Mine Automation and Data Analytics

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Week - 8

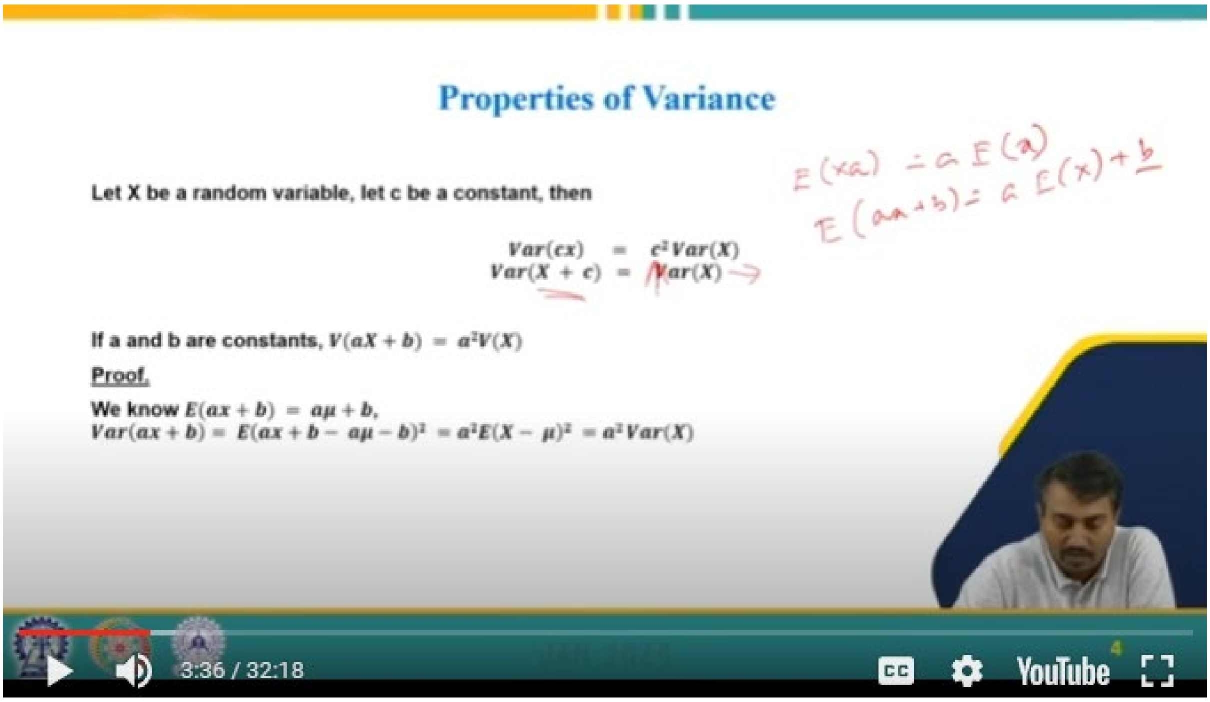
Lecture 38: Discrete Random Variable Part II

Welcome back to my course on mine automation and data analytics. Today, we will discuss the second part of the discrete random variable. In the first part, which is in 19a, we addressed the variance and then the expectations and will further progress on this concept. So, in this lesson, we will cover the following. We will discuss the properties of variance and the standard deviations and then start on the distributions, Bernoulli distribution, binomial distribution, and uniform distributions. So, let us see some of the properties.

In the last lesson, we have discussed some of the properties of expectation. For example, we have discussed that the expectation of ax plus b is equal to a into the expectation of x plus b like that or the expectation of x and y is expected x plus y is equal to the expectation of x plus y. Let us see whether those properties are also valid for the variance or not. So, here, the variance for the Cx is C, which is a constant.

So, when C is a constant, the variance is C square of the variance x. So, this is important to note. Earlier, the expectation of xa was equal to an expectation of a. The variance is square. This is the difference and the variance of earlier expectation of ax plus b into expectation x plus b.

So, for the variance, it is a constant. So, it is independent. Variance remains the same variance x. So, if a and b are constant, this same principle will apply. So, it would be a square variance of ax that is Vx.



So, the proof expectation of ax plus b equals a mu b. Mu is the expectation value of x. So, the variance of ax plus b is nothing but the expectation of ax plus b minus a u minus b whole square. So, here, b, b will be sorted out and will go out. So, a is standard x minus mu.

So, a is a constant, and that is multiplied; a square is coming out, and a square is multiplied. So, x minus mu square, this is the variance. So, this is nothing but a square variance of x. Variance of the sum of two random variables. The sum of random variables' expected value equals the sum of individual predicted values.

In other words, if x and y are two random variables, then the expectation of x plus y e of x plus y is equal to ex plus ey, which we have already seen. For the variance, the variance of x plus x var x plus x is equal to var of 2x, which is nothing but 2 with the square. So, four var x is not, var of x, and var of x is not. That is, four var of x is not equal to 2 var of x. So, is this statement always true in all cases? It is not.

So, for independent random variables, we have already discussed independent variables or independent events. So, random variables x and y are considered to be independent if the knowledge of the value of one of them does not alter the probability associated with the others. It is similar to the independent event. So, one event is unrelated to another, or the outcome of one event does not influence the other. So, here, for the random variables, even random variables, it does not alter the probability of different random variables based on the change of the other variables.

So, rolling a die, the sample space is 1, 1 up to 6, 6. There are 36 spaces, starting from 1, 1 to 6, 6. When a die is rolled twice, this is the number of samples, so a total of 36. Now, x is the outcome of the first dice. Then, y is the outcome of the second dice.

So, now discuss that knowing x or xi does not change the probability of y taking any value of 1, 2, 3, 4, 5, 6, which means three is coming in one rolling die case. For the second, it is not dependent at all. So, this can be considered independent random variables. So, x and y are independent random variables. So, for independent random variables, the variance of x plus y, var x plus y equals var of x and y, the same as expected.

So, this is an important consideration we have to remember. We are now rolling a die twice. So, let x present the outcome of one fair die and y present the result of another. We observe that the expectation of x and y is equal to 3.5 because, for equally likely events, we have seen the expectation is n plus one by 2.

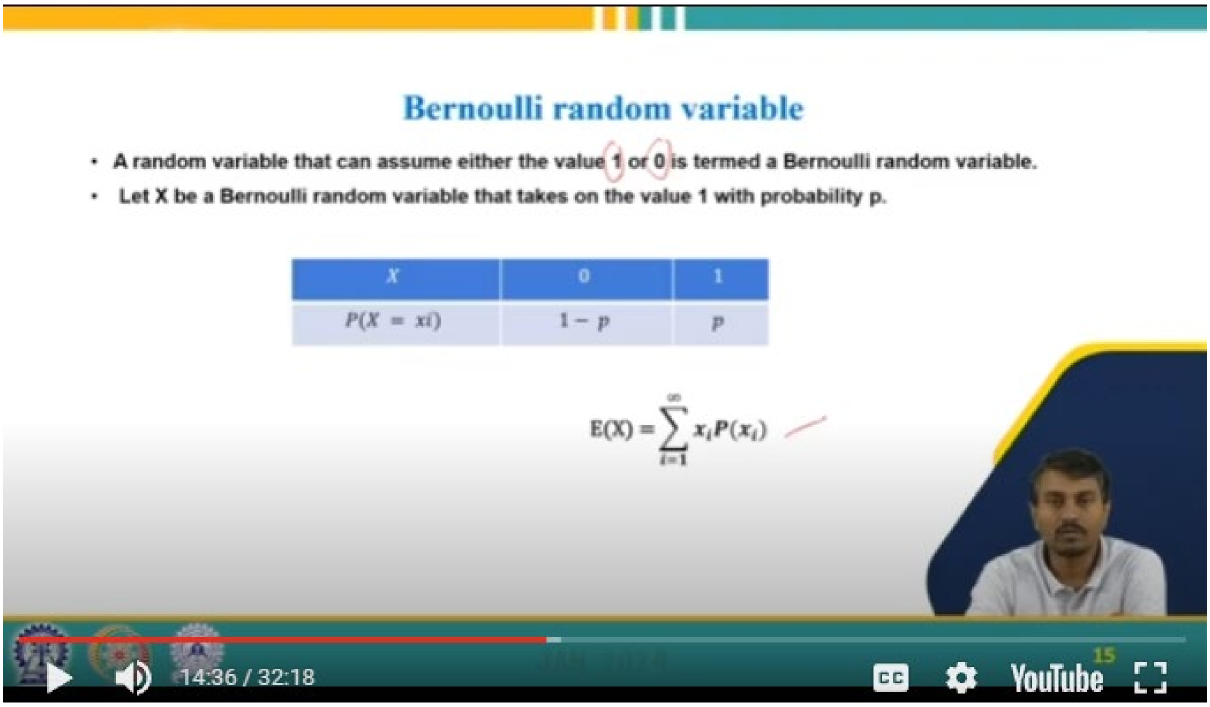
So, 6 plus 1 divided by two is 3.5. The sum of the outcome of both dice rolled together as donated x plus y is calculated to have that variance x, and y is 2.9 based on the expectation x minus mu formula. Since x and y are independent, we compute the var of x plus y, which is similar to the var of x and var of y, which is 5.

83, which aligns with the result obtained by applying the computational formula. The variance of the sum of many independent random variables. So, that variance of x plus y is equal to that of x plus y for the independent event, or independent random variables is applicable for several independent random variables. So, the outcome stating the variance of the sum of independent random variables equals the sum of variance applies not only to 2 but to any number of random variables. So, let us see x1, x2 up to xk is the discrete random variable.

Then the variance of xi in i is from 1 to k summation is equal to the variance of xi summation of 1 to k variance of xi 1, x2, x2 up to xk—standard deviation of random variables. So, the quantity standard deviation of x s dx equals the root over of variance x. We must consider the positive square root of the variance as a standard deviation for the random variable. Similar to the expected value, the standard deviation is expressed in the same unit as a random variable—properties of standard deviation.

So, let x be a random variable, c be a constant, then the standard deviation of cx, c is a constant. It is coming out similar to the expectation c into the standard of x, the standard deviation of x. For the standard deviation of x plus c, c is a constant. So, that is negated. So, the standard deviation of x plus c equals the standard deviation of x.

Bernoulli's distribution is the Bernoulli trial. A trial or experiment where the outcome can be categorized as either success or failure is referred to as a Bernoulli trial. So, the sample space is all about success and failure. So, when this kind of categorization exists, it is a Bernoulli trial. So, let x represent a random variable that takes the value one if the outcome is a success and 0 if the result is a failure.



So, x is referred to as Bernoulli random variables. Let us see the example. I was tossing a coin head and tail. So, success is the head, for instance, and failure is the tail. Experiment 2, rolling a die, 1, 2, 3, 4, 5, 6 is a sample space.

So, getting a 6 is a success. So, the probability is one by 6, and failure getting another number is five by 6. Any other 1, 2, 3, 4, or 5 is not a success but a failure—Non-Bernoulli trial. It is selecting a person at random and inquiring about their age. Does it fall under the Bernoulli trial? It will come out differently whenever you choose a person to inquire about their age.

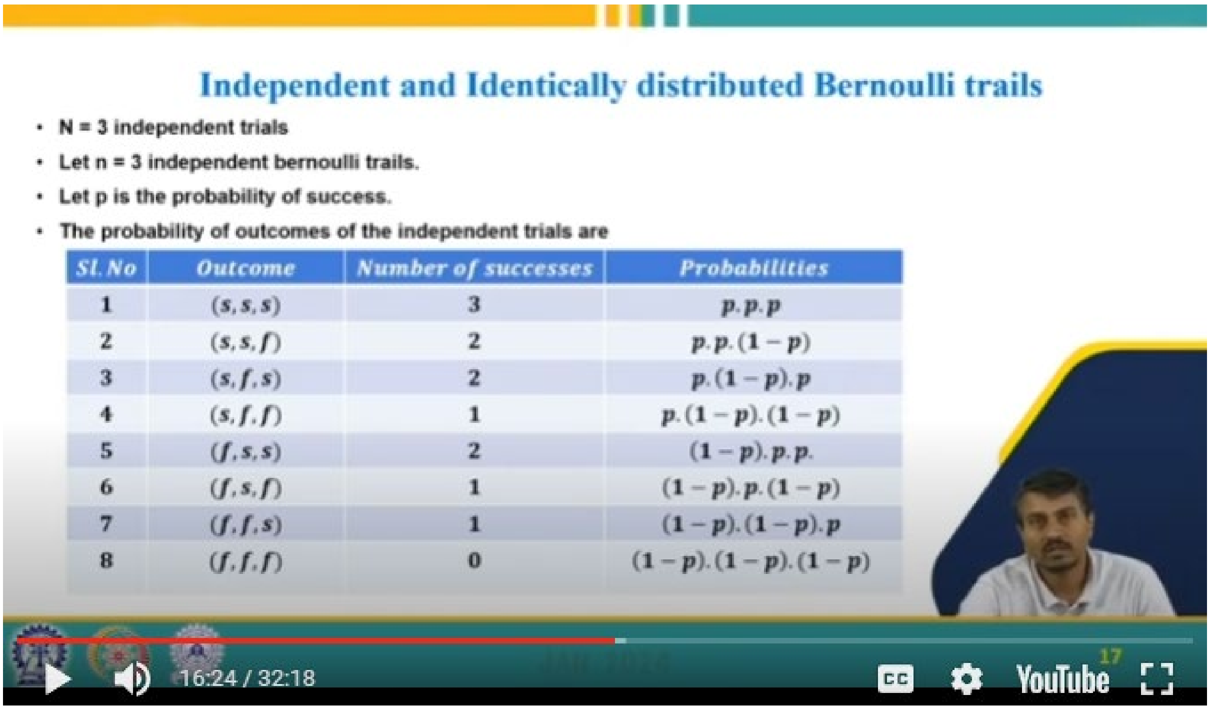
There is no success or failure kind of outcome will come. Only two outcomes will not come. So, this experiment does not qualify as a Bernoulli trial because it does not entail only two possible outcomes. Now, Bernoulli random variables. A random variable that can assume either value 1 or 0 is termed a Bernoulli random variable.

So, x is 0 or 1, and the probability for 1 is p. For 0, the probability is one minus p. An expectation we have already seen that expectation e of x is equal to xi p xi summing over i is equal to 1 to infinity. So, x is a Bernoulli random variable that takes on the value 1 with a probability of p, and the probability distribution of the random variable is as follows. The expectation is this. So, the expectation is 0 into one minus p, and one into p equals p.

So, for a Bernoulli random variable, the expectation is p. For variance, it is p minus p square, p minus p square. So, that is p into one minus p. Variance of Bernoulli distribution. The maximum variance occurs when p equals half when success and failure are equally likely.

So, in simpler terms, the most uncertain Bernoulli trials are characterized by the most significant variance resembling the tossing of a fair coin. Independent and identically distributed Bernoulli trial, n equals three independent trials. So, p is the probability of success, and c is the outcome. Possibly success, success-success, success-success, failure, success-failure, success-failure, success-success, failure-success, failure-failure, failure-failure. These are the possible outcomes of the independent events.

So, several successes here are 3, the number of successes is 0, several successes are 1, several successes are 1, several successes are 2, several successes are 1, and the number of successes is 2, 2. So now, these are the probabilities you can calculate because the likelihood of success is P. So success P is success success success P P P P Q success failure P into one minus P for a mutually independent event we know the probability. Based on that, we can now summarize the number of successes as 0, 1, 2, and 3.

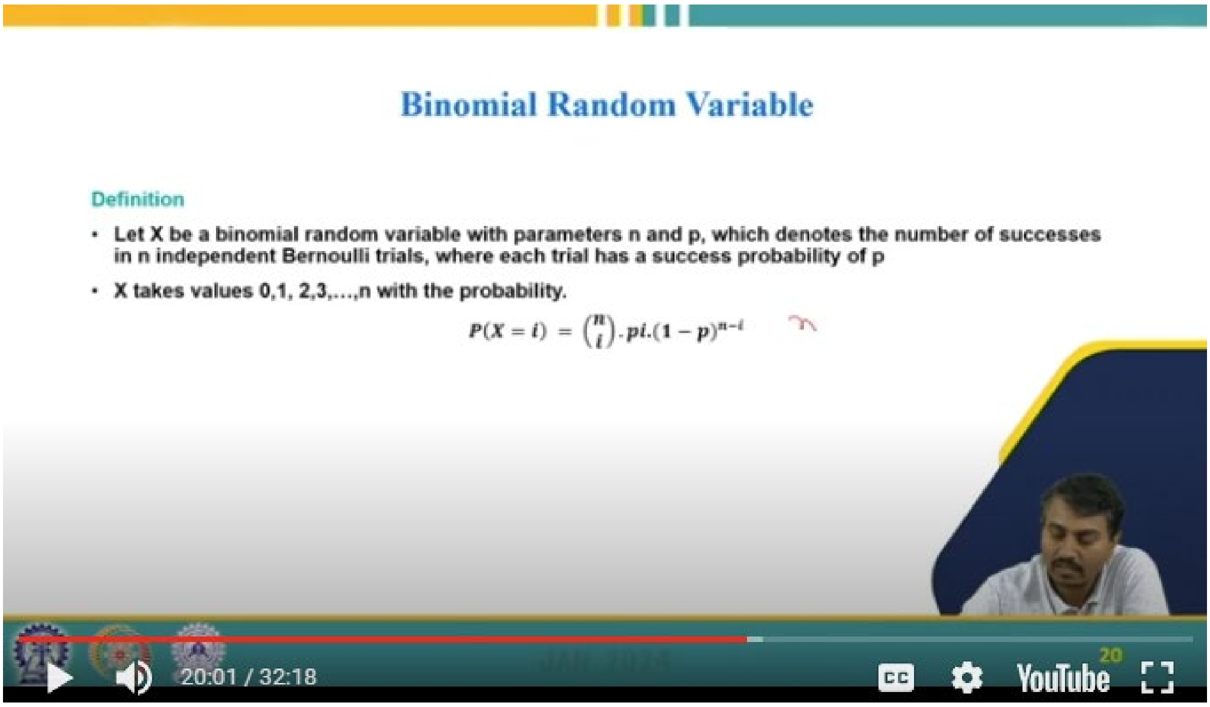


Then we can prepare another table. So, n independent trials that are three, P is the probability of success, and x is equal to several achievements in 3 independent trials. So summing over that 0 success is one minus P to the power cube to the remains that is cube root, cube degree that is 3 degrees, and the 3 number of success three that is three times success coming consecutively that is P cube. So here it shows some distribution. So now you want to fit that in a distribution and see some fantastic results. So n independent trials x is equal to the number of successes.

So consider any outcome that results in total success. So the outcome will be a total of I success and n minus I failure. Similarly, the probability of I success and n minus I failure is P to the power I into one minus P n minus I because both are independent events. So many different outcomes result in success, and n minus one failure is equal to n c i. So the probability of I success in n trials is that x i equals n c I into P to the power I and one minus P n to the power n minus i.

$$P(X = i) = \binom{n}{i}.\, pi.\, (1 - p)^{n-i}$$

So n c i P to the power i one minus P to the power n minus i. Bernoulli binomial random variables. So let X be a binomial random variable with parameters n and P, which denotes the number of successes in n independent Bernoulli trials where each trial has a success probability of P. So X takes the value of 0, 1, 2, 3, n with the likelihood that you can distribute. So P of x i equals n c i P to the power i into one minus P to the power n minus i.



So, let us take the example of tossing a coin thrice. So this example we have already seen. So here, success is the head, and failure is the tail. So X is the random variable that counts the number of heads in a toss in the tosses.

So n is equal to 3, and probability is always half. So now calculate that. 0, n c 0 that is half to the power 0 into one minus half is equal to 1 minus half is equal to half to the power n minus 1, n minus i is equal to 3 because n is 3, i is equal to 0. Similarly, this is the same value: one by

8, 3 by 8, 3 by 8, and 1 by 8. So, this binomial distribution fits the probability mass function we have already calculated.

The shape of the PMF for the same n and different P. So, for P less than half, it is right-skewed. For P, which is equal to half, it is symmetrical, and P, greater than 5, is left-skewed. So we demonstrate that the same for n equals four and different P.

So let us see. So, if P is equal to 0.3, it is right-skewed. The number n is also less; n is equal to 4. For n is equal to 4, but P is equal to 0.5 half; this is more or less symmetrical, it is symmetrical.

For n is equal to 4, but P is equal to 0.8, it is left-skewed. So, these are essential properties of the binomial distribution. When P is more significant than 0.5, it is always left skewed. For the larger n, we will see again with the P less than 0.

5, P is equal to 0.5 and more significant than 0.5. So here, n is large, and n equals 40, 40 times this experiment is conducted. So here it is right-skewed, but the distribution looks regular or symmetrical for larger values. But it is right skewed because the right side is a tail. For the symmetrical, larger n, P is equal to 0.5, and for the left skewed, larger n and more significant P greater than 0.

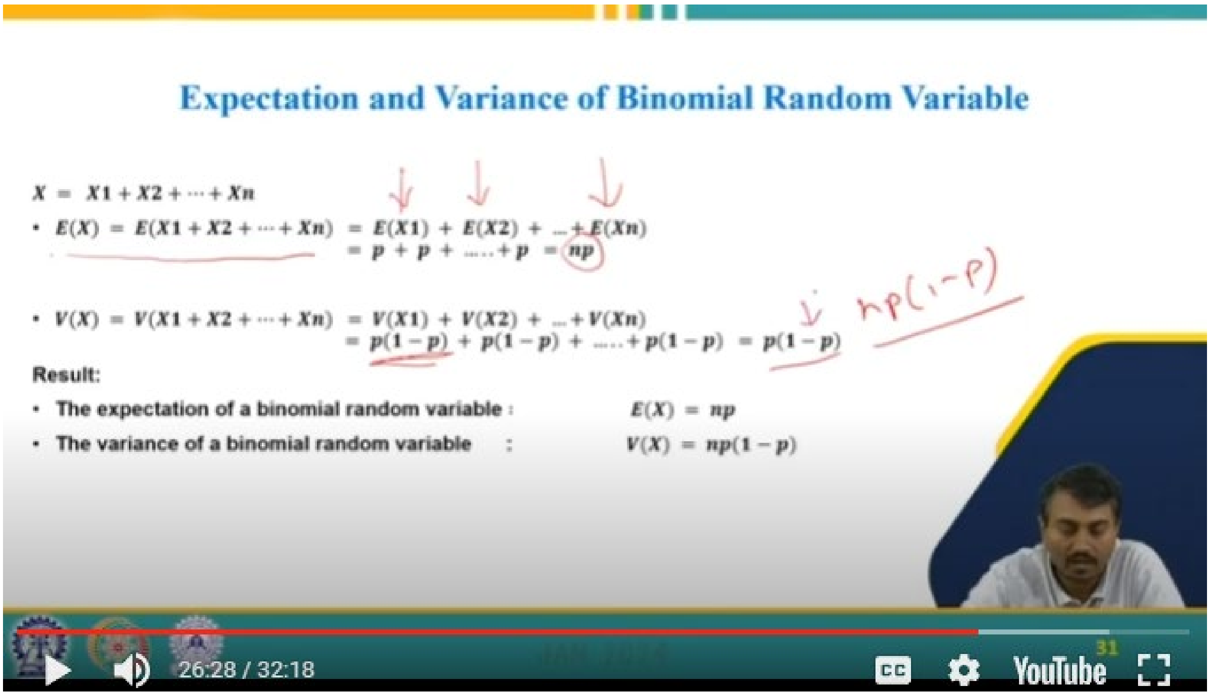
5. It is a left skewed. So, the effect of n and P on the shape of the distribution is that for small n and small p, the distribution is right skewed. For small n and large P, the distribution is left-skewed. The distribution is symmetric for small n, and P equals 0.5. As n becomes large, the binomial distribution tends towards symmetry.

So, what is the expectation and variance of a binomial random variable? So binomial random variables represented by X bin n, P, this is the representation, equals the number of success in an n independent trials when each trial is booming with the probability of P. And we represent X of xi, x1, x2, x3 up to xn. So where xi is equal to 1 if trial i is a success and is equal to 0 if trial i is a failure. So the probability of xi is equal to 1 is equal to P, and the probability xi is equal to 0 is equal to 1 minus P.

The expectation of a binomial random variable : $\boldsymbol{E(X) = np}$

The variance of a binomial random variable : $\boldsymbol{V(X) = np(1-p)}$

So, 0 is a failure, and 1 is a success. So, for that, we will calculate the variance and expectation. So X is equal to x1, x2 up to xn. So, the expectation is summing over this expectation. So the expectation for each is P, and then there is n, so this is total nP, the expectation of X. So variance is the variance of x1, x2, x3; the variance is P into one minus P.

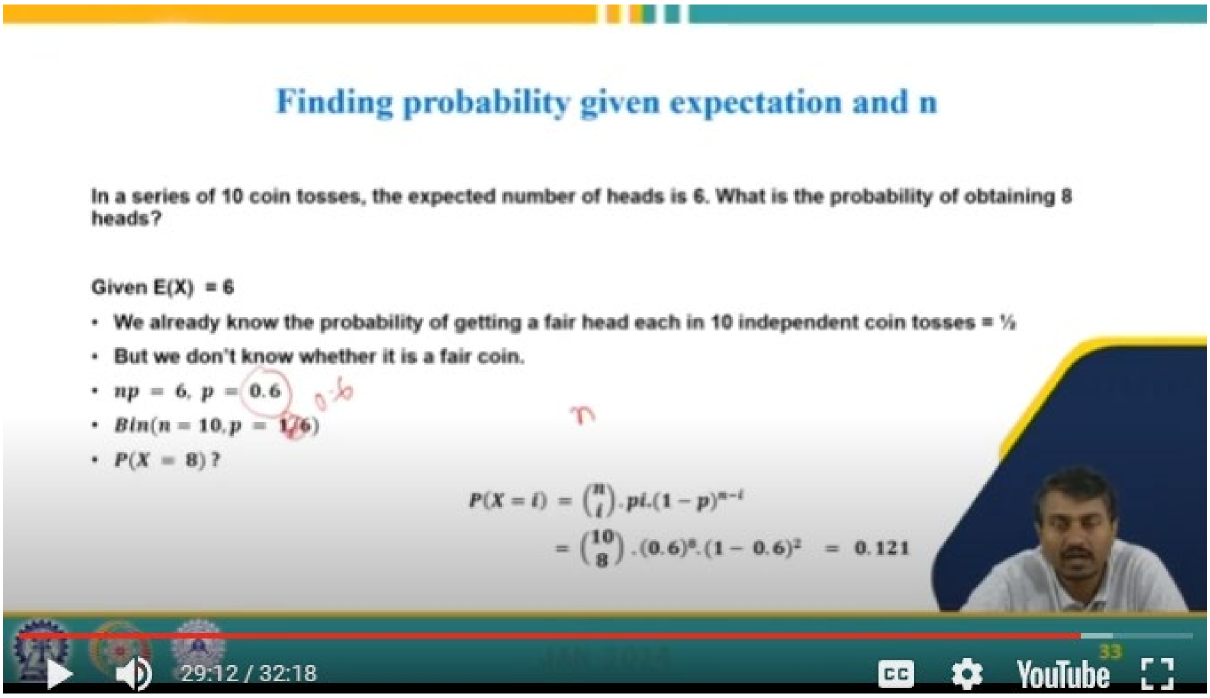


So the total would be nP into one minus P; this is a correction. So the expectation of a binomial random variable Ex is equal to nP, and the variance of a binomial random variable is nP into one minus P. Example: tossing a coin 500 times. So, if a fair coin is tossed 500 times, what is the standard deviation of the number of times a head appears? So x is the number of heads for 5000 coins tosses. It is 500, so it should be 500. Then, X of the binomial distribution is 500 half because it is equally likely half.

So the expectation is nP, n into P, this is 2500, variance nP into one minus P that is 1250. Standard deviation is the root over of variance positive square root of the variance, so that is 35.35. It is finding probability given expectation and n.

So, the expected number of heads in a series of 10 coin toss is 6. What is the probability of obtaining eight heads? So, given that Ex is equal to 6, we already know the likelihood of getting a fair head each in 10 independent coin tosses, which is half. But we need to find out whether it is a fair coin. So, nP is equal to 6, and P is equal to 0.

6. So the n binomial equals 10, and P equals 0.6; this would be 0.6. n equal to 10, and P is equal to 0.6. So, what is the probability that X is equal to 8? So X equals 8, but nCi, n is the 10, and nCi equals 10C8 and 0.



Six is to the power 8, and 1 minus 0.6 to the power ten minus 8 equals 2, which is square. So that is coming to 0.121. Uniform random variables. A discrete uniform random variable is a type of random variable that characterizes the outcome of a finite set of equal probable values.

In essence, it indicates a scenario where each potential outcome has an identical likelihood of occurrences. The term uniform is employed because the probability is uniformly distributed across the range of possible values—uniform random variable. So, let X be equal to a random variable equally likely to take any values 1, 2, or 3 up to n. So the probability mass function will be listed like this X of 1, 2, 3, n, the probability is one by n, one by n, one by n for all.

So the expectation is n plus one by 2, and the expectation of X square is n plus one into two n plus one by 6. So, the variance is n square minus one divided by 12. So, this is a critical observation variance for the uniform random variables. Example of uniform random variables.

So, let us take a fair six-sided die as an example. When rolling the die, the potential outcomes are 1, 2, 3, 4, 5, and 6. So because each face has an equal probability of landing face-off, the random variable representing the outcome of the die roll adheres to a discrete random, discrete uniform distribution with n equal to 6. The probability of obtaining any specific number, such as 3, equals one by 6. Example 2. Another instance is selecting a card randomly from a thoroughly shuffled standard deck of 52 cards, each with an equal probability of 1 by 52.

So these are the references and the things we have covered. So, we have covered the properties of variance and standard deviation. We discussed the Bernoulli distribution, then the binomial distribution, and the uniform distribution. Thank you.