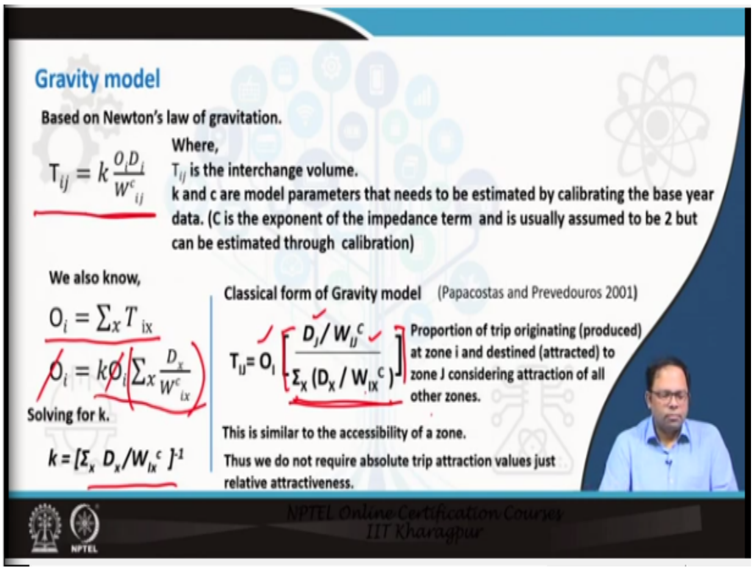


## Gravity model:

Gravity model is based on Newton's law of gravitation. The total number of trips $\mathbf{T_{ij}}$, which is also known as the interchange volume, is determined using the formula given below:

$$T_{ij} = k.\frac{O_i.D_i}{W_{ij}^c} \tag{1}$$

**k** and **c** are model parameters which are estimated by the process of calibration using base year data. '$\mathbf{W_{ij}}$' is the impedance between zone **i** and **j** and '**c**' is the exponent term of impedance, which is usually assumed to be 2 but can be also estimated through calibration. As already discussed, $\mathbf{O_i}$ is the sum of all the trips originating at **i**, and ending at all other zones. Replacing $\mathbf{T_{ix}}$ with the result from gravity model, the formula for '**k**' can be obtained.

$$O_i = \sum_x^n T_{ix} \tag{2}$$

***Substituting the equation (1) in equation (2);***

$$O_i = k.O_i \sum_x^n \frac{D_x}{W_{ix}^c} \tag{3}$$

$$k = \left[\sum_x^n \frac{D_x}{W_{ix}^c}\right]^{-1} \tag{4}$$

***Substituting the equation (4) in equation (1);***

$$\boldsymbol{T_{ij} = O_{i}.\left[\frac{D_j/W_{ij}^c}{\sum_x^n D_x/W_{ix}^c}\right]} \tag{5}$$

The **equation 5** given above is the classical form of the gravity model where number of trips from zone **i** to zone **j** ($\mathbf{T_{ij}}$) is derived as the ratio of the total number of trips ending at zone **j**, divided by the '**c**$^{\mathbf{th}}$' power of the inter-zonal impedance between zone i and j ($\boldsymbol{W_{ij}^c}$) to the sum of ratio of attractions to all the zones divided by their respective '**c**$^{\mathbf{th}}$' power of the inter-zonal impedance between zone i and all other zones ($\boldsymbol{\sum_x^n D_x/W_{ix}^c}$) and then multiplied by the total number of trips originating at zone **i**. So, $\mathbf{T_{ij}}$ a proportion of all trips originating at zone **i** and is dependent on the proportion of trips ending at zone **j** with respect to the total trips attracted to all the zones. These kind of measures, as already covered in other section, are used to evaluate accessibility of a zone as well where only the relative attractiveness of zones are required instead of the actual figures. So, the absolute values of trip generation, and the relative share of trip attractions are good enough to estimate the trip distributions.

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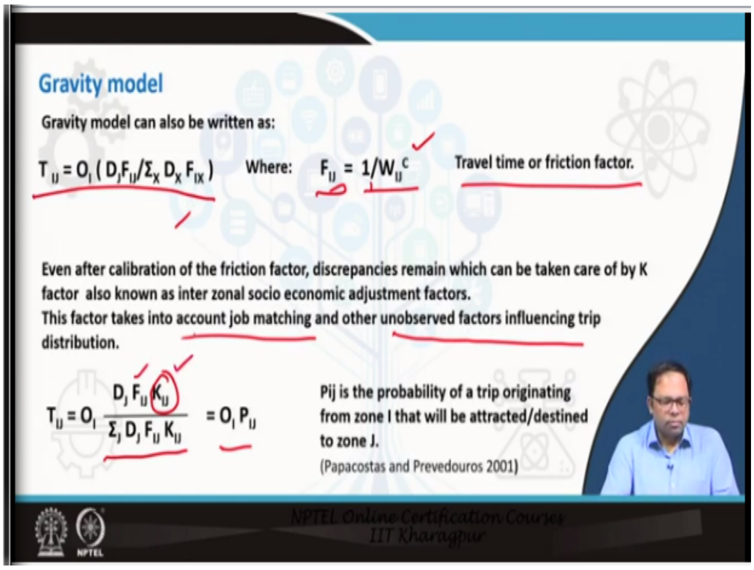


Gravity model can be re-written as follows:

$$\boldsymbol{T_{ij} = O_{i}.\left[\frac{D_jF_{ij}}{\sum_x^n D_xF_{ix}}\right];\ \text{Where } F_{ij} = \frac{1}{W_{ij}^c}} \tag{6}$$

**$F_{ij}$** is called the travel time or friction factor. Calibrating a gravity model or calibrating the value of '**c**' essentially means to calibrate this particular friction factor **$F_{ij}$**. Calibration is required because, starting the estimation with **c=2**, the total number of trips attracted and total number of trips generated will not essentially match. Calibration is done in order to match the total trips generated and total trips attracted.

Even after many iterations of calibration, the model may not have a condition where the number of trips attracted match with the number of trips generated. This might be due to the exclusion of some other factor (other than the impedance included) as trip distribution does not necessarily depend exclusively on inter-zonal impedances. In such cases, another factor **$K_{ij}$** is introduced in the model and calibrated like it is done with **$F_{ij}$**. For example, if general trip distribution is done for all work trips, a particular segment of employees and a particular segment of jobs is not identified. In that case there would be some differences from zone to zone. That makes it necessary to include another factor that would take care of such unobserved factors influencing trip distribution.

$$\boldsymbol{T_{ij} = O_i . \left[\frac{D_j F_{ij} K_{ij}}{\sum_x^n D_x F_{ix} K_{ix}}\right]} \tag{7}$$

$$\boldsymbol{T_{ij} = O_i . P_{ij}} \text{ ; Where } \boldsymbol{P_{ij} = \frac{D_j F_{ij} K_{ij}}{\sum_x^n D_x F_{ix} K_{ix}}} \tag{8}$$

The equation 7 shows the new factor **$K_{ij}$** alongside **$F_{ij}$** in the formula for number of trips originating at zone i and ending at zone j. The whole formula can be simplified and written as represented in equation 8, where **$P_{ij}$** represents the probability of a trip originating at zone i, and ending at zone j.

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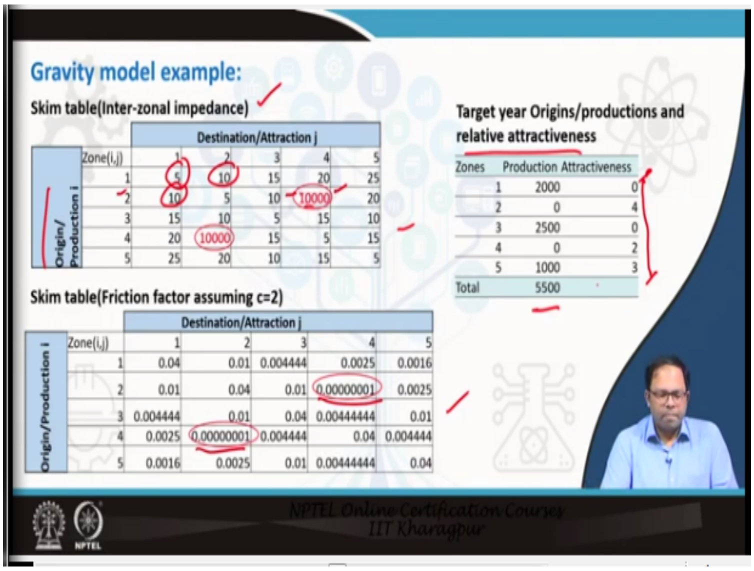


## Gravity model example:

| A. Zones | Productions | Attractions |
|---|---|---|
| **1** | 2000 | 0 |
| **2** | 0 | 4 |
| **3** | 2500 | 0 |
| **4** | 0 | 2 |
| **5** | 1000 | 3 |
| **Total** | **5500** | |

| B. Skim Table (inter-zonal impedances) | Destination/ Attraction | | | | | |
|---|---|---|---|---|---|---|
| | **Zones** | **1** | **2** | **3** | **4** | **5** |
| **Origin/ Production** | **1** | 5 | 10 | 15 | 20 | 25 |
| | **2** | 10 | 5 | 10 | 10000 | 20 |
| | **3** | 15 | 10 | 5 | 15 | 10 |
| | **4** | 20 | 10000 | 15 | 5 | 15 |
| | **5** | 5 | 25 | 20 | 10 | 5 |

| C. Skim Table (Friction factor; c=2) | Destination/ Attraction | | | | | |
|---|---|---|---|---|---|---|
| | **Zones** | **1** | **2** | **3** | **4** | **5** |
| **Origin/ Production** | **1** | 0.04 | 0.01 | 0.0044 | 0.0025 | 0.0016 |
| | **2** | 0.01 | 0.04 | 0.01 | 0.00000001 | 0.0025 |
| | **3** | 0.0044 | 0.01 | 0.04 | 0.0044 | 0.01 |
| | **4** | 0.0025 | 0.00000001 | 0.0044 | 0.04 | 0.0044 |
| | **5** | 0.0016 | 0.0025 | 0.01 | 0.0044 | 0.04 |

In the above example, there are three tables; A, B, and C having data related to five zones. Table A is the table of trip productions and attractions for each of the five zones 1,2,3,4,5. It can be seen that the zones 1 and 3 only produce trips; zones 2 and 4 only attracts trips; and zone 5 attracts as well as produces trips. The trip attractions are less in number as compared to the total productions because these are not absolute attractions but relative ones. Table B is the skim table of inter-zonal impedances between each pair of the 5 zones. These inter-zonal impedances could be anything; cost, time, etc. Among these values, for the pair of zone 2 and 4, in either direction the impedance is very high (1000). This implies that nobody travels between these two zones. Instead of putting 0, in order to maintain the consistency in estimation, an unrealistically high impedance value is set to ensure no trips are distributed between these two zones. Similar approach is taken in cases where network links are not available in between two zones to represent the pairwise inaccessibility. The table C has the friction factors which are estimated using the impedances. The estimates of the friction factor in table 'C' have been made assuming c = 2.

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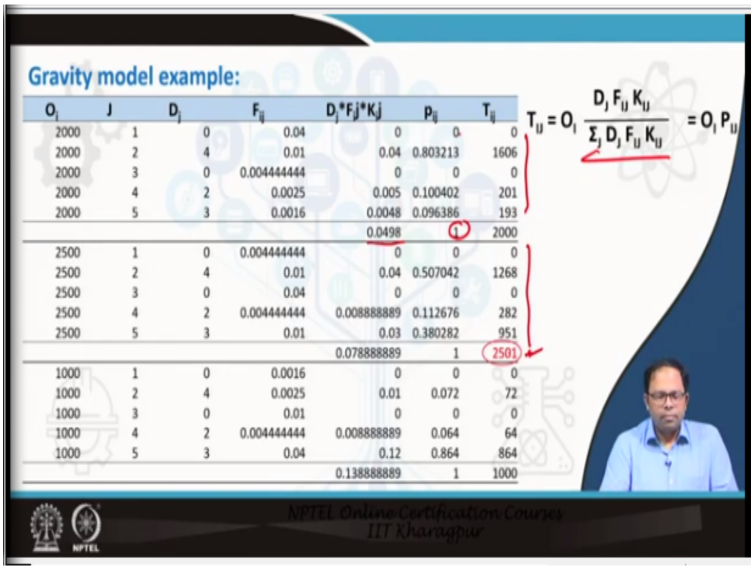


| $O_i$ | J | $D_j$ | $F_{ij}$ | $D_j{*}F_j{*}K_j$ | $p_{ij}$ | $T_{ij}$ |
|---|---|---|---|---|---|---|
| 2000 | 1 | 0 | 0.04 | 0 | 0 | 0 |
| 2000 | 2 | 4 | 0.01 | 0.04 | 0.803213 | 1606 |
| 2000 | 3 | 0 | 0.004444444 | 0 | 0 | 0 |
| 2000 | 4 | 2 | 0.0025 | 0.005 | 0.100402 | 201 |
| 2000 | 5 | 3 | 0.0016 | 0.0048 | 0.096386 | 193 |
| | | | | 0.0498 | 1 | 2000 |
| 2500 | 1 | 0 | 0.004444444 | 0 | 0 | 0 |
| 2500 | 2 | 4 | 0.01 | 0.04 | 0.507042 | 1268 |
| 2500 | 3 | 0 | 0.04 | 0 | 0 | 0 |
| 2500 | 4 | 2 | 0.004444444 | 0.008888889 | 0.112676 | 282 |
| 2500 | 5 | 3 | 0.01 | 0.03 | 0.380282 | 951 |
| | | | | 0.078888889 | 1 | 2501 |
| 1000 | 1 | 0 | 0.0016 | 0 | 0 | 0 |
| 1000 | 2 | 4 | 0.0025 | 0.01 | 0.072 | 72 |
| 1000 | 3 | 0 | 0.01 | 0 | 0 | 0 |
| 1000 | 4 | 2 | 0.004444444 | 0.008888889 | 0.064 | 64 |
| 1000 | 5 | 3 | 0.04 | 0.12 | 0.864 | 864 |
| | | | | 0.138888889 | 1 | 1000 |

In the table shown above, the trip distribution for the three production zones 1,3, and 5 are estimated. For each zone, there are seven columns; $\mathbf{O_i}$ represent the total zonal productions; **J** represents the zone to which the trips are being attracted to; $\mathbf{D_j}$ represents the relative attractiveness of the corresponding zones in column J; $\mathbf{F_{ij}}$ is the friction factor for travel between origin zone to corresponding destination zone; $\mathbf{D_jF_{ij}K_{ij}}$ represents the weighted distribution of trips from origin zone to each of the destination zones calculated by multiplying $\mathbf{D_j}$,$\mathbf{F_{ij}}$,and $\mathbf{K_{ij}}$ (here $\mathbf{K_{ij}}$=1); $\mathbf{P_{ij}}$

represents the probability of trips originating in the origin zone and ending at each destination zone calculated by dividing each $D_jF_{ij}K_{ij}$ by its column total $\Sigma D_jF_{ij}K_{ij}$; $T_{ij}$ is the number of trips originating in origin zone and ending at the destination zone, which is estimated by multiplying the total trip origins and the probability of trips ending at each destination zone.

After the table is estimated for all the three production zones, the sum of $T_{ij}$ values for each of these zones should give the total trips originating from the respective zones. But in case of zone 3, $\Sigma T_{ij}$ is 2501, but $O_i$ is 2500. The mismatch arises due to some rounding off at some stage in the calculation. In real scenario, there may be 300-400 zones and depending upon the stage at which rounding off is done, a small rounding off may result in very high values. Such errors must be avoided.

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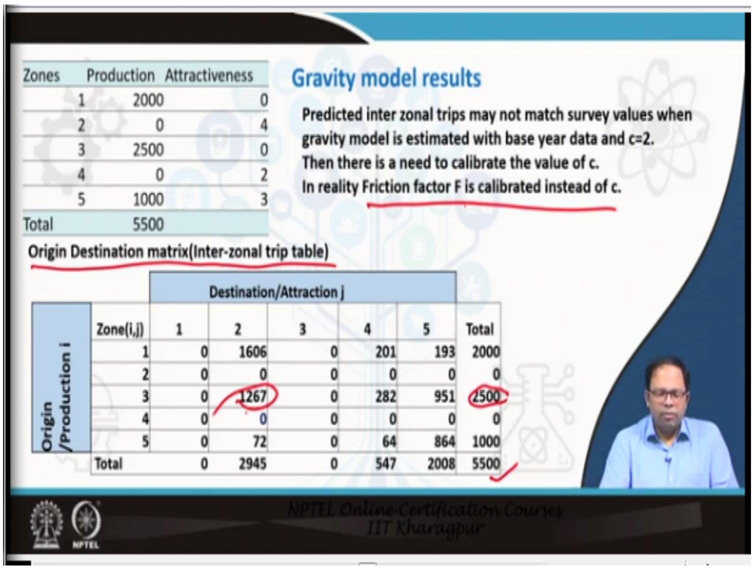


**Results of the gravity model example.**

After correcting the rounding up error in the estimation, the following OD matrix or the inter-zonal trip table has been generated.

| **D. Origin-Destination matrix** | **Destination/ Attraction** | | | | | | **Total** |
|---|---|---|---|---|---|---|---|
| **Origin/ Production** | **Zones** | **1** | **2** | **3** | **4** | **5** | |
| | **1** | 0 | 1606 | 0 | 201 | 193 | 2000 |
| | **2** | 0 | 2 | 0 | 0 | 0 | 0 |
| | **3** | 0 | 1267 | 0 | 282 | 951 | 2500 |
| | **4** | 0 | 0 | 0 | 0 | 0 | 0 |
| | **5** | 0 | 72 | 0 | 64 | 864 | 1000 |
| **Total** | | 0 | 2945 | 0 | 547 | 2008 | 5500 |

$\mathbf{T_{ij}}$ for zone 3 to zone 2 was reduced by 1 to 1267 in order to correct the rounding off error and to get the $\mathbf{O_i}$ value for zone 3 as 2500. Although the predicted inter-zonal trips match with the survey values for the target year in this example, for some other year it may not match since value of c=2 may not be appropriate. In such cases the value of **c** needs to be calibrated by actually calibrating the friction factor F instead.

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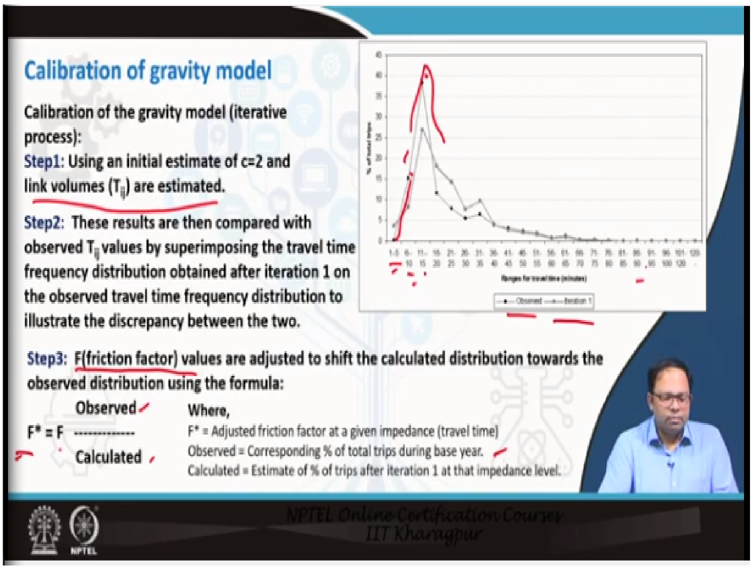


**Calibration of gravity model:**

Calibration of gravity model is an iterative process that has different steps. In the **first step**, based on the base year data on productions and attraction, estimation of link volumes or inter-zonal trips ($\mathbf{T_{ij}}$) is done by taking the value of c=2. In the **second step**, a histogram or distribution of trips is plotted by segregating the trips based on the travel time (or impedance to be more accurate) and the difference in the predicted and observed values are noted. For example, in the example shown, predicted (calculated) values of $\mathbf{T_{ij}}$ and the observed values of $\mathbf{T_{ij}}$ are plotted in the graph, with the travel times (1-5 minutes, 6-10 minutes, 11-15 minutes, etc.) on the x-axis. It is evident that there is a mismatch between the two plotted values for different travel times. The aim of the calibration process is to eradicate the differences in the predicted and observed values. This is done by adjusting the value of '**c**'. In order to do this, the values of fraction factor '**F**' is adjusted (or replaced) by a new factor '**F***' for each impedance range. So, for each travel time segment, new friction factors are estimated.

$$\boldsymbol{F^* = F.\left[\frac{Observed\,\%\,of\,trips\,in\,base\,year}{Estimated\,\%\,of\,trips\,after\,iteration}\right]} \qquad \mathbf{(9)}$$

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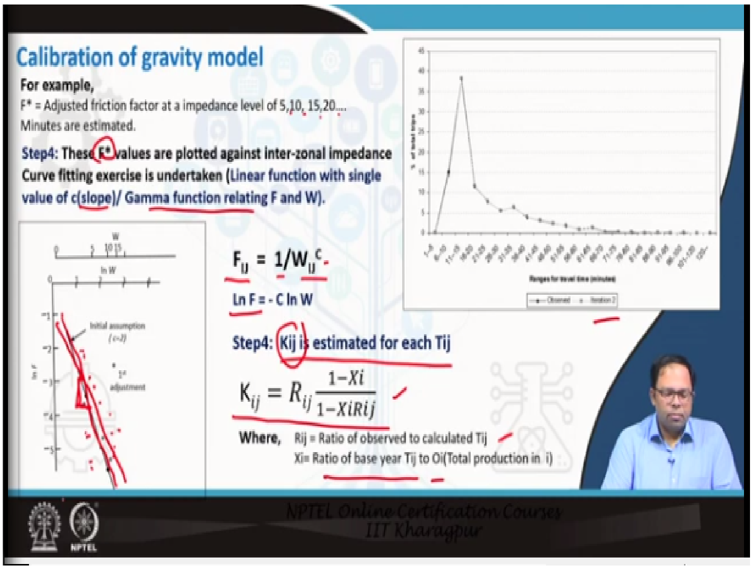


***From equation 4;***

$$\boldsymbol{F_{ij} = \frac{1}{W_{ij}^{c}}} \qquad \mathbf{(10)}$$

***taking natural logarithm on both sides;***

$$\boldsymbol{\ln F = c.\ln W} \qquad \mathbf{(11)}$$

In the **third step**, the F* values are plotted against the inter-zonal impedance W. Instead of directly plotting the two values, the equation is modified as shown in equation 11, and a plot is made with the values of '**ln F**' vs. '**c.lnW**'. As shown in the figure, a plot between the two assuming c=2, is supposed to be a straight line, but the values observed do not lie on the line entirely and is scattered in the space. A curve fitting exercise is undertaken to find the value of '**c**' for which the line '**c.lnW**' can explain the variability of the scatter plot in a better way. In other words, a suitable value of 'c' is found that can represent the observed values in a better way. The curve fitting exercise could be a linear function with a single slope like of **c**, or it could be something like a gamma function relating F and W.

Using the new values of 'F' the next iteration is undertaken, in which the trips are distributed again as done in step 1, consequently the values are plotted as described in step 2. In the example, the second iteration shows a perfect match in the data. For some other data, there may be a need to carry out a couple of more iterations like this. There may also be a requirement to include a new factor which can be undertaken as the **fourth step**. In this step, a new factor $\mathbf{K_{ij}}$ is estimated for each $\mathbf{T_{ij}}$ i.e. for each inter zonal volume. The following formula is used to estimate $\mathbf{K_{ij}}$.

$$K_{ij} = R_{ij}.\frac{1 - X_i}{1 - X_i R_{ij}} \qquad \begin{aligned} X_i &= \text{Ratio of base year } T_{ij} \text{ to } O_{ij} \\ R_{ij} &= \text{Ratio of observed to calculated } T_{ij} \end{aligned} \qquad \textbf{(12)}$$

Similar iterations may also follow the fourth step and this is how a trip distribution gravity model is calibrated.

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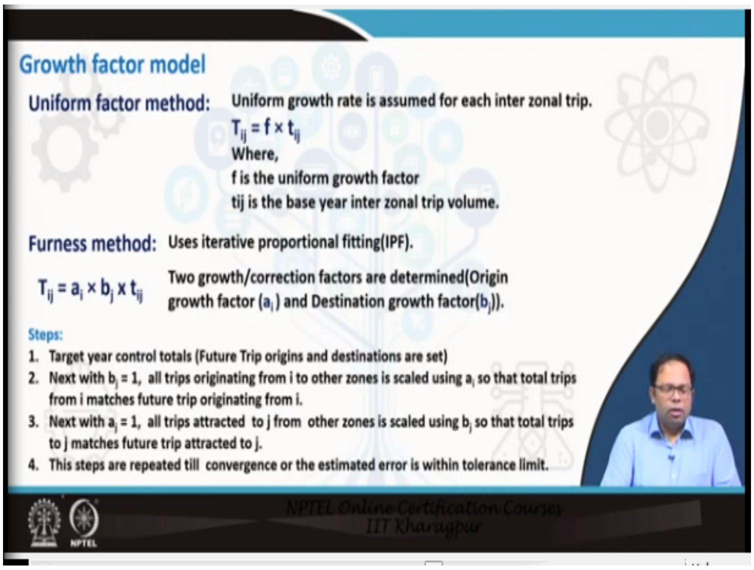

## Growth factor model:

There are many types of growth factor model. Some of them which are discussed in this section are; uniform growth factor model, Furness method, and Fratar model.

**Uniform growth factor model:**

$$T_{ij} = f.t_{ij}. \quad \begin{aligned} f &= Uniform\ growth\ factor \\ t_{ij} &= Inter-zonal\ trip\ volume\ of\ base\ year \end{aligned} \tag{13}$$

In uniform growth factor model, a constant growth factor '**f**' is multiplied to each of the base year inter-zonal trip volumes ($\mathbf{t_{ij}}$) to obtain the predicted inter-zonal trip volumes for the target year ($\mathbf{T_{ij}}$).

**Furness method:**

Furness method, which is a very popular method, uses iterative proportional fitting (**IPF**). In this method the base year OD matrix, and the target year control totals are already at hand. Control totals means the total number of trips produced and the total number of trips attracted by each zone.

$$T_{ij} = a_i.b_j.t_{ij}. \quad \begin{aligned} a_i &= Origin\ growth\ factor \\ b_j &= Destination\ growth\ factor \\ t_{ij} &= Inter-zonal\ trip\ volume\ of\ base\ year \end{aligned} \tag{14}$$

Based on the total trips generated by each zone, an origin growth factor '$\mathbf{a_i}$' is estimated by dividing the target year trip generation by the base year trip generation for each zone. Similarly, a destination growth factor '$\mathbf{b_j}$' is also estimated using the target year attractions and the base year attractions for each zone. This method has several steps which is performed iteratively. In the **first step** of the iteration, '$\mathbf{a_i}$' values for all the zones are estimated as the ratio of respective target year $\mathbf{O_i}$ to the respective base year $\mathbf{O_i}$. In the **second step**, assuming '$\mathbf{b_j}$' to be 1, the '$\mathbf{T_{ij}}$' values for all the zones are estimated using equation 14, to match the target year $\mathbf{O_i}$ for each zone. In the **third step**, the '$\mathbf{T_{ij}}$' values are used to estimate the zonal attractions, which now serves as the modified base year attractions. In the **fourth step**, '$\mathbf{b_i}$' values are estimated by dividing each of the target year zonal attractions totals by the modified zonal attraction totals. In the **fifth step**, each of the '$\mathbf{T_{ij}}$' values are re estimated using equation 14, assuming '$\mathbf{a_j}$' to be 1. In the **sixth step**, each of the modified '**T**$_{ij}$' values are used to obtain the total trips generated by each zone. In the **seventh step**, the gap between the target year generation and the generation obtained from the matrix after step six, for each zone, is estimated. If the differences are found to be within the tolerable range, the iteration is stopped and the values of the factors hence obtained are considered final. In case the differences are beyond the tolerable range, next iteration is undertaken starting with the first step through the seventh step, with the matrix obtained at the end of the sixth step in the previous iteration.

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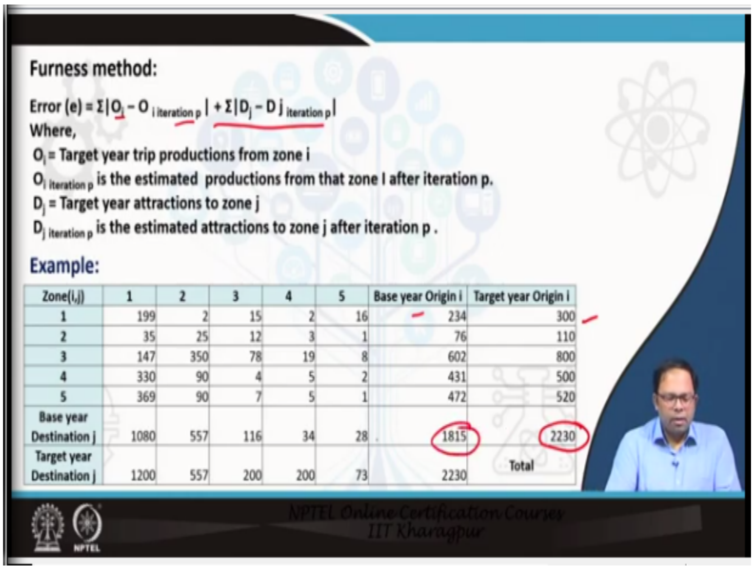


$$Error\ (e) = \sum |O_i - O_{i\ iteration\ P}| + \sum |D_j - D_{j\ iteration\ P}| \tag{15}$$

$O_i$ = *Target year trip production from zone i*

$O_{i\ iteration\ P}$ = *Estimated trip production from zone i in* $P^{th}$ *iteration*

$D_j$ = *Target year trip production from zone j*

$D_{j\ iteration\ P}$ = *Estimated trip production from zone j in* $P^{th}$ *iteration*

The error after each iteration is calculated using the formula shown in equation 15. The error is the addition of, the sum of the differences between target year trip origin totals and estimated trip origin totals after an iteration, for each of the zones; and the sum of the differences between target year trip attractions totals and estimated trip attraction totals after an iteration, for each of the zones.

In the given example, there is a dataset of trip distribution of five zones; 1,2,3,4, and 5, with the target year zonal origin and destination totals. The total number of trips in the base year is 1815, and that of the target year is 2230.

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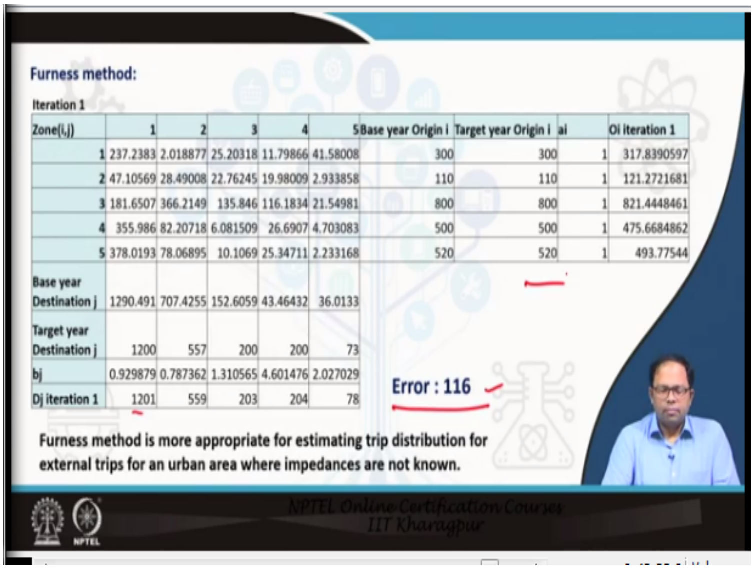


| Zone(i,j) | 1 | 2 | 3 | 4 | 5 | Base year Origin i | Target year Origin i | ai | Oi iteration 1 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 237.2383 | 2.018877 | 25.20318 | 11.79866 | 41.58008 | 300 | 300 | 1 | 317.8390597 |
| 2 | 47.10569 | 28.49008 | 22.76245 | 19.98009 | 2.933858 | 110 | 110 | 1 | 121.2721681 |
| 3 | 181.6507 | 366.2149 | 135.846 | 116.1834 | 21.54981 | 800 | 800 | 1 | 821.4448461 |
| 4 | 355.986 | 82.20718 | 6.081509 | 26.6907 | 4.703083 | 500 | 500 | 1 | 475.6684862 |
| 5 | 378.0193 | 78.06895 | 10.1069 | 25.34711 | 2.233168 | 520 | 520 | 1 | 493.77544 |
| Base year Destination j | 1290.491 | 707.4255 | 152.6059 | 43.46432 | 36.0133 | | | | |
| Target year Destination j | 1200 | 557 | 200 | 200 | 73 | | | | |
| bj | 0.929879 | 0.787362 | 1.310565 | 4.601476 | 2.027029 | | | | |
| Dj iteration 1 | 1201 | 559 | 203 | 204 | 78 | | | | |

Starting the first iteration, in the first step, '$\mathbf{a_i}$' values are estimated by as $\frac{300}{234}$; $\frac{110}{76}$; $\frac{800}{602}$; $\frac{500}{431}$; and $\frac{520}{472}$. In the second step, each of these '$\mathbf{a_i}$' values are multiplied by the $\mathbf{t_{ij}}$ values in their respective rows. In the third step, the new $\mathbf{T_{ij}}$ values are added column wise to get the new $\mathbf{D_j}$ values as 1290.491, 707.4255, 152.6059, 43.46432, and 36.0133. In the fourth step, '$\mathbf{b_j}$' values are estimated as $\frac{1200}{1290.491}$; $\frac{557}{707.4255}$; $\frac{200}{152.6059}$; $\frac{200}{43.46432}$; and $\frac{73}{36.01330}$. In the fifth step, these '$\mathbf{b_j}$' values are multiplied with the $\mathbf{T_{ij}}$ values in respective columns. In the sixth step, the row-wise sum gives the trip origin totals after the first iteration. The seventh step is the step for calculation of error, where equation 15 is used. The resultant error in the given example is 101. The iterations are carried out till a value of error is reached which is acceptable by the analyst.

Furness method is appropriate for estimating trip distribution for external trips i.e. trips coming from outside the area under consideration or for an urban area where impedances are not known. But the data for the target year productions and attractions must be obtained to be able to use this method.

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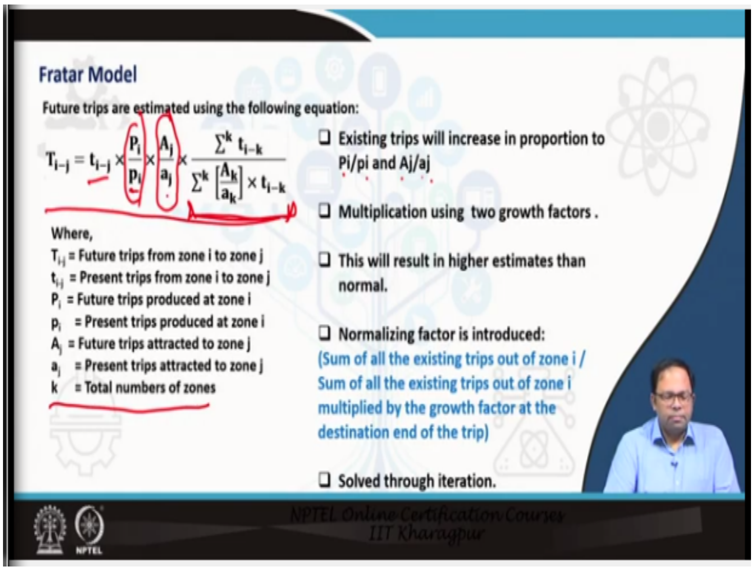


**Fratar model:**

Fratar model is another growth factor method which is also popular. In this model, future trips are estimated using the equation shown below.

$$T_{ij} = t_{ij} \times \frac{P_i}{p_i} \times \frac{A_j}{a_j} \times \frac{\sum_k t_{ik}}{\sum_k \left[\frac{A_k}{a_k}\right] \times t_{ik}} \tag{16}$$

$T_{ij}$ = *Future trips from zone i to zone j*

$t_{ij}$ = *Present trips from zone i to zone j*

$P_i$ = *Future trips produced at zone i*

$p_i$ = *Present trips produced at zone i*

$A_j$ = *Future trips attracted to zone j*

$a_j$ = *Present trips attracted to zone j*

$k$ = *Total number of zones*

Where, $\mathbf{T_{ij}}$ is the base year inter zonal trip volume between zone i to zone j; $\frac{P_i}{p_i}$ is the proportion of increase of trip generation with respect to existing trip generation for zone i. $\frac{A_j}{a_j}$ is the proportion of increase of trips attracted to zone j with respect to existing trip attraction of zone j. So, the two ratios described are a type of growth factor for generation and attraction respectively. If both of these growth factors are multiplied, the result will be higher than expected. So, a normalizing factor is introduced, which is the ratio of sum of all trips generated by zone 'i' in base year; to the sum of trips leaving zone i, each multiplied by the growth factor of the trips attracted to the destination zones. This method also requires iterations like the Furness method, till satisfactory results are achieved.

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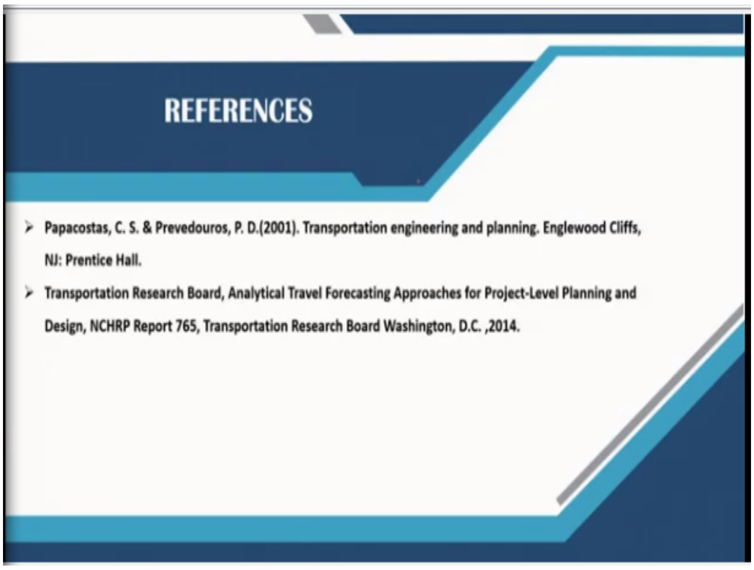


These are some references that can be looked into for further reading.

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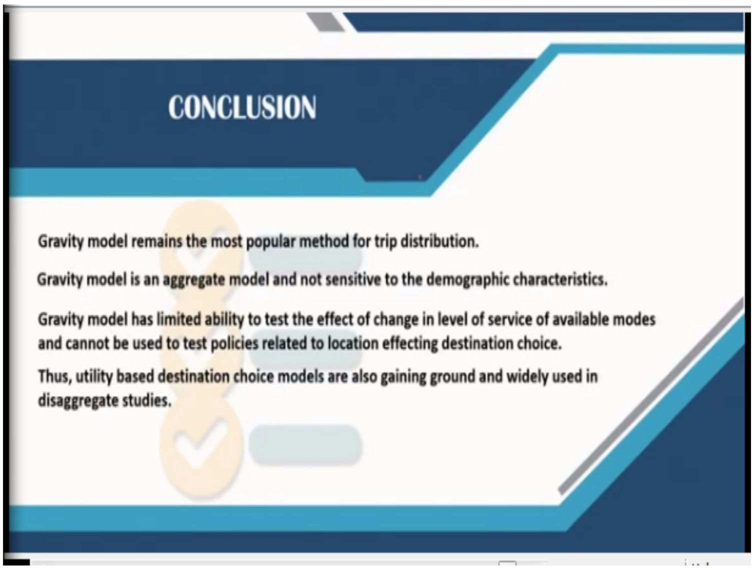


In the conclusion, gravity model remains the most popular method for trip distribution model. It is an aggregate model and not sensitive to demographic characteristics. In order to develop a model which is sensitive to socio-economic and other demographic characteristics, a destination choice model which is utility based is preferred. These kinds of model are gaining popularity in disaggregate choice modelling. Gravity model has limited ability to test the effect of change in level of service of available modes and cannot be used to test policies related to location effecting destination choice, which is a limitation of this model.