Mine Automation and Data Analytics

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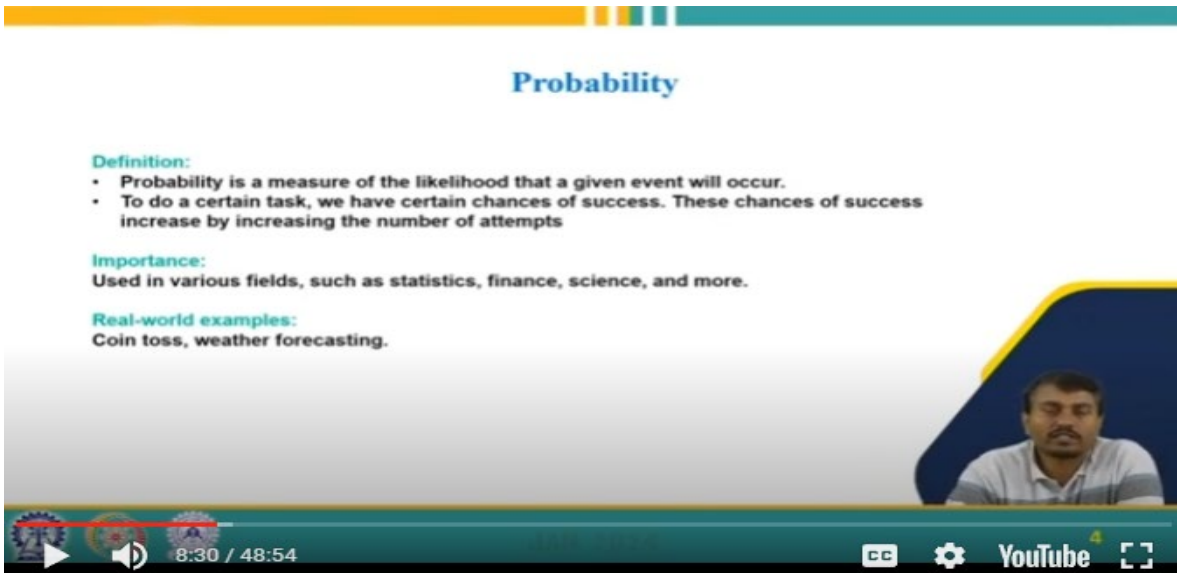
Week - 07

Lecture 35: Introduction to Probability and its associated terms

Welcome back to my course in mine automation and data analytics. Today, in this lesson, we are going to discuss a new concept: the concept of probability. This is a beneficial concept, and it would be used for different process estimations to understand the sequence of processes or a particular event in a streamlined flow of processes. Understanding this probability concept would be beneficial in terms of predicting accidents, predicting the maintenance time or maintenance frequency of the machines, and other situations that, from time to time, require management attention in the mine, mainly when different sensors are working and different sensors sometimes some of the sensors might malfunctions. So these are random events, and if we observe this kind of phenomenon for a more extended period, we will find a similarity in the occurrence of this kind of error in the sensor or tools in the parts of the machine. So, these are all mathematical terms we have to define and understand.

That is why the concept of probability is required for the mining engineers to know how to run the mine efficiently and optimize different processes involved in the mine. So, in this lesson, we will cover the basic terms of probability. We have to know many terms, and we have to habituate the different terms. You have to know the meaning of different terms that would be useful for us. What are the types of probability we have to understand? Conditional probability, marginal probability, and joint probability are the fundamental concepts of probability that we have to understand.



So, what is probability? Probability is a measure of the likelihood of a given event that will occur. For example, there is data that for the last five years, the number of accidents in a mine was a reportable accident. Three accidents occur a reportable accident. So, in five years, you can expect the accident frequency earlier than five years ago, and based on that next five years, you can be optimistic that we will take action. We will take some corrective action, and we can avoid that. Still, it is tough to predict when the accident will happen, but this is the general frequency of the accident in the mine. So it is a chance, so it is an estimation of chances precisely predicting would be difficult, but chances we can expect and we can understand and for a similarly for the machine also when the machine will be a breakdown that is very difficult to predict but when several machines are working in a mine site and based on the database of the last few years of working in similar conditions that breakdowns occur maybe in six months in a six month one breakdown occurs, for example, this is the frequency so you are expecting that in the next six month, one breakdown is expected okay one or more or less than that like that. So this is, but which machine will break down? This is very difficult to say, but this understanding will help the miners and the management to take some corrective action so that we can avoid that or defer that based on the corrective actions.

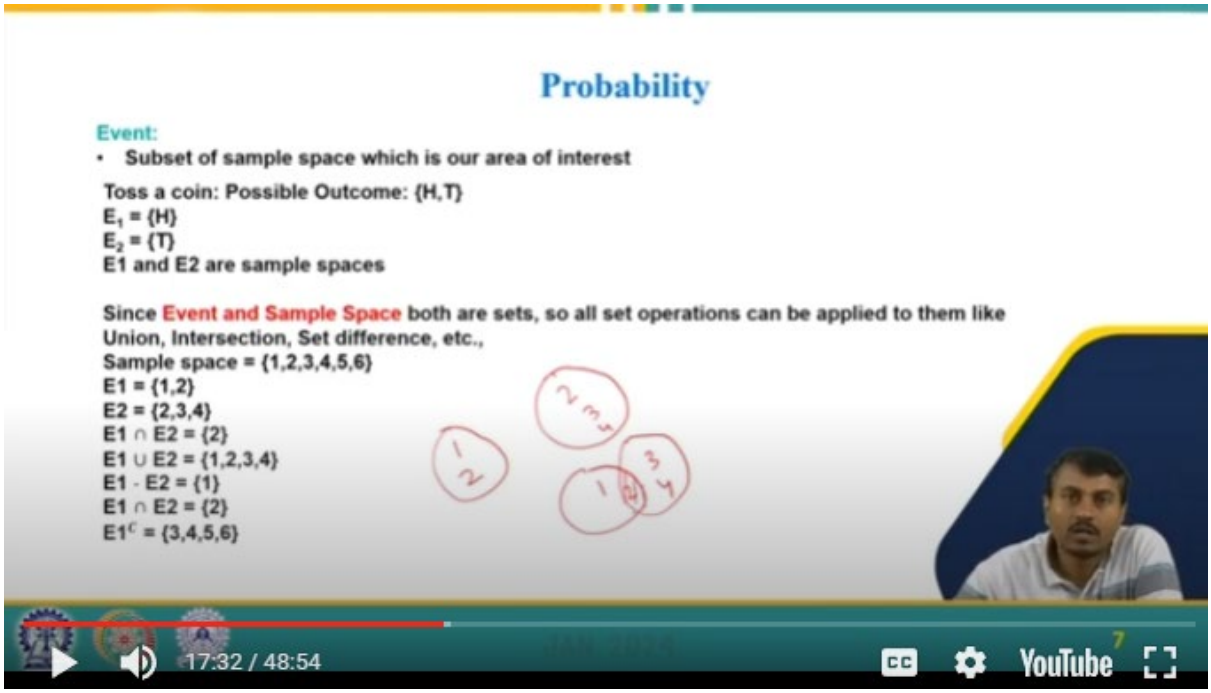
So, these exercises to understand this process are essential to running the mine smoothly. So, we have certain chances of success in doing a specific task. For example, we are working on giving a good amount of support or an adequate amount of support and trying to avoid root falls okay, but even then, some small chunk of rock falling at the floor may happen but with the increased effort and increased supervision and erecting proper and suitable amount of support system might result in a lesser frequency of root fall on the floor. So, this is a chance phenomenon so we can play on that. We can use these chance factors to understand the health of the mines situation of the mines in a better way. These particular mathematics of chances calculating the chances of probability are constructive, and in finance calculations and the scientific method estimation or the science scientific field, probability has a considerable role to play, and it is playing.

So in this particular lesson we will try to give you some of the famous example that we basically follow tossing a coin or rolling a dice okay those are the very famous example that it is very easy for all of us to understand yes how the event are randomly occurring and what are the chances that these event will happen like that and one thing that another thing nowadays you all might have seen in the in the smartphone that it is giving a prediction that there is a 70% chances of rainfall today but you observe that rainfall was not occurring okay it happens but sometimes it also happens that there are 60% chances of rainfall and rainfall is occurring so if you consider over a more extended period of time this data then you will understand okay in these kind of similar situations the chances of the rainfall is 70% not strictly for that day but over a period of time for a more extended period of time if you observe the data and match it will it will pretty match with the overall curve of the nature of the rainfall data. This is valuable information, and nowadays, particularly weather forecasting in agriculture plays a significant role, so ordinary people also now understand the usefulness of this probability based on this particular prediction of the weather forecasting data.

So similarly, mining is a big engineering field, and we have abundant scope to utilize this science to understand the process in the mining sector better. So, probability lets us see that we know the possible outcome but do not know the exact outcome. That is the fundamental notion of probability. We can say that this is possible, this is possible, or this is not possible, but exactly telling it is it is not it is not deterministic. That is why there is a difference between the deterministic and probabilistic approaches.

From the Newtonian approach we know that yes if the velocity of a of a car is 5 kilometer per hour after an 1 hour it will reach 5 kilometer away from the point of origin origin it will reach because it is a deterministic one v into t this basically that displacement it take in one hour time but there may be the possibility that this is a static phenomena this is similarly in the mining case event that the I know that this is the dumper speed and it will reach this this amount of distance in this amount of time this is a deterministic approach but in a real case scenario in a real case scenario what will happen it has been observed that during the travel of that vehicle some accident occur in that particular route so it would it would disturb the track or the travel of that particular vehicle in that particular route or there might be some amount of rock fall occur on that particular haul route so that is been obstructed the travel of that particular car through that it may it may happens or it may happen that suddenly there was a brake failure on the vehicle so there are possible chances though in a real case scenario you have to consider all these conditions so when you consider all these conditions you are not very sure that if the velocity the speed of the car is 5 kilometer per hour okay it will certainly reach 5 kilometer in one hour

you can say it possibly but there are chances that it might not able to reach 5 kilometer exactly considering all this factor so and mining is all about different unprecedented event or some kind of unwanted event may occur and certain kind of things that is not under control may happen because we are working in the nature so considering this situation it is very very well thing that we have to consider all these consideration all these parameters and then based on that predicting the possibility of reaching this particular target step by step like that so that is a more scientific way to understand and to execute the plan of operations and that is why it is it is required for the mining engineers to understand the concept of probability and understanding different factors how it is impacting the other factors there are dependency there are there are some event that is that is mutually exclusive not dependent there are some event that is dependent on each other so considering all these things it is an wise thing that mining engineers should also understand the concept of probability



So let us see some example they are tossing a coin possible outcome is head and tail what is the exact outcome it is very difficult whether it would be head or tail yes it will be head it is very difficult to say but there are possibility that head may come or tail may come rolling a dice there are six outcome possible one two three four five six but exactly telling yes I will roll a die and yes six will appear that is very difficult to predict but I can say what are the chances that six will occur or for a period of time if I if I roll a die for a period of time more extended period of time might be based on the observation it will found that yes the chances of coming six is like this

another example box containing red green and blue balls the possible outcome is red green ball red green and blue ball so ball is picked randomly which one you are choosing because you are not observing that you are randomly selecting the ball whether it would be red whether it would be blue whether it would be green you do not know but you can certainly say the chances of coming over a more extended period of time if we exercise this experiment you can say that so that is basically the the concept of probability and we have to understand that so a head and tail

we have discussed one two three four five six is the outcome of the rolling dice and the red green blue is the outcome of the of the process so under these conditions I can pretty well say yes what is the probability of coming H or H it is basically half because head and tail head and tail hey either head or either tail there is only two possible outcomes so there is a 50% chance so for a fair amount of time if we exercise it tossing a coin hundred observation you may find that observation is coming 48 52 49 51 or 50 50 even that may go so based on that overall for a more extended period of time it will say yes 50% chance similarly for the other cases as well.

So let us discuss some of the terms subset of a sample space which is our area of interest event okay so here head and tail is the possible outcome now take the set for the event that is head head is occurring so event 1 is the head event 2 is the tail so event 1 and event 2 are the sample space you are gonna need to is the sample space of this particular phenomenon of tossing a coin so since event and sample space both are set basically finally it will form a set number of based on the number of outcome so all set operations can be applied to them like Union intersection and set difference etc for the sample space 1 2 3 4 5 6 rolling a die these are the possible outcome and for example event 1 is 1 & 2 even 2 is basically 2 3 4 these are the set for example now event E1 and E2 there is there is an intersection so what will happen so event 1 is 1 & 2 event 2 is 2 3 4 so intersection of E1 and E2 is basically 1 2 3 4 so this is the 2 so this is 2 okay now Union so Union is 1 2 so E1 and E2 1 2 3 4 so 1 2 3 4 is the Union 1 2 3 4 is the Union and the difference difference is in a set this is 1 this is 2 3 4 so when there is a difference difference is this so E1 minus E2 is equal to only 1 because this is removed now intersection you have seen now E1 complement so what is E1 complement so E1 is E1 complement is so what is the possible outcomes 1 2 3 4 5 6 E1 is 1 & 2 so E1 complement is not E1 happening that the the in the set that is the event that is comprises E1 that is not occurring C complement so it is basically 3 4 5 6 in a rolling dice case so probability of not E1 that is basically E1 C is basically this.

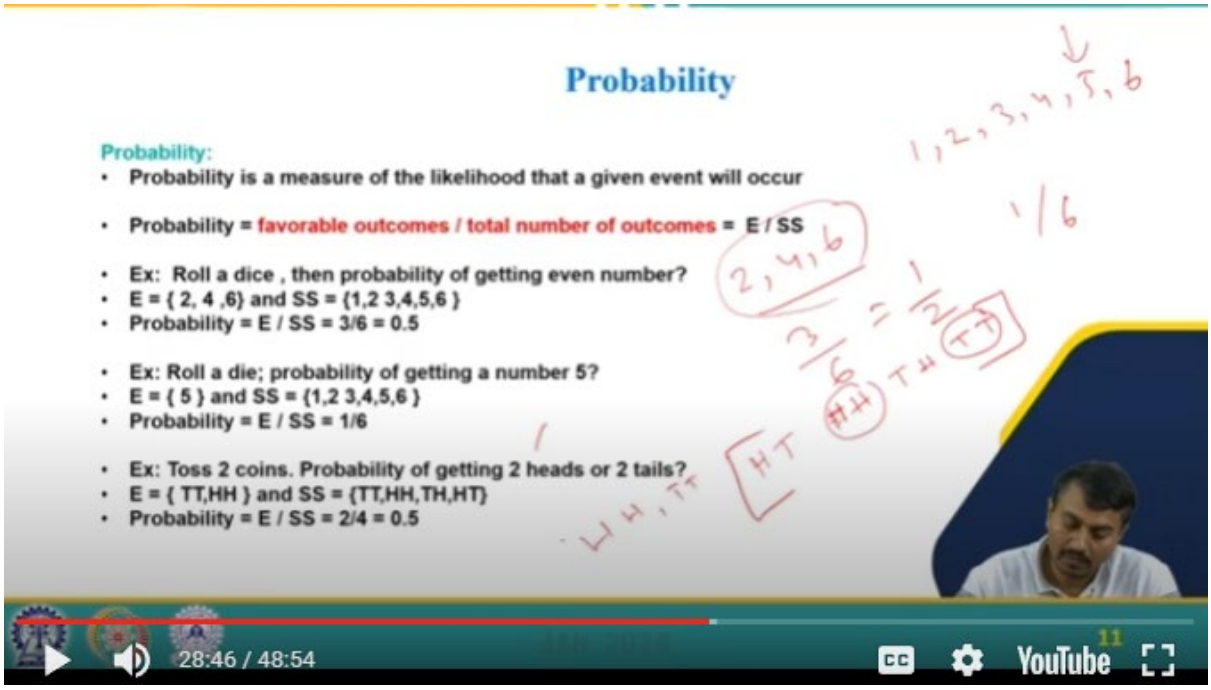
Mutually exclusive event this is very very important concept to understand when the two event are mutually exclusive for example head and tail head and tail event E1 is the head event E2 event E2 is the tail does it happen that head and tail both will occur simultaneously no it is not possible so E1 E2 intersection there is nothing so E1 and E2 are mutually exclusive event because E1 and E2 is nothing so this is basically the concept so that mutually exclusive event that it will not occur whether it will occur head or it will occur tail head and tail will occur both it is not possible so this is the concept of mutually exclusive event basic terms so let us familiarize with some basic terms one is the sample space sample space is the set of all possible outcome of an experiment all possible outcome so for the dice example 1 2 3 4 5 6 this is the total set for the head and tail head and tail is the set coin toss and head head and tail so these are the possible set so now when you see the flipping a rolling die to the two number of die so what are the possible outcome when one in the first case one another in the two one another so one in two one in three one in four one in five one in six for example two into two two two three two four two five two six like that these kind of total set so there are possibility that 36 number of outcome will come so that 36 upper total outcome is the sample space for rolling to die now.

What is the event event is basically a subset of the sample space for example getting a head getting a head you on out of these is 1 by 2 out of these two ok probability now probability on event the likely of a of an event occurring so here occurring is half so it can also be said that number of favorable outcome by total outcome for example we are not calculating that what is the chances of there is an accident so what is the chances of accident we are we are calculating in terms of number of days so in a 365 days in a five years total five number of accident occurs 365 into 5 this is the total number of days and out of these only five number of accidents occur five days accidents occur five number of times in five days so possible

number of outcome that is five out of the total possible outcome means outcome in that is that is in a whole day so there are chances of happening an accident in a day ok so this is 5 divided by 5 into 365 so that 1 by 365 is the chances of happening an accident in a day so this is basically the example so you can also understand that 100 number of toss you have done and you observe 49 times is the head as occurring so what is the probability of head for that particular condition 49 by 100 so this way we can basically calculate the likelihood of an event

Types of probability, so there are a few terms we also have to familiarize one is based on equally likely outcomes. For example, when rolling a die or tossing a coin, if I assume that the coin is fair, the coin does not have any partiality, it is fair then there is a chance that yes occurring of the head and occurring of the tail is 50-50 similarly for rolling a die if this is very similar and there is no partiality or any abnormality on the shape of the block so coming one coming six is equally likely ok so we can calculate the probability of occurring that even by the number of outcomes that may go and the total number of outcomes so total number of outcome in die 1 2 3 4 5 6 and coming three is only one so one by 6

Empirical probability based on the observed outcome: ok, so it is based on a more extended period. You are observing some phenomena that in an in a transport route in transport route you have found that in a day there are five traffic violations, five on day one and six on day two, again five on day three on day four, it is four and five six seven that is this is varying ok so total number of observations that is average you can find out so the probability of finding and traffic violation in an in a particular number of trial in a case you can find out from this data subject of probability it is based on the notions judgment so this is used in the game or predicting the outcome that who will win in a game possibly Brazil other Argentina is a very favorite example who will basically on the World Cup so there is basically on the based on the people notion so that also you can calculate



So let us see some of the example so probability is a measure of the likelihood that a given event will occur ok so the favorable outcome is the event that is we are concentrating on and the total number of outcome is a total set so here the example is given roll a dice and then the probability of given even number so even number is what even number is two four six total outcome is one two three four five six so probability of getting an even number so there are three possibility three and there are total six possibility is equal to half rolling a diet what is the probability of getting number five so out of these getting a five is only once so one by six toss to coin probability of getting two head or two tail so tossing a two coins so what will happen it may happen that head in the second case tail again in the second case it may happen that it is head that is second is head and for the second for the another possibility that it is tail then it is head then it is tail it is tail.

These are the possible outcomes of tossing two coins. Now we want to find out whether getting two heads or two tails, so this is the only two heads, this is two tails, so head and tell the two times it may happen so two and this is four two divided by four is going to happen this is the concept so suppose you roll a six-eyed die and what is the probability of rolling three so similarly one by three one by six because three is only once so there is only one favorable outcome of these samples as one two three four five six.

Now see some few more concept so probability of sample space sample space is the total one so probability of the sample space is always one because all the event occurring all these particular outcome in totality is always one and there are some formula that we have to remember this is one probability of E1 union E2 okay and that is basically probability of E1 probability of E2 and minus probability of E1 intersection E2 similarly for the probability of E1 E2 and E3 union it is probability of E1 probability of E2 probability of E3 and then minus intersection of E1 E2 minus E2 E3 intersection minus E3 E1 intersection plus intersection of E1 E2 E3 now in case of mutually exclusive event we have seen in the mutual exclusive event probability of E1 intersection E2 was 0.

$$\mathbf{P(E1 \cup E2) = P(E1) + P(E2) – P(E1 \cap E2)}$$

$$\mathbf{P(E1 \cup E2 \cup E3) = P(E1) + P(E2) + P(E3) – P(E1 \cap E2) - P(E2 \cap E3) - P(E3 \cap E1) + P(E1 \cap E2 \cap E3)}$$

so for a mutual exclusive event the union is E1 and E2 is basically summation of E1 and E2 because E1 and E2 intersection is 0 similarly for E1 E2 and E3 intersection for the mutual exclusive event it is a summation of E1 E2 and E3. So let us see some examples: total space in the sample for a rolling dice 1 2 3 4 5 6 E1 is the event of 1 2 3, and E2 is the event of 4 and 5. Okay, now the question is, what is the E1 and E2 union probability? Hence, E1 and E2 are mutually exclusive because this number of sets happens 1 2 3 4 5 6, which is not there. There is no mutually same thing here: 1 2 3 is a separate outcome, and 4 5 is an individual outcome. Hence, the E1 and E2 intersection is 0, so E1 and E2 in the union is the summation of this. This is three by 6, and this is two by 6, so this is five by 6 is the total probability of E union,

E2 union E1, and E2 union tossing two coins E1 is the head, E2 is the tail, what is the probability of E union E2. Hence, E1 probability is one by 4, and E2 probability is one by four since these two events are mutually exclusive. Hence, one by four is equal to 2 by 4, which is equal to 5. Now, the concept of probability another is the probability of E complement C? Okay, it is one by the probability of E. Okay, this is to be understood so that this formula will be helpful. Using this formula, let us see one example: the probability of getting a good rank in the gate is 0.

9 probability of not getting a good rank is then 1 minus 0.9 that is 0.1 okay let us proceed conditional probability this is a very important concept there are two events A and B and the probability A slash B is basically probability of A given B means probability of happening A when B is happening so what is the probability of happening A given B is basically denoted as probability of A divided by B for similarly for probability B divided by A is basically probability of happening B given A so let us see one example getting a prime number from a dice is 2 3 5 and getting a even number from a dice 2 4 6 what is the probability of P A divided by B that is probability of A given B okay so now the sample space would be B is already happening that is 2 4 6 now the sample space would be 2 4 6 for the probability of happening B is already happening 1 by 3 so B so A set is 2 3 5 and this is 2 4 6 so what will happen so 3 5 2 this is 4 6 this is the 2 so this is basically 1 out of this 3 so 1 by 3 so probability of A happening B is basically 1 by 3 well it you're happening a already happened B is 1 by 3 so these are the formula probability A by B is equal to probability of A union A intersection divided by probability of B similarly for the B B by A is equal to probability of B intersection A divided by probability of A

$$\mathbf{P(A/B) = P(A \cap B) / P(B)}$$

$$\mathbf{P(B/A) = P(B \cap A) / P(A)}$$

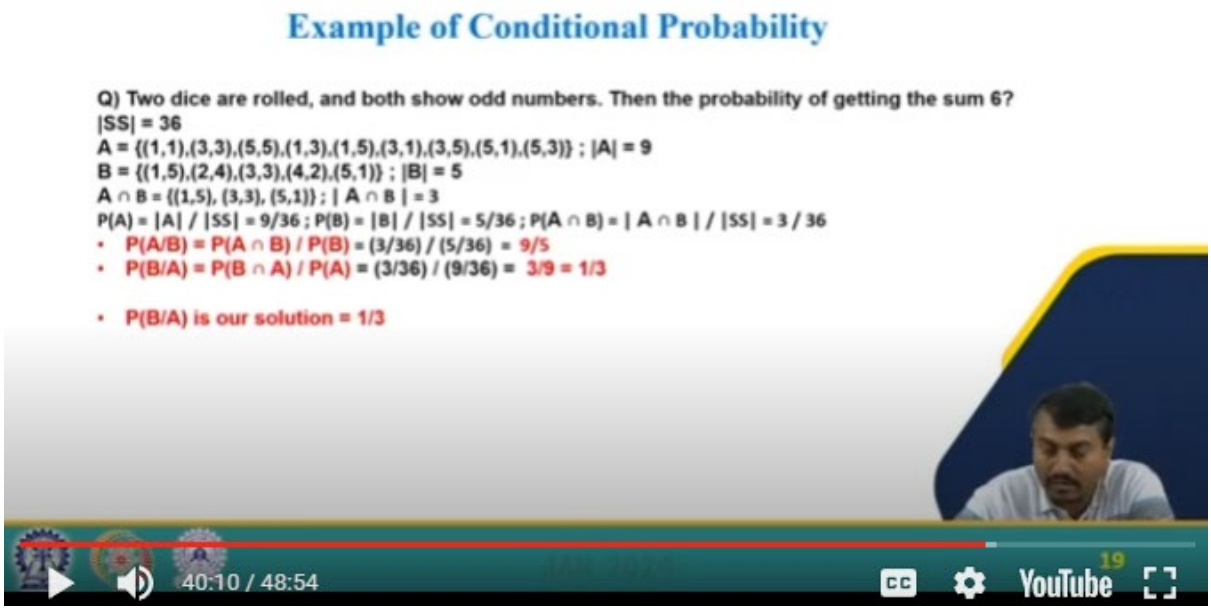
Definition probability of an event given that another event has occurred so formula you have seen the probability of A given B is equal to the probability of A and B intersection divided by P (B), so the probability of getting ahead in a second toss given that the first resulted at the tail. Hence, this is the conditional probability example. So this is basically the steps to solve the questions of a probability concept so find the total sample space first then see which is event A find which event B then the finding the probability of probability of A probability of B and then probability of P probability of A intersection B and then based on that we can calculate probability happening A already happened B or similarly for the probability B happening already happening A example of conditional probability so when two dice are rolled so the total sample space is 36 another question is the first dice shows 4 then

what is the probability that the sum is 6 the first dice 1 2 3 4 5 6 second dice 1 2 3 4 5 6 so A the event A dice showed 4 so dice showed 4 for the first case so this is the case 4 1 4 2 4 3 4 4 4 5 4 6 now the event B is basically sum is 6 1 6 1 5 2 4 3 3 4 2 and 5 1 so this is the only this is the only 1 4 2 A intersection B 4 2 so probability of A is what 1 by 6 that is correct probability of B probability of B is what that is only 1 out of the 5 sample space so 1 2 3 4 5 so this is 5 by 36 and probability of AB A intersection B that is only 1 that is 1 by 36 and now the probability of AB is equal to probability of A intersection B here it is written that is 1 by 36

and probability of B is already calculated there are 5 number of outcome out of the 36 so it is basically 1 by 5 probability of B already happened A is basically 1 by 6 what is the correct solution to this question so

It is P probability of B happening A okay so this is 1 by 6 but what if the question asks that the probability that dice is showing 4 given the dice sum is 6 that is 1 by 5 that is probability A happening A when already happening B so this is basically the some conceptual thing that we want to clear this is another kind of example two dice are rolled both shows odd numbers then the probability of getting the sum is 6 total set is 36 A is basically comprises of 1 1 3 3 and like this total is 9 B is comprises of these 5 so A intersection B is basically only 3 so probability of A is basically happening total in the case 9 divided by 36 that is rightly written then probability of B it is 5 divided by 36 so intersection AB that is only 3 by 36 so probability of A divided by B is equal to 9 by 5 probability B divided by A is basically is equal to 1 by 3 so probability B divided by A is our solution 1 by 3 that is probability of getting some 6 because probability in the earlier case also it is not that 9 by 5 it is it should be there is a correction record it should be 3 by 5 it should be 3 by 5 instead of 9 by 5 it should be 3 by 5

Properties of conditional probability another few concept that is when A is a subset of B when A is well within the B B is the total set now A is inside it this is the B this is the total the B so in that case probability AB , A slash B that is probability of happening A under condition B that is P intersection B divided by P(B) is equal to P (A) divided by P(B) and likelihood of happening B under condition of A is equal to probability intersection BA because BA it is composed total divided by P(A) is 1 so probability of B intersection A is basically the whole set A and probability of A is 1 or total that is this is probability of A probability of A that is 1 now if A or B are mutually exclusive event then P(A / B) is equal to 0 and PB by A is equal to 0 so similarly for P A union B divided by C is equal to P (A) happening A under condition C plus P happening B under condition C minus P A intersection B divided by C that is A intersection B happening under condition of C similarly for the P A complement happening under condition B is equal to 1 minus P A happening under condition of B.



Marginal probability this is also very important concept so to understand the marginal probability let us consider a scenario of with two random variables A and B the joint probability A union intersection B represent the probability that event A and B occurs together the marginal probability of A denoted as probability of A focuses solely on the probability of event A happening regardless of the occurrence or non-occurrence of the event B so this marginal probability says that some event happening regardless of anything happening in that particular case so mathematically marginal probability is obtained by summing or integrating in all these cases continuous in random variable the joint probability over all possible values of the other variables so formula is summing over all possible be P A intersection B or probability of A is equal to integration all possible be P intersection B probability under B so in similar terms you marginalize and sum out the unwanted variable in this case variable B to obtain the probability distribution of the variable of interest that is variable A

$$\boldsymbol{P(A)} = \sum_{\boldsymbol{all\ possible\ B}} \boldsymbol{P(A \cap B)}$$

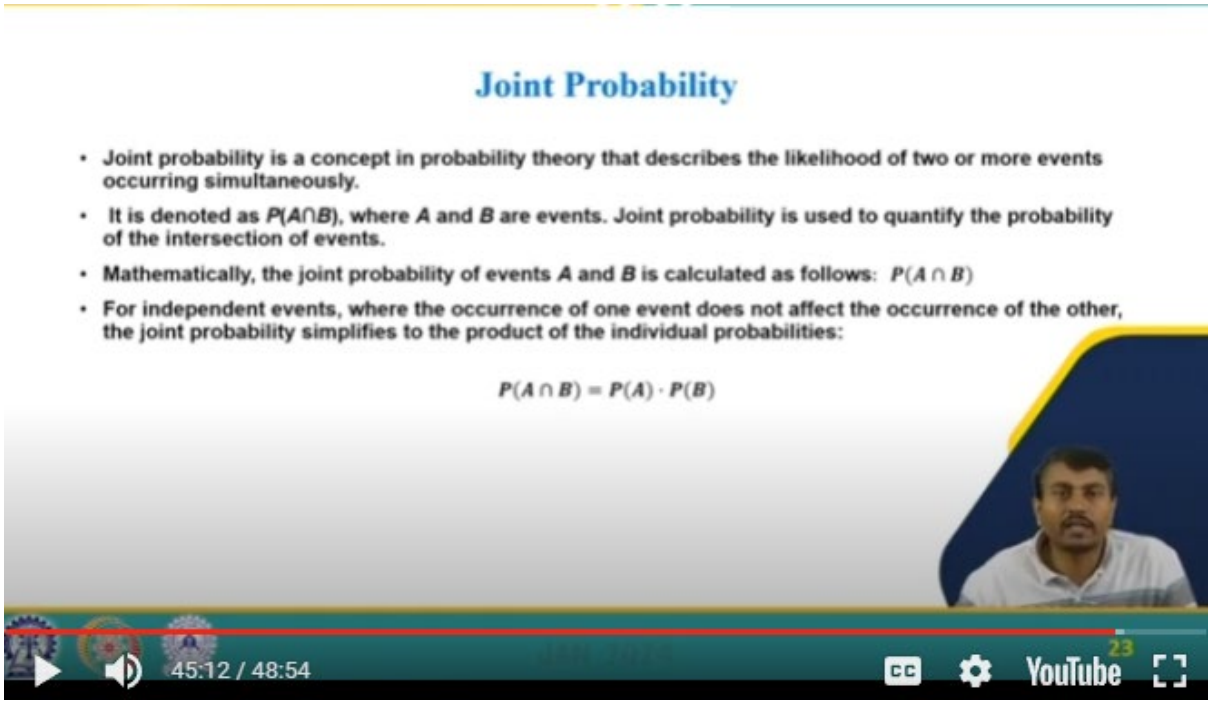
$$\boldsymbol{P(A)} = \int_{\boldsymbol{all\ possible\ B}} \boldsymbol{P(A \cap B)\ dB}$$

So, marginal probability refers to the possibility of a single event or outcome occurring without considering other events. It is derived from a joint probability distribution that describes the probabilities of combinations of events. So, marginal probability is a fundamental concept in probability theory and is used in various statistical analyses and machine learning algorithms, especially when dealing with multiple variables and their interactions.

Joint probability so joint probability is also a very important concept joint probability is a concept in probability theory that describe the likelihood of two or more events occurring

simultaneously it is denoted at P A intersection B where P A and B are the events and joint probability is used to quantify the probability of the intersection of the events so mathematically the joint probability of event A and B is calculated as follows P A probability of A intersection B for a mutually independent event or independent event when one event does not affect the other so joint probability is basically is this a probability of A intersection B is equal to multiplication of probability A and probability B okay for example when occurring suppose the head it is happening in the second case it is still happening for the for the two toss you are doing so these are not dependent that whether it is head is coming here is tail is coming that is not dependent on each other so probability A intersection B is basically multiplication of the coming out of head and coming out of tail here is that is a probability

So however the event are dependent meaning the occurrence of open one event affects the occurrence of other the joint probability is calculated using the conditional probability formula so conditional formula we have already seen so P A intersection B is the joint probability event of P A and B P A and P B marginal probability events of P A and B respectively P A P A happening under condition B that is conditional probability of P A event P A event given that the event of P B has occurred similarly for the P B A conditional probability of event P B given that event P A has happened so let us see some solved cases suppose we have two fair coins one is red another is blue so you want to calculate the conditional probability joint probability and marginal probability of the outcomes so let's see we are interested in probability of getting a red coin given that we flip two heads so probability are slash under condition of happening two heads H H so number of outcomes where both coins are head or one is red okay and the total number of outcome are both coins are heads so number of outcome are both coins are head and one is red that is one okay total number of outcome when both coins are head that is only one so probability of R divided by H H is equal to one for the joint probability the joint probability is the probability of two events happening together so let's a is the event getting a red coin R and the blue is the getting two heads H H so probability of A intersection B is equal to probability R and H H so number of outcome are both coins are head so both coins are head and one is rate is one and total is total possible outcome that is P A intersection B is 1 by 4



For the for the marginal probability is the probability distribution of the single event occurring without reference to any other events so let's calculate the marginal probability of getting a red coin that is R so probability of R is basically number of outcomes where one coin is red and the total number of outcome so total number of outcome where one coin is rate that is 2 R R and R B and total number of outcome R R R B B R and B R so this is 4 so 2 divided by 4 that is half so these are basically the example for this case where two fair coins one is red and another is blue conditional probability is 1 joint probability is 1 by 4 and marginal probability is 1 by 2 so these are the references so let me summarize what we have covered in this particular lesson we have provide the basic terms of probability like random experiment sample space event mutual exclusive event that are discussed and the different types of probability that is conditional probability then marginal probability and joint probability and we have solved using the formula for the conditional probability marginal probability and joint probability thank you.