Engineering Mechanics

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Module - 07
Lecture - 03
Rotational Motion – III

In the previous lectures we saw how we apply the concept of angular momentum to solve rigid body problems. We mainly focused on conservation of angular momentum for body is moving about a fix axis. In this lecture we are going to start with the motion when an external torque on a body is applied.

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For example let me show you. I will take this scale and hold it on horizontal position like this.

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Leave it, take on horizontal position and leave it. What I would like to know is when it reaches here ignoring friction what is this angular speed. So, the problem that I am solving is I have a scale like this.

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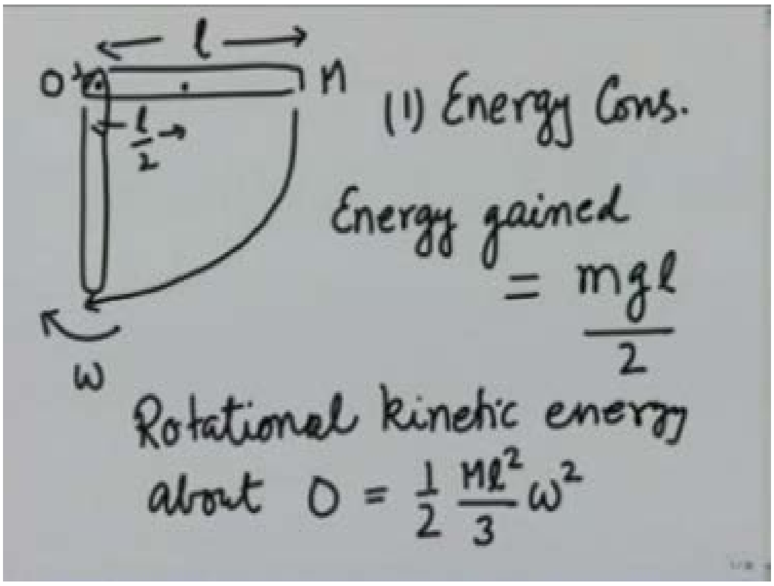

It can rotate about this point O for example, and when we leave it, it drops down like this. I would like to know what is this angular speed omega, when it comes here. I will solve this problem in 2 ways show you what a wrong way. So, let the length of the scale be l. So, that the center of mass is at l by 2 if it is a uniform scale. Let this mass be M. So, first let me apply energy conservation right away.

When the scale falls it is center of mass drops down by a height of l by 2. So, the energy gained should be equal to m g l divided by 2. All of this energy gets transformed into it is kinetic energy which in this case is the rotational kinetic energy. Energy about point O this is point O, I remind you and this is going to be equal to 1 half I about point O which you see in the previous lecture is M l square over 3 times omega square. The 2 should be equal.

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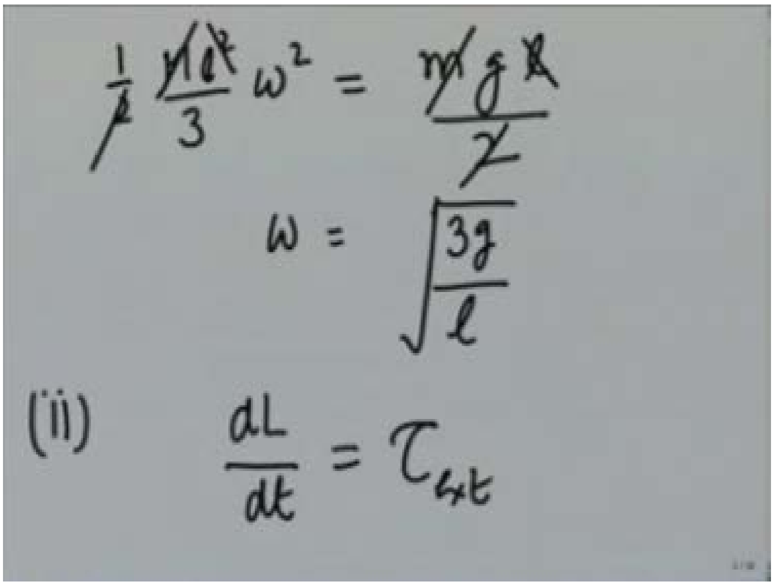

Therefore, I have One half M l square over 3 omega square is equal to M g l divided by 2. This M cancels this 2 cancels 1 of the l's cancels. Therefore, I get omega equals 3 g over l square root. So, this is the answer that comes off energy conservation. Second way is by applying the equation that d L d t is equal to external torque and let us apply that.

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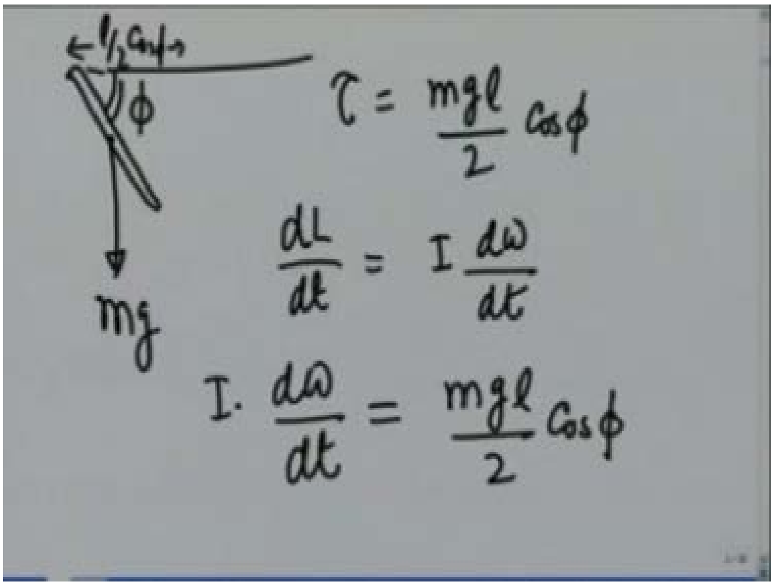

Let the position of the rod be at an angle phi from the horizontal. At that point all the force acts here mg and therefore, the torque that is being applied is going to be equal to m g l divided by 2, cosine of phi because this distance is l over 2 cosine of phi. This should be equal to d L d t which is nothing but I about point O d omega d t. Therefore, I should have I which is moment of inertia d omega over d t is equal to m g l divided by 2 cosine of phi.

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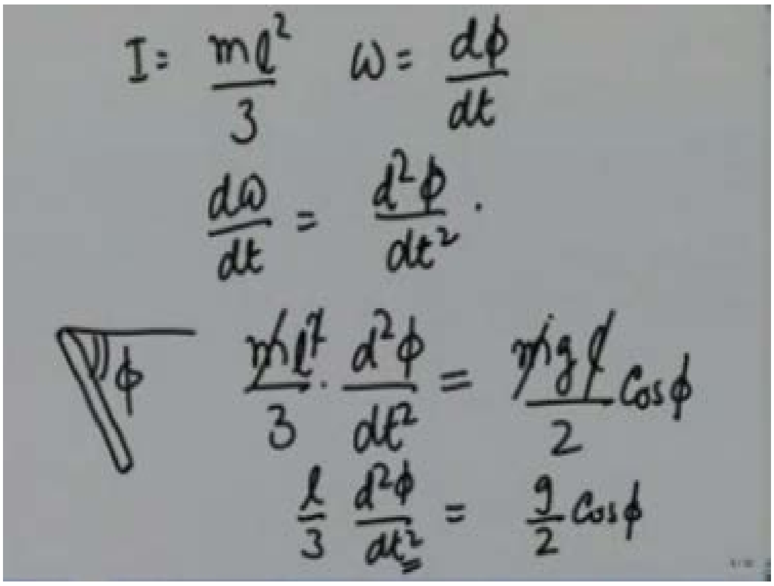

As I already have mentioned I is ml square over 3 omega is d phi over d t. Therefore, d omega over d t is d 2 phi over d t square and therefore, the equation when the rod is at an angle phi from the horizontal is going to be m l square over 3 d 2 phi over d t square, which is I alpha or I times the angular acceleration is equal to m g l over 2 cosine of phi. L cancels m cancels and therefore, I get l over 3 d 2 phi over d t square is equal to g over 2 cosine of phi. Since the left hand side involves t, we will use an old trick that we have used earlier in deriving energy conservation.

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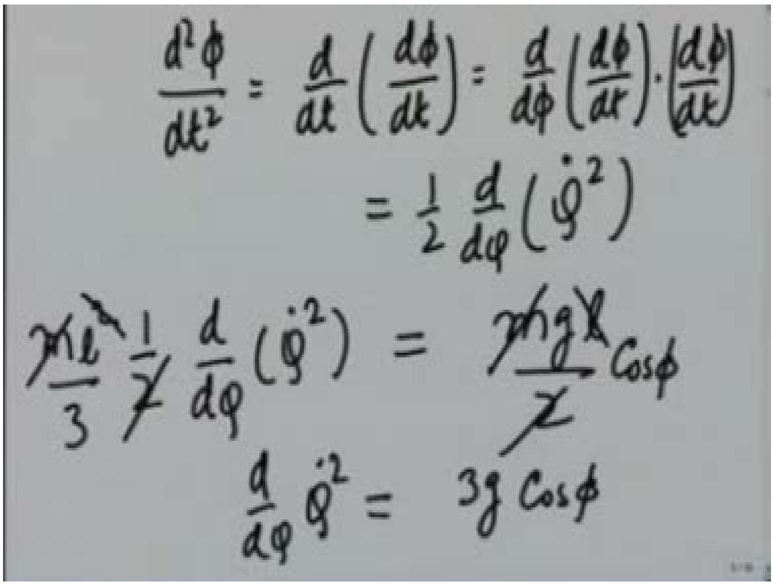

That is to write d 2 phi over d t square as d over d t of d phi over d t which is nothing but d over d phi of d phi over d t times d phi over d t which can be written as 1 half of d over d phi of phi dot which is d phi or d t square. Therefore, the earlier equation becomes 1 half d over d phi dot square which is nothing but d 2 phi over d t square is equal to m g l over 2 cosine phi. There was, I here which is ml square over 3.

I had simplified earlier I will do it again m cancels this 2 cancels 1 of the l's cancels. Therefore, I get d over d phi of phi dot square is equal to 3 g cosine of phi.

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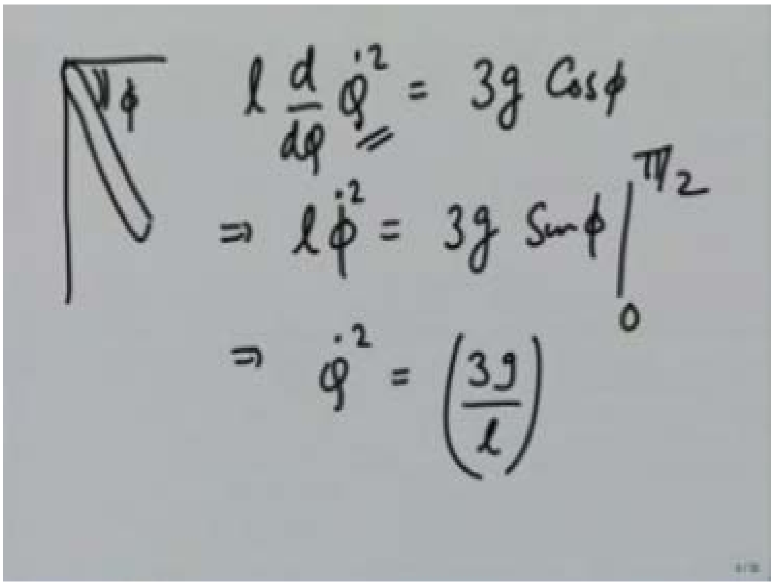

Therefore, integrating which is d phi dot square equals 3 g cosine of phi. There is an l also here. So, I can write l here this is for a rod which is of length l and this angle is phi. This gives you l phi dot square is equal to 3 g sin of phi starting from 0 to angle pi by 2 when it comes to the vertical. This gives you phi dot square equal 3 g over l which is the same answer as obtained earlier.

So, I have used energy conservation directly and I have also integrated the equation of motion, which essentially by this transformation comes to energy conservation law energy conservation by itself. Now, you may ask at this point.

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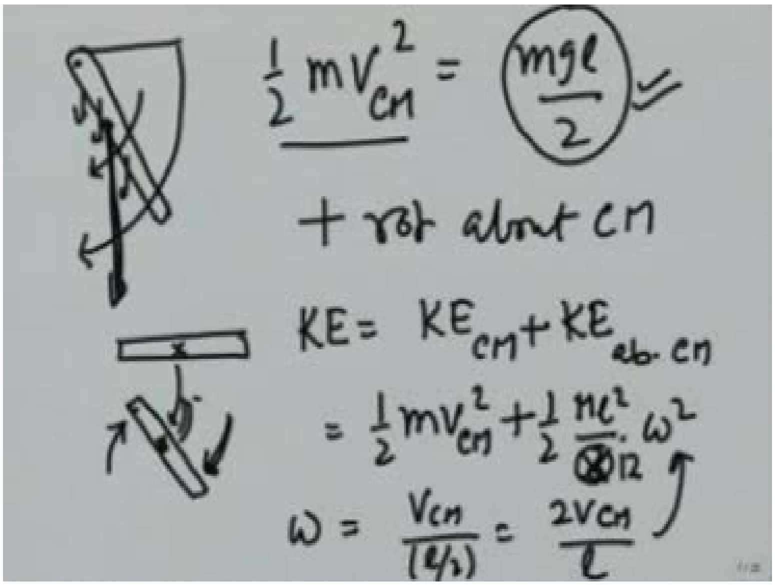

Why did not I do one more thing why could not I do that 1 half mv center of mass square is equal to m g l by 2 when the rod came from here to here. After all it is the center of mass which is moving and it moves with a speed V c m and this should be equal to the gained gain energy because the potential energy drop what is wrong with this. What is not here is that you have to be careful when there are extended bodies involved, when velocities are distributed over the body when forces are distributed over the body that what all is happening.

For example here if I start with this position here is the center of mass when the rod comes in this position, not only has the center of mass moved. You see with respect to the center of mass the body has also rotated like this. So, in coming down like this the scale is performing 2 motions. The center of mass is moving and the scale as you can see is changing it is orientation with respect to an axis passing through the center of mass. It is rotating and it is rotating exactly by the same angle, as it was rotating about O.

So, actually the motion is 1 half V c m square plus rotation about C M. If you calculate the kinetic energy it is always KE of CM plus KE about CM, it will come out to be 1 half mV CM square plus 1 half I about CM which will be M l square over 3 omega square. Omega by definition here is going to be V c m divided by l by 2 which is 2 V c m divided by l, about center of mass should be 12 naught 3, about it is 3 about point O about CM it is twelve.

If you substitute these numbers and you get precisely the answer that you got earlier. What you may ask at this point is, but what about the torque. Torque the forces are also distributed, but I took torque to be acting at the CM because the torque depends linearly on the distance from the pivot point. It is average comes out to be as if it is acting on the center of mass, but not when I calculate the moment of the velocity because that becomes r times r omega it depends on it as r square.

So, you have to be careful when we are talking about rigid bodies and distribution of forces and velocity over the body, that we take proper consideration of rotation and angular momentum also. As an extended example of this let me now take just booklet.

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I will keep its printed side on top.

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Drop it I will keep the printed side on top and drop it you see the printed side has gone down. Let me do it again I am keeping the printed side up dropping it roughly from 1 meter and let us see what happens the printed side goes down.

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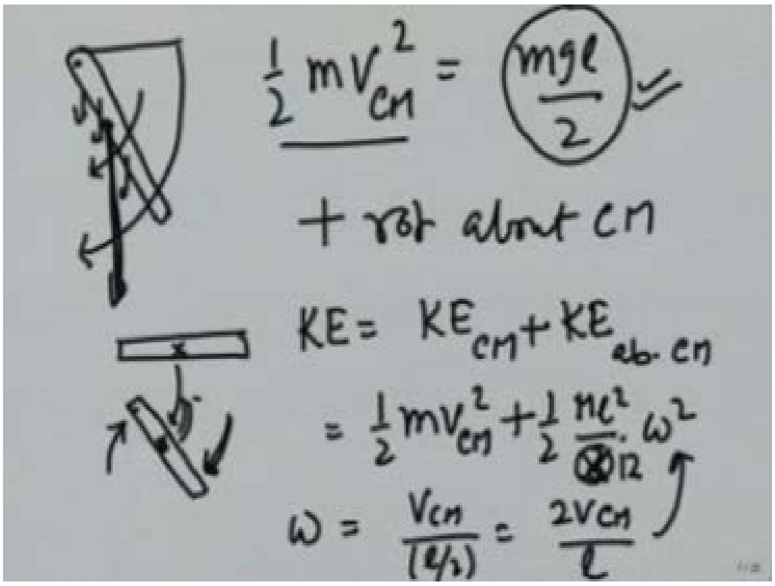

Can I explain this in fact there is a very famous book based on such a thing the book is known as why the toast lands jelly side down. It has been generally observed if you apply jam or butter or jelly on your toast, it if it drops from your hand it roughly the jelly side always goes down. Let us try to understand this using our rotational dynamics.

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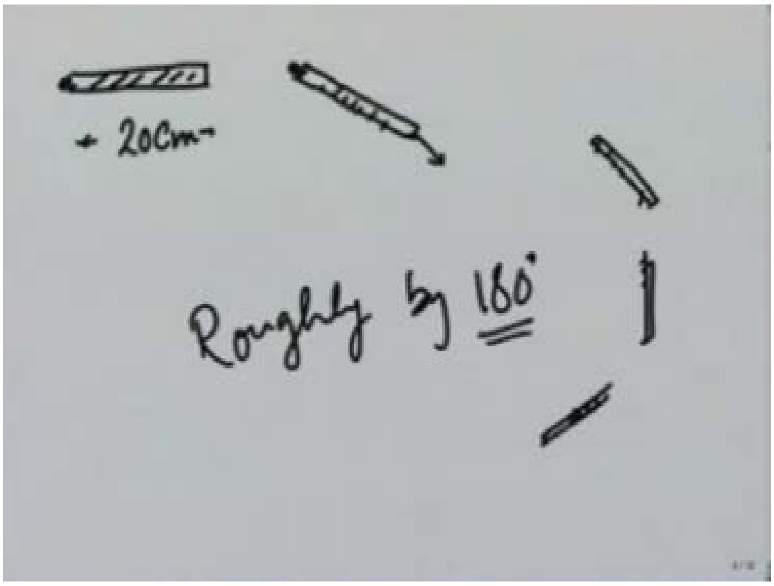

So, roughly I will take this book let which is let us say of the order of magnitude about twenty or fifteen centimeters wide. I was holding it here, what it did was first it went down like this it rotated about this point. Until it reach an angle when it slipped off. Once it slips off it starts falling down and falling down it rotates. What we observed it rotates roughly by 180 degrees. Why roughly if even if it is 150 it will fall down the other side down even if it 200 will fall down the other side now. So, let us try to understand this.

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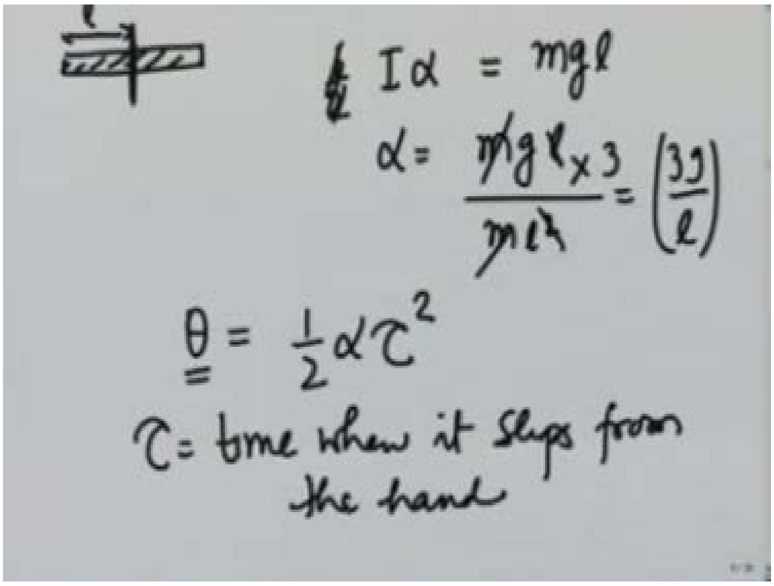

First step is when the booklet is like this, let the length the distance of the center of mass from the pivot point where I am holding it be l. Then it experiences a force mg at the center of mass and a torque m g l. Therefore, it should have an angular acceleration which should be 1 half which should be I times alpha. Alpha would be equal to m g l divide it by I which is ml square over 3, 1 l drops out m drops out. So, the angular acceleration is roughly 3 g over l.

It covers an angle theta which will be equal to 1 half alpha tau square. Well let me write tau as the time when it slips from the end. So, theta is a free alpha tau square what should be the value of theta. We know from mechanics that if a body is at an angle.

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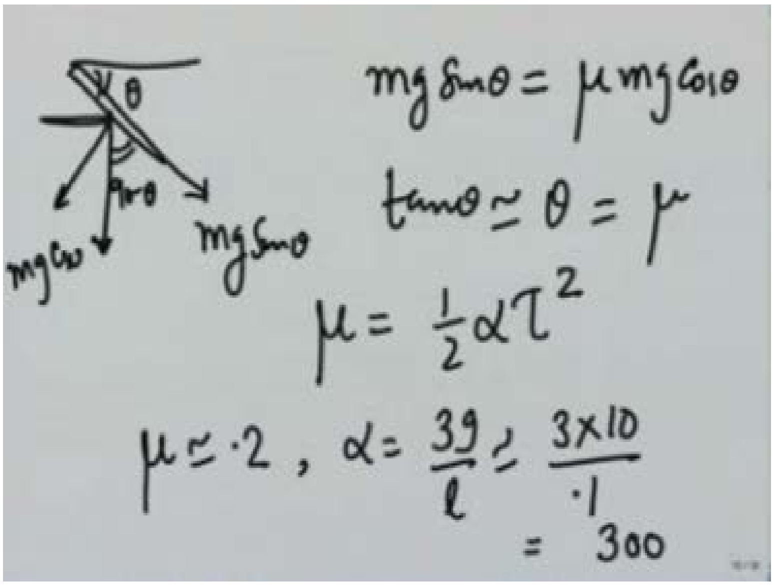

Such that this is theta this is mg theta 90. This is 90 minus theta this force would be mg sin theta this force component would be mg cosine theta. So, if mg sin theta roughly becomes mu times, mg cosine theta, then the body is going to slip off my hand. Therefore, I can write tangent theta roughly equal to theta for theta small equals mu. So, when mu equals 1 half, alpha tau square this this booklet is going to slip off. Let us take a typical value mu equals roughly point 2, alpha is given to be 3 g over l, which I will take to be roughly equal to 3 times ten over point 1 meters. So, that comes out to be roughly 300.

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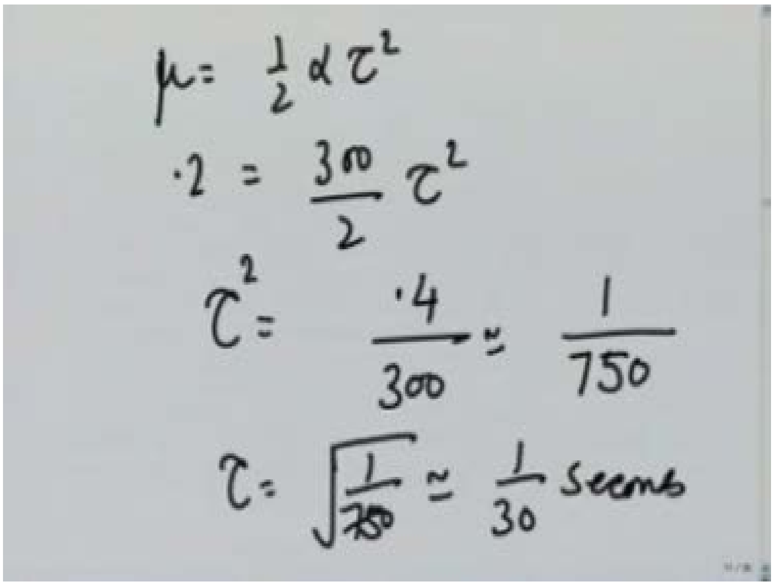

Therefore, I have mu which is equal to 1 half alpha t square point 2 equals 300 divided by 2 whole square or tau the time for it to slip off is going to be point 4 over 300 roughly 1 over 750 tau square. Therefore, tau it was 1 over 750 which I can write as maximum 1 over 30 seconds.

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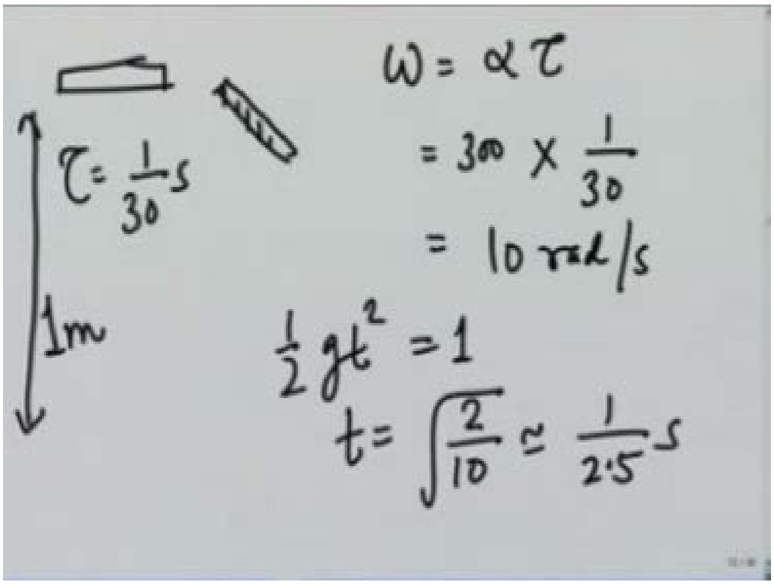

So, we have come up to the point, where the toast or this booklet came here slipped off. This time tau is roughly 1 over 30 seconds. In this time it has gained an omega which is alpha tau and after it slips off that omega remains at constant and makes it rotate. So, this is roughly equal to 1 over 30 times 300 which is equal to 10 radiance per second. If this dropped by 1 meter, then the time taken for it to drop could be given by 1 half g t square equals 1 t equals 2 over 10 square root. Which is roughly 1 over 2.5 seconds.

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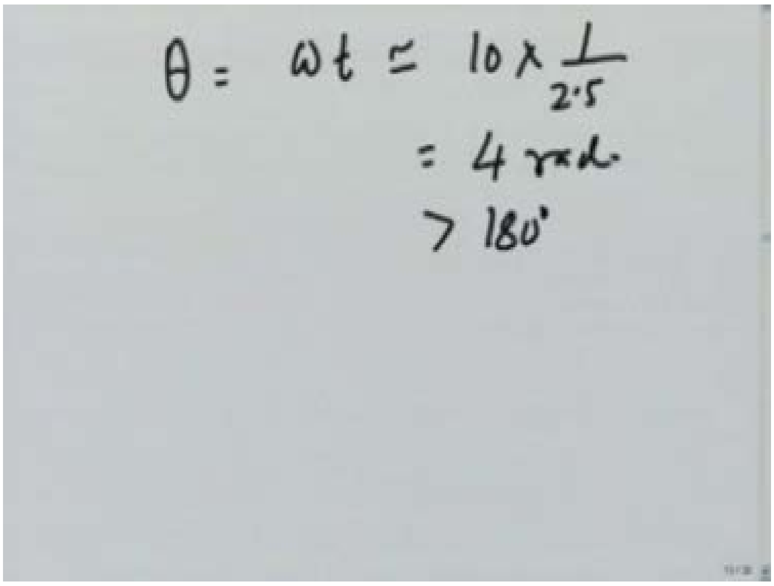

Therefore, you see that the angle by the time it comes down, would be equal to omega times t which is 10 times 1 over 2 point 5 which is 4 radiance. Slightly greater than 180. So, that is roughly the angle through which the book rotates and you see it drops the other way. This is an interesting problem when we applied the ideas learnt. So, far having done the problem where the axis was fixed.

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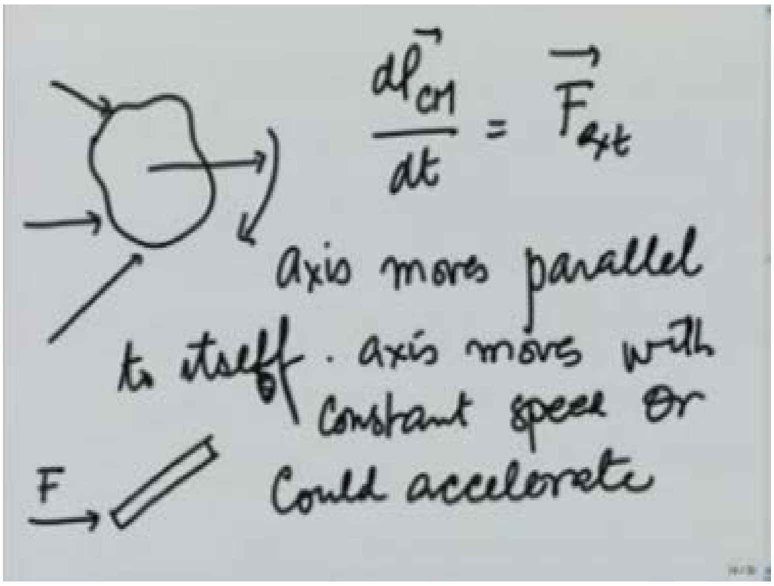

Let us now move on to problems where if I am given a rigid body and I hit it with a force. It is center of mass could move according to d p c m divided by d t equals force external, but also because the forces are distributed over the body, the body itself could change orientation as it moves. For simplicity to start with we will be confined with motion where axis moves only parallel to itself. For example, it could be a rod on a horizontal table being hit by a force F. Then the axis would move parallel to itself to the axis will be perpendicular to the table. Also the axis moves by with constant speed or could accelerate. We want to see how to describe the motion of the rigid body in this case.

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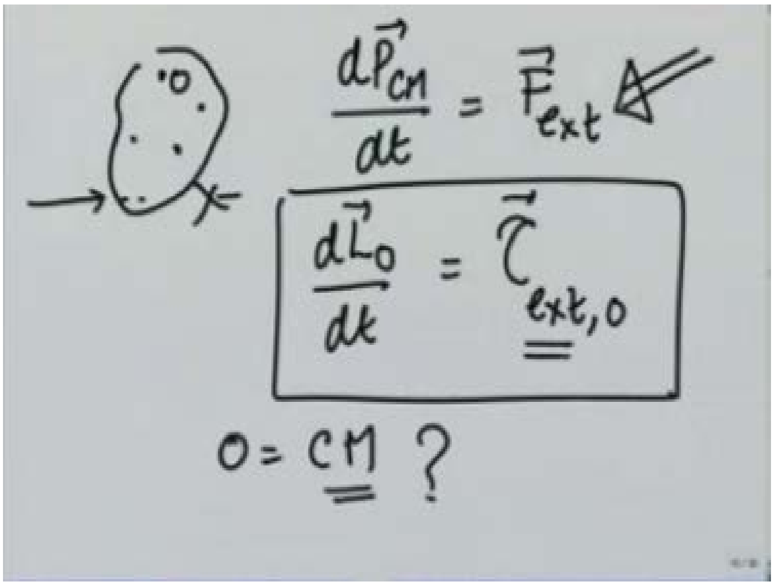

So, let us take a rigid body apply say for force here. One thing is very clear that the center of mass moves according to this equation, that we had already derived. What about the torque, there is a torque about any given point which is not in line of the force and this should be equal to d L d t about that point. Let us call this point O, could O be any point, should O be CM, should O be a particularly point specified by certain conditions we do not know, but we do wish to apply this equation to describe it is orientation change.

This equation we know for sure describes the motion of the center of mass. Usually you must have seen in your twelfth grade or so that O is taken to be the point center of mass, but why that is the question and that is what we wish to answer now. Remember our goal is to apply d L over d t equals tau only external when a body is moving.

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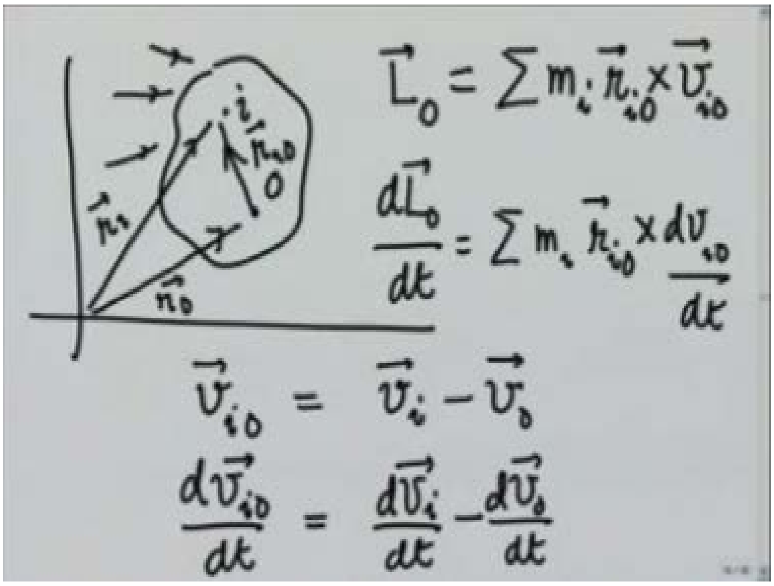

So, let us take a general point O and a body that is being applied by several forces. Let us calculate the angular momentum L about O. This by definition is going to be summation m i r i with respect to O cross v i with respect to O. Where I is some general point. Therefore, d L o d t and so far we have been doing that, when we take the first derivate d r o d t that gives you 0 when crossed with velocity.

So, it comes out to be summation m i r i with respect to o cos d v I o over d t. Let me show you this is r i o. This is r I, that is the coordinate of the point with respect to an outside origin and this is r o. Now, v i with respect to o is nothing but velocity of I th point with respect to outside point minus velocity of o. Therefore, d v i o over d t is going to be equal to d v i over d t with respect to outside point minus d v o over d t.

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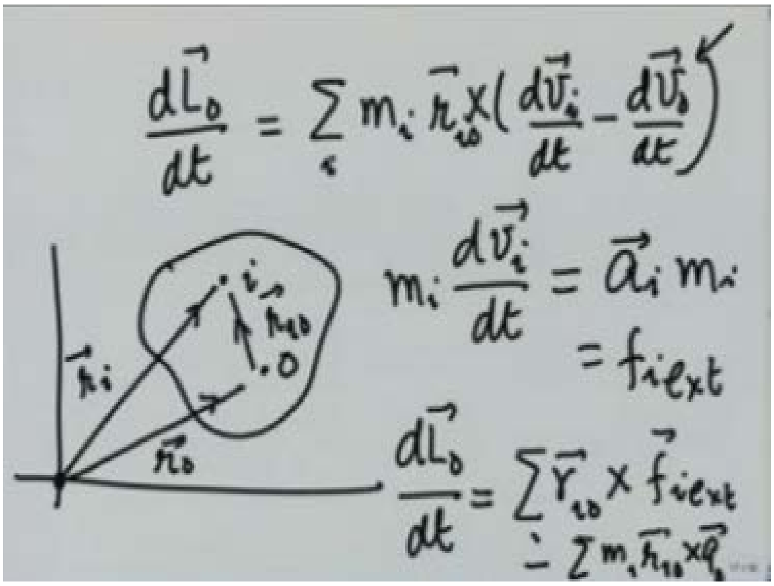

Therefore, we get d L o over d t, that is rate of change of angular momentum with respect to point O is equal to summation i m i r i o d v i over d t minus d v o over d t and I forgotten to put a cross sign here. Let me remind you again, this is my reference plane, this is my body. This is point I, this is some arbitrary point O, this is r o was derivative using a v o this is r i and this is r I o, d v i d t is equal to acceleration of point I, with respect to the outside point, which is an initial point. Therefore, m i times this is f i external.

Therefore, I can write d L o over d t is equal to summation r i o cross f i external minus summation m i r I o crosses this is nothing but the acceleration of point O acceleration of point O.

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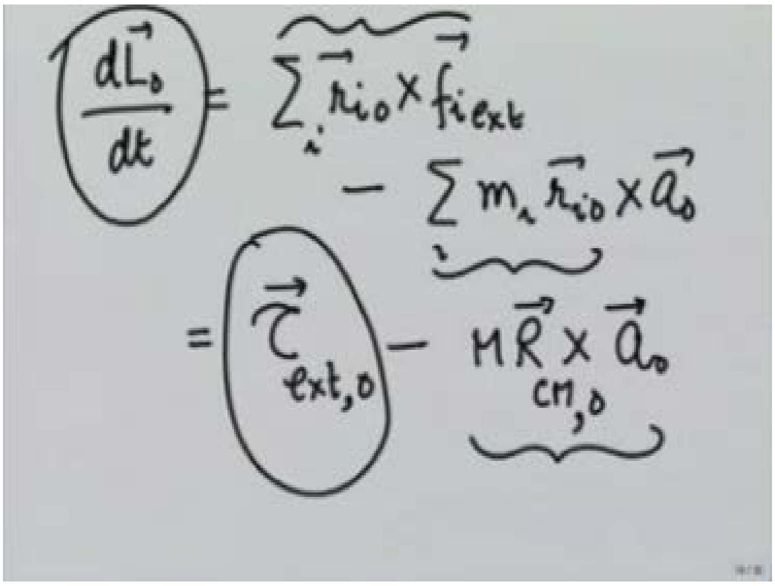

So, what we obtained now is that the rate of change of angular momentum, when angular momentum is taken about an arbitrary point O, which could be in the body which could be accelerating or anything is equal to summation i r i o cross f i external only, minus summation i m i r i o cross acceleration of point O itself. This by definition is the external torque with respect to point O no matter what the point O is doing.

So, this is tau external with respect to point O minus a o is a constant. As far as summation over I is concerned and this is nothing but M times R of CM with respect to O. So, if I want that d L d t be equal to this only, this term must vanish let me remind you again what we are doing.

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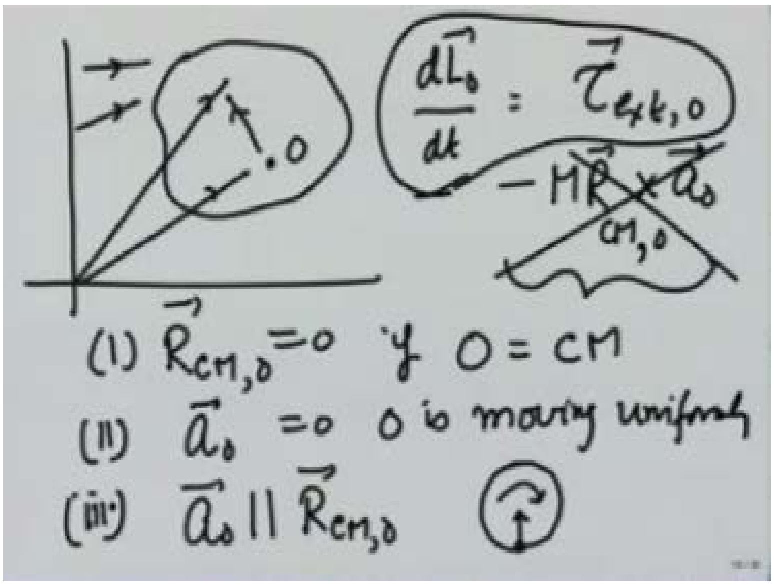

What we are doing is, we are taking a rigid body. Taking an arbitrary point O on this applying some force on the rigid body, calculating the torque due to this force with respect to point O point O could be accelerating moving whatever. Then when I analyze this, what I find is that d L o d t is equal to tau external minus MR of CM with respect to O cross acceleration of O. I want this term to be 0 I want to choose this point O in such a manner that the last term vanishes. So, that with my eyes closed I can always apply d L over d t equals tau external only.

This would vanish under 3 conditions number R c m O itself is 0 and that will be case, if O the point itself is center of mass. This is what you have been doing in your twelfth grade or whatever, that you have been taking torques about center of mass and a mean F been applying d L about CM divided by d t is equal to tau external about the CM or this should be about O. Number 2 acceleration of point O itself be 0, that means, O is moving uniformly. Third, that acceleration O be parallel to R c m o, that is the point O is accelerating towards the CM itself.

This also you have been doing in your twelfth grade. For example, in a wheel which is rolling, you usually take point about torque about this point and apply d L d t equals tau. This point is actually accelerating towards the CM. So, therefore, your answers come out to be correct. So, there are only 3 points in a body, in which case this term vanishes and I can apply with my eyes closed that d L over d t is equal to tau external about that point. We will take a few examples using this equation. So, what we have learnt in general.

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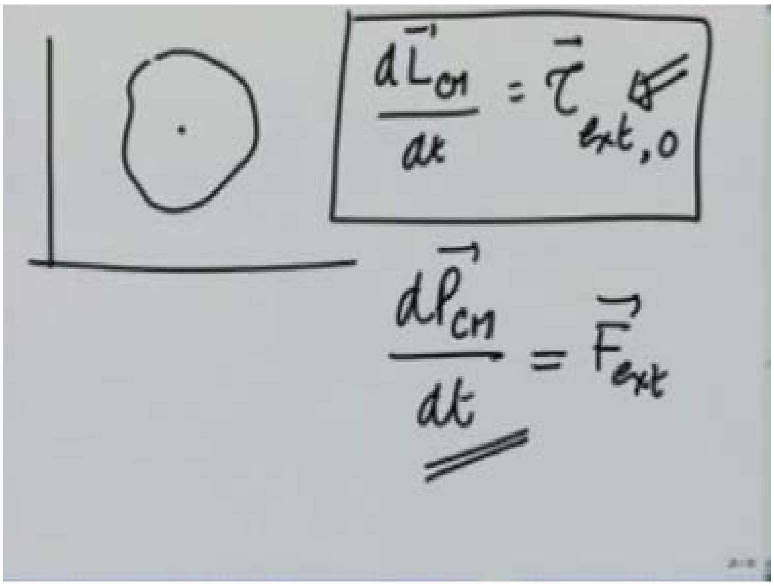

So, far I am restricting myself to only that motion in which the axis moves parallel to itself. That d L about CM d t is equal to tau, external only no other forces need to be taken into account. So, pseudo forces or anything can be applied safely. The motion of the center of mass itself is given by d P CM over d t is equal to F external. These 2 equations are sufficient to describe the entire motion of the body. In that how it is translating that is given by this and how it is changing its orientation is given by this.

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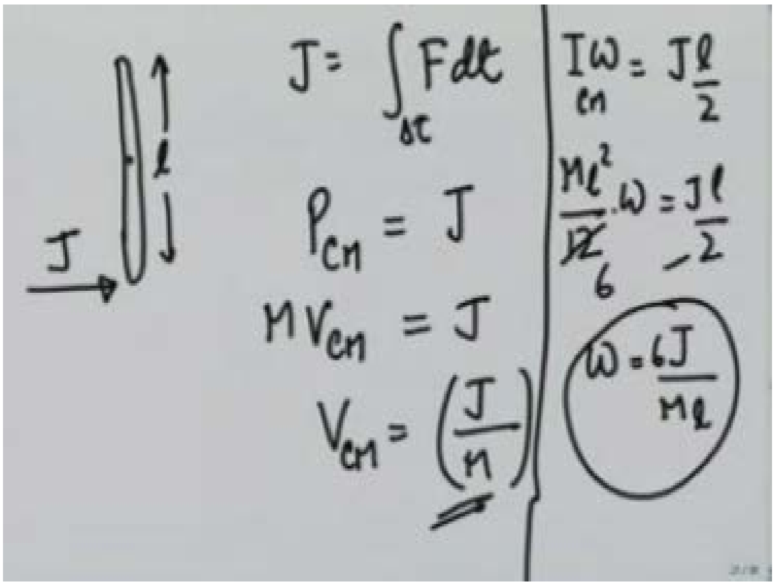

As the first simple example of this. Let us take a rod uniform of length l. Let us hit here with an impulse J, impulse as you would recall is nothing, but integral F d t where F is applied for a very short time. So, this is a very short time delta tau let us say. By the moment equation you get that the momentum gained by the rod should be equal to P c m equals J. Therefore, M V c m should be equal to J. So, after the rod is held at a center of mass should move with this velocity.

How about the omega that it obtains. As we have already seen I can safely write. Let me do it right here that the angular momentum it obtains about the center of mass should be equal to the impulse torque impulse direct gains of all the center of mass and that will be J l over 2. Therefore, M l square over 12 times omega should be equal to J l over 2 and it ends up getting omega equals J over M l times 6. So, it rotates as it moves with this angular speed. It moves with this a center of mass moves with this speed. Let us take another example.

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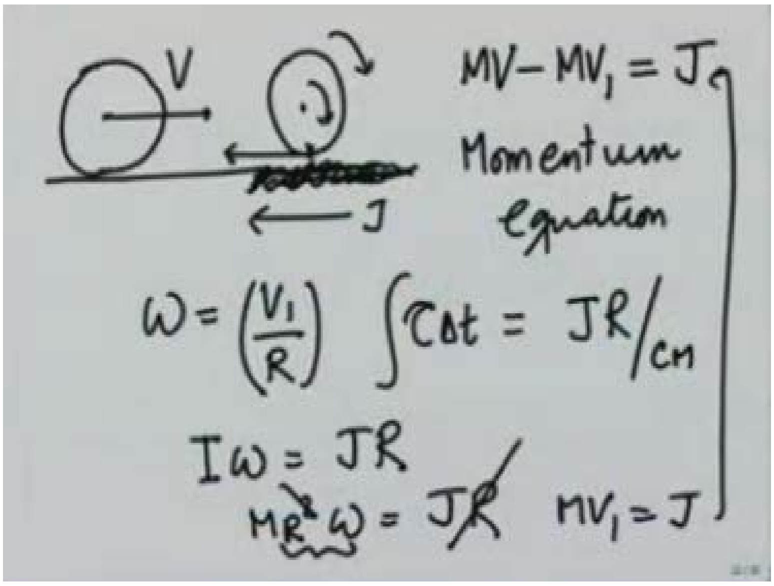

Let us say there is a wheel which is thrown this way with a velocity V. It is moving on a smooth surface there is no friction here. Suddenly it encounters a rough surface. The surface is so rough that starts rolling right away. So, here it was just moving there was no rotation there is no rotation of the wheel about the center of mass. As soon as it hit this rough place it started rotating rolling.

So, at its lowest point was stationary at that point. What about it is rolling speed. So, let us see, what has happened the moment it hits this rough patch it gets an impulse J in this direction initially if it was moving with momentum V. Is moment now gets reduced because of this impulse. So, that is that comes from momentum equation. Not only that if it is rolling, then it should as also be rotating about the center of mass with the speed V 1 over R. Therefore, it has gained an angular momentum about the center of mass, where does this angular momentum come from.

That angular momentum comes from the impulse that it brought here. So, the torque impulse it got is equal to J times R with respect to center of mass. So, the angular momentum it should obtained about the center of mass should be equal to J times R and this tells you I is MR square omega is equal to J times R, R cancels. R times omega is V 1. So, MV 1 equals J.

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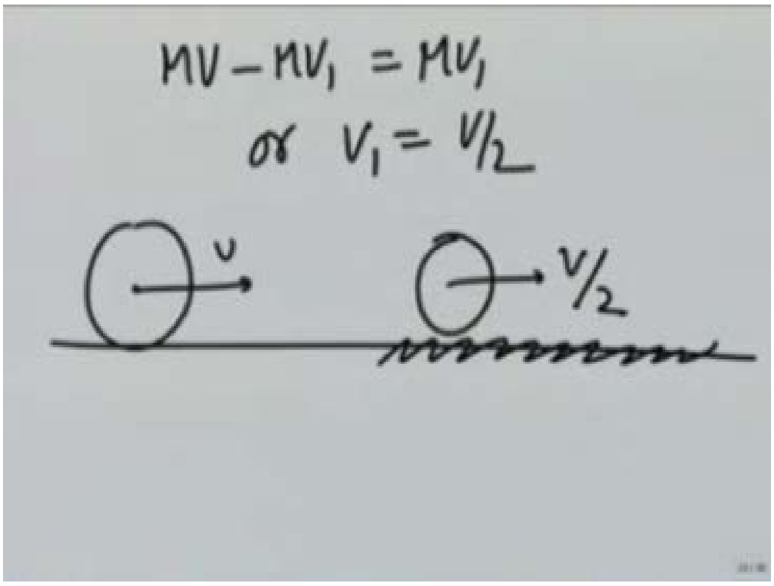

Therefore, putting this in this equation I get MV minus MV 1 equals MV 1 or V 1 equals V over 2. So, as soon as this wheel which is moving the speed v, without rolling on a smooth surface hits a rough surface here, it will start rolling with a speed V over 2.

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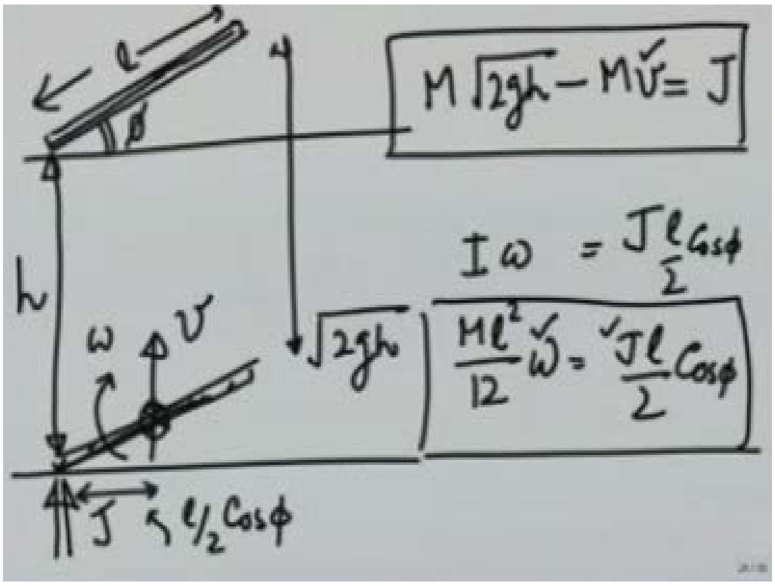

As a third example let me take a rod and you must have seen this. If you sometimes drop things. Let this initial angle from the horizontal be phi and let this height from the ground be h and let it drop you must have seen it happen at home. If you drop something like this, it goes back and forth and you hear a voice tuck tuck tuck tuck tuck tuck. So, what I want to know is after it hits the floor and suppose no energy is lost, what will be its rebound velocity and what will be its rebound omega.

When it drops from a height h, when reaches here as velocity is going to be the square root of 2 g h. When it hits here, there is an impulse that acts here let that be called J. So, I am going to have square root of 2 g h times the mass of the rod minus MV, this is loss and the momentum that should be U to J. That is from momentum equation.

Similarly, now I apply the torque equation about the center of mass, which again the safest point to apply it for. This J gives you a torque impulse this distance is going to be l over 2 cosine of phi, if the length of the rod is l. Therefore, torque impulse is going to be J l divided by 2 cosine of phi and this must give it an angular moment about center of mass, I omega or Ml square over 12 omega is equal to J l over 2 cosine of phi. That is equation number 2. I have 3 unknown v, omega and J. So, I need 1 more equation and that comes from energy conservation.

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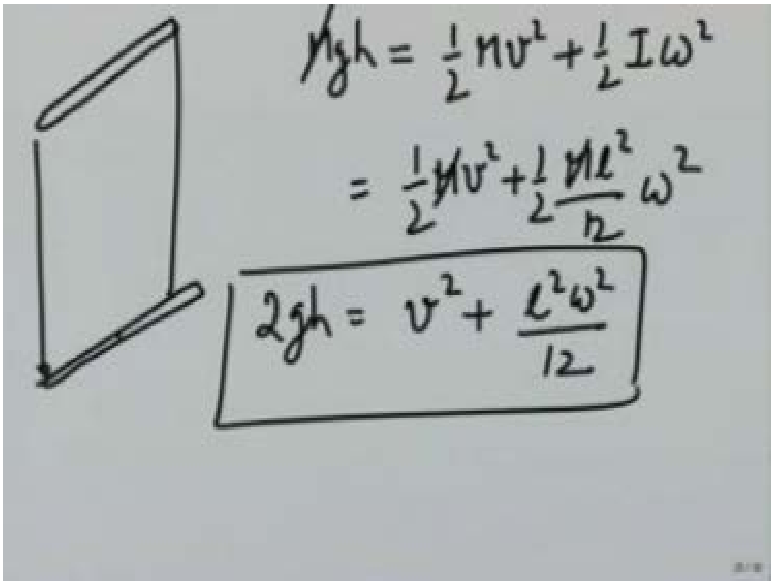

The energy conservation tells me that the energy that it gained, while being toped. The kinetic energy it gained is going to be equal to M g h and that should be equal to its energy right after we and that is going to be 1 half MV square plus 1 half I omega square, which is equal to 1 half MV square, plus 1 half Ml square over 12 omega square.

M cancels and therefore, you get 2 g h equals v square plus l square omega square over 12. So, now I have got 3 equations to solve to get my answers the 3 equations are.

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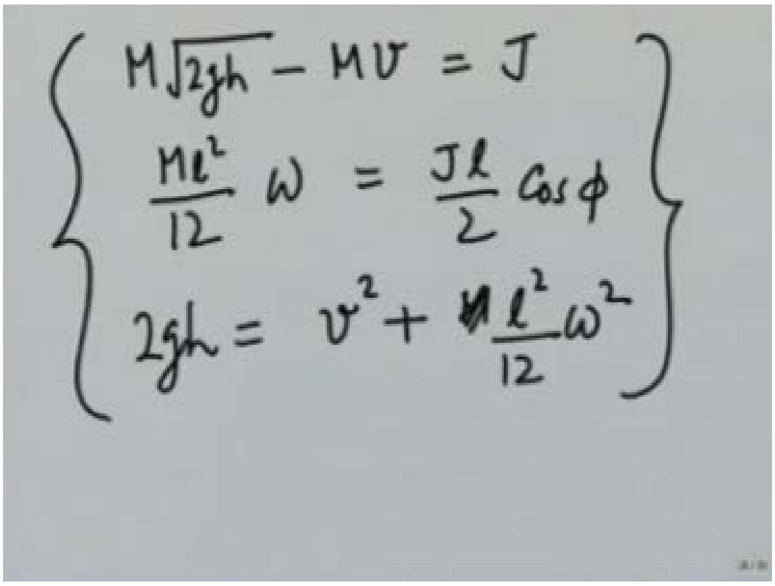

M square root of 2 g h minus MV equals J, then I have M l square over 12 omega equals J, l over 2 cosine of phi. Third equation I have is 2 g h is equal to v square plus l square over 12 omega square. Substituting things property I can get my answers from this and this I leave as an exercise for you to solve. As a last example I take is a very interesting 1 and very famous one.

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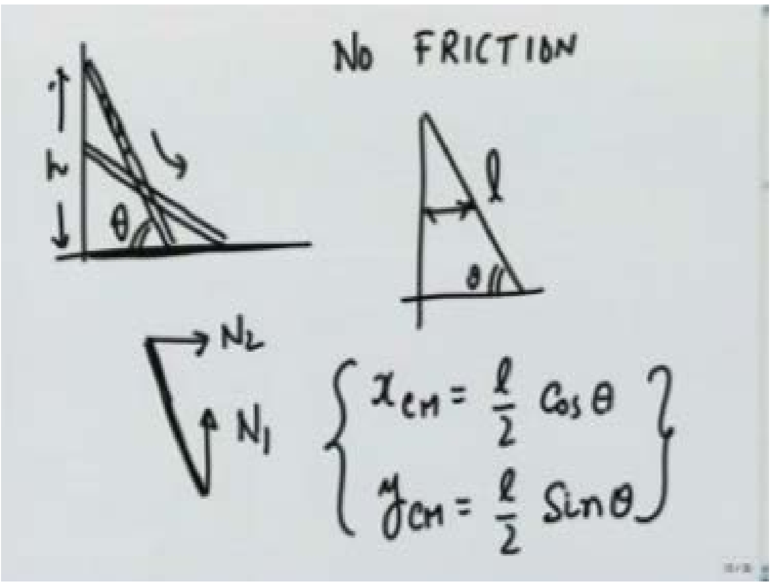

It is if I have a ladder leaning against a wall and on the floor here. Suppose, there is no friction. In the ladder, obviously, would start slipping like this. The question is, if initially it was at height h, at what height does the upper end of the ladder leave the wall. If I make a free body diagram for the ladder, there is going to be a reaction N 1 here and N 2 here. The point at which N 2 vanishes is going to be the point where the upper end has left the wall.

So, what we want to determine is N 2 and make it 0 to find this out. Let us see what is happening as the ladder comes down. As the ladder comes down it is center of mass has moved down, not only that, the ladder has rotated about the center of mass. So, it has both a transitional as well as a rotational motion. However, all I need to describe is complete motion is this angle it makes with the floor because the center of mass of the ladder is going to be given by this itself.

So, let us make this picture again. There is a ladder of length l, if this angle is theta center of mass is right here. You can see that x c m is going to be l over 2 this is x c m l over 2 cosine of theta and y opposite center of mass is going to be l over 2 sin of theta.

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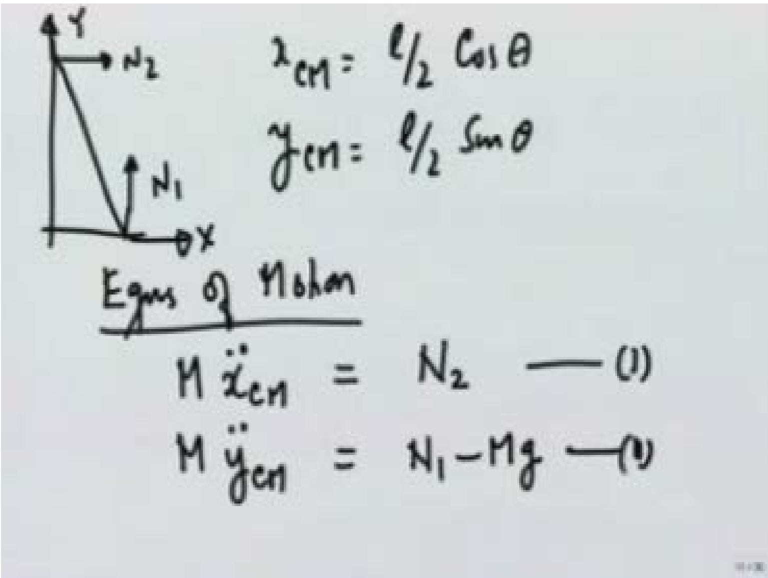

So, for this ladder I have x c m is equal to l over 2 cosine of theta and y c m is equal to l over 2 sin of theta this is N 1 this is N 2. How about the equations of motion mass of the ladder times x c m double dot is. Obviously, going to be N 2 this is X direction this is y direction. Mass of the ladder y c m double dot is going to be equal to N 1 minus Mg.

That is for the translation of the center of mass, equation 1 equation 2. On the other hand it also rotates as it moves and it is rotating like this.

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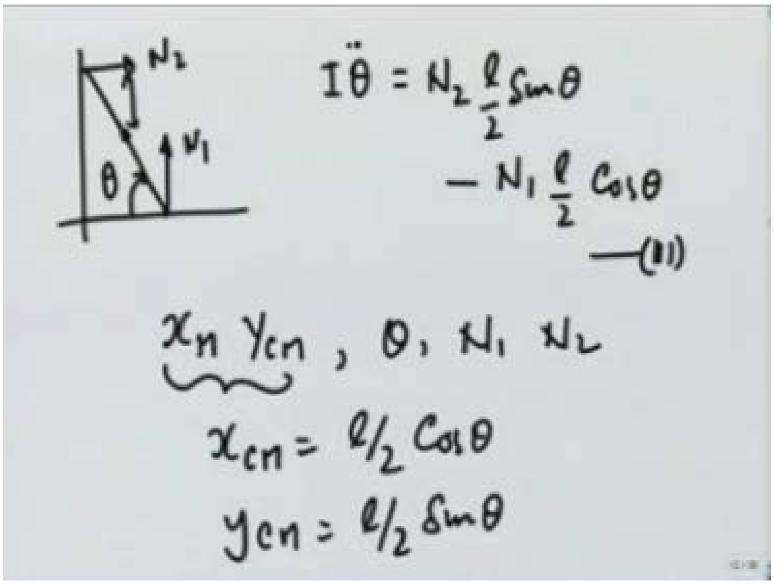

For the rotational motion N 1 N 2 since we are taking theta like this, we will write the equation that way. I am going to have I theta double dot about the center of mass is going to be equal to N 2 times this, distance which is l over 2 sin theta minus N 1 l over 2 cosine of theta. This is 1 equation number three. So, I have 3 equations, but unknown seem to be more x c m y c m theta N 1 and N 2. However, x c m and y c m are not independent because as we saw x c m is equal to l over 2 cosine of theta and y c m this equal to l over 2 sin of theta.

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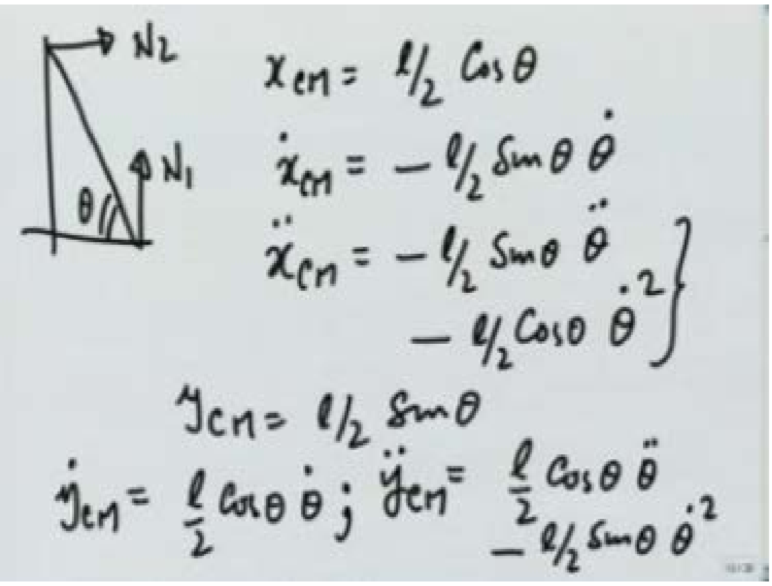

Let us now write expressions for N 1 and N 2. Let me just make sure that N 1 and N 2 are the ones that I am taking, x c m is equal to l over 2 cosine of theta. Therefore, x c m dot is equal to minus l over 2 sin theta, theta dot and therefore, x c m double dot is equal to minus l over 2 sin theta, theta double dot minus l over 2 cosine theta, theta dot square.

Similarly, let us derive y c m double dot, y c m is equal to l over 2 sin theta. Therefore, y c m dot is equal to l over 2 cosine of theta, theta dot and y c m double dot is equal to l over 2 cosine of theta, theta double dot minus l over 2 sin theta, theta dot square. Therefore, we can write expressions for N 1 and N 2.

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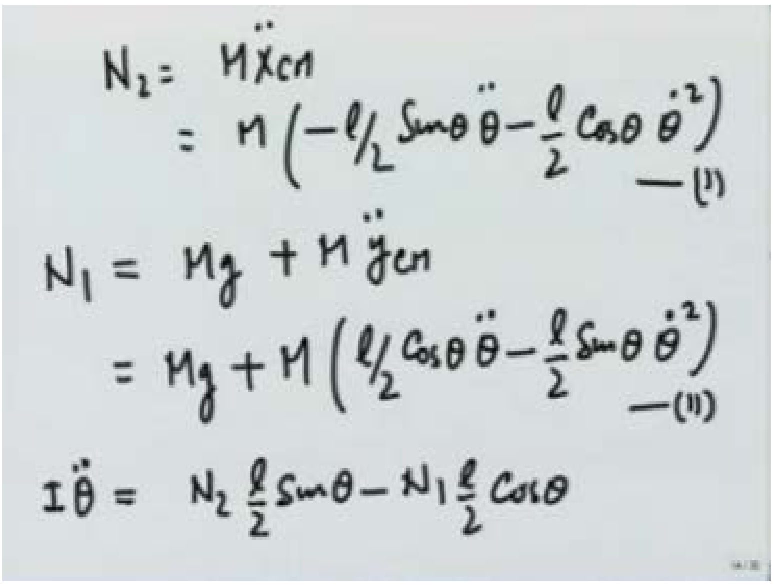

N 2 would be equal to M times x c m double dot which is M minus l over 2 sin theta, theta double dot minus l over 2 cosine theta, theta dot square. N 1 would be equal to Mg plus M y c m double dot which gives me Mg plus M l over 2 cosine theta, theta double dot minus l over 2 sin theta, theta dot square. Let us see, if I substitute it for y theta double dot correctly y CM double dot correctly and I get.

So, these are equations that give me N 1 and N 2. Similarly, I theta double dot is equal to N 2 l over 2 sin theta minus N 1, l over 2 cosine theta as we wrote earlier let us check that.

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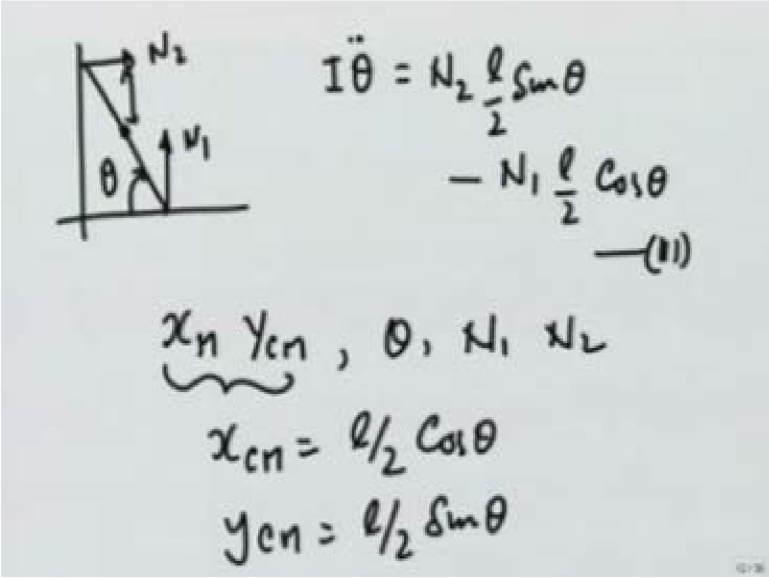

I theta double dot is N 2 l over 2 sin theta minus N 1 l over 2 cosine theta. Now, if I substitute for N 1 and N 2 here let us see what do we get.

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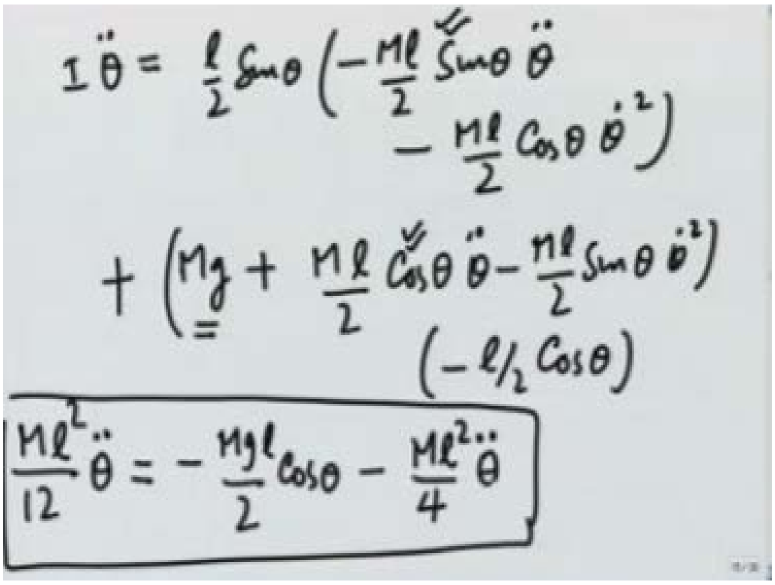

We get, I theta double dot is equal to l over 2 sin theta N 2 is minus Ml over 2 sin theta, theta double dot minus Ml over 2 cosine theta, theta dot square plus Mg plus Ml over 2 cosine theta, theta double dot minus Ml over 2 sin theta, theta dot square times minus l over 2 cosine of theta.

Let us see, what do we get when we add these two. So, on the left hand side I get Ml square over 12 theta double dot is equal to minus M g l over 2 cosine of theta that takes care of this term. Then I am going to have minus Ml square by 4 cosine square theta coming from here, minus Ml square by 4, sin square theta coming from here. That will give me minus Ml square over 4 theta double dot that takes care of this term and this term.

Finally, I am going to have plus Ml square by 4 sin theta, cosine theta, theta dot square from here and plus Ml square by 4 sin theta cosine theta, theta dot square from here. So, that term cancels. So, this is my final equation when I substitute it for N 1 and N 2 into the equation for torque.

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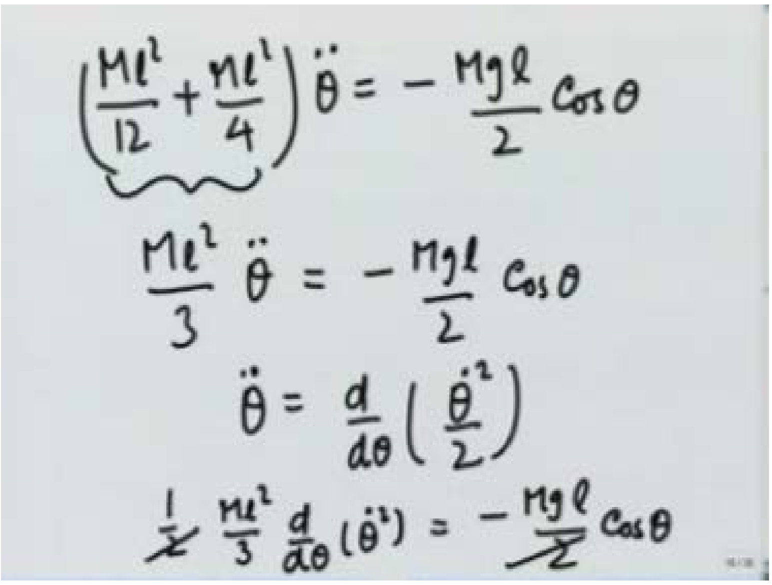

If I take this over to the other side I am going to get Ml square over 12 plus Ml square over 4, theta double dot is equal to minus M g l over 2 cosine of theta. Let us check that if that is correct and that is correct. This is Ml square over 3, theta double dot is equal to minus M g l over 2, cosine of theta. I want to integrate this equation and therefore, I am going to write theta double dot as equal to d over d theta of theta dot square over 2.

Therefore, this equation becomes 1 half Ml square over 3 d over d theta, theta dot square is equal to minus M g l over 2 cosine of theta.

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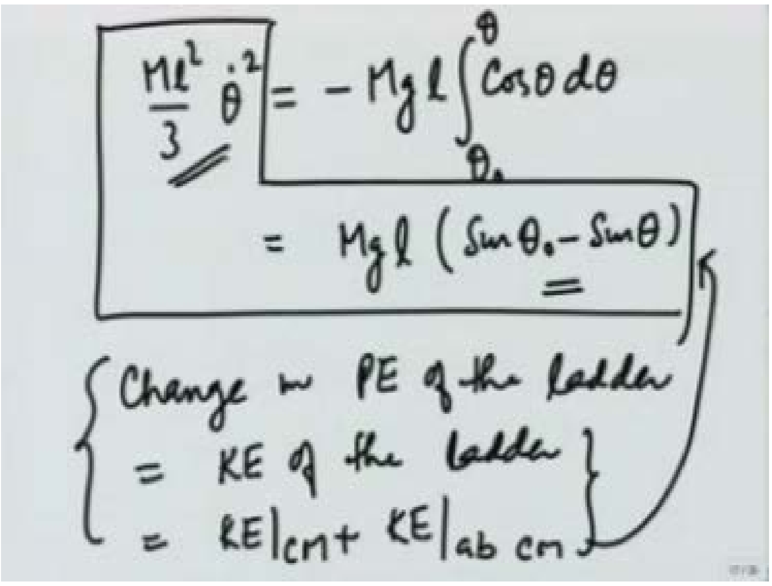

That this 2 cancels and that gives me and that is M g l sin theta 0 minus sin theta. This is the final equation that I get. Note here this is really nothing but the energy conservation equation. I would leave it for you to find out that. The change in PE of the ladder is equal to the KE of the ladder which is equal to KE of c m plus KE about c m would directly lead you to this equation, but I wanted to derive this equation using equations of motion.

Nonetheless this equation is now the equation that would lead to the answer. So, please try this that derive this directly from the energy conservation equation. This divided by 2 would give you the change in the potential energy and these 2 terms together would give you this term.

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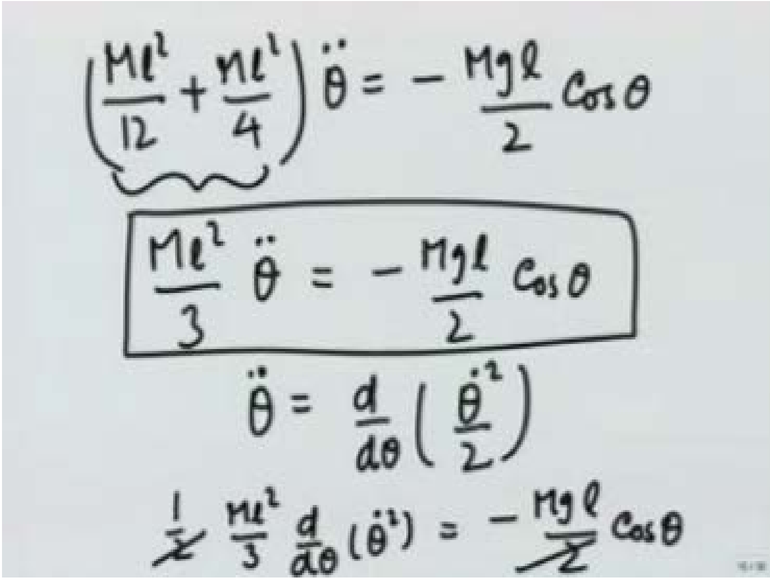

So, I have Ml square over 3 theta dot square is equal to M g l sin theta 0 minus sin theta. So, I get the value of theta dot square. Similarly, I already have Mg, the value for theta double dot and if I substitute this and the value for N 2 I will get an expression for N 2. Let us go to the expression for N 2. N 2 was equal to minus Ml over 2 sin theta, theta double dot minus Ml over 2 cosine theta, theta dot square and let us substitute.

Let us cancel this, let us cancel M and this gives me theta dot square is equal to 3 g over l sin theta 0 minus sin theta. Similarly the equation for theta double dot which we had derived earlier this one. Ml square over 3 theta double dot equals minus M g l over 2 cosine theta let us write that. Ml square over 3 theta double dot is equal to minus M g l over 2 cos theta. Let me make sure this is the equation.

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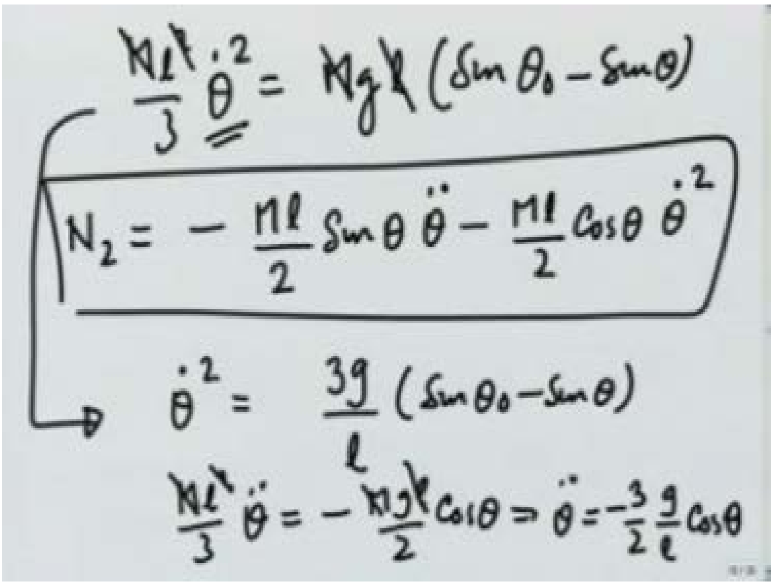

Ml square over 3 always minus m g l over 2 cosine theta. Gives me M cancels, gives me theta double dot is equal to minus 3 divided by 2 g over l cosine of theta. Substituting these in the expression for N 2 here we get.

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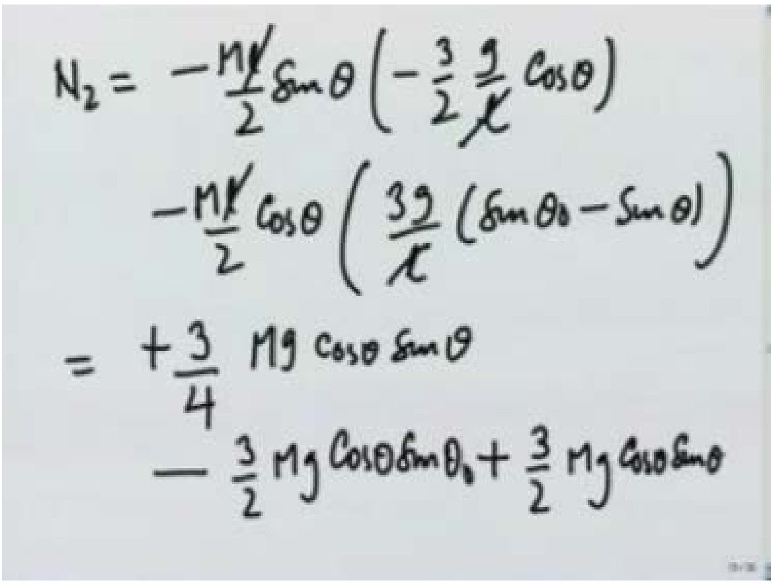

N 2 is equal to minus Ml over 2 sin theta. Let me substitute for theta double dot which is minus 3 over 2. G over l cosine of theta and the next term is minus M l over 2 cosine theta. Minus M l over 2 cosine of theta and for theta dot square I get 3 g over l sin theta 0 minus sin theta. 3 g over l sin theta 0 minus sin theta. So, this gives me minus 3, this l cancels 3 over minus minus plus 3 over 4 3 over 4 Mg cos theta sin theta minus. This l also cancels 3 over 2 Mg cos theta sin theta 0, minus minus plus 3 over 2 Mg cos theta sin theta.

Let us add all of them together and we get N 2 is equal to minus 3 over 2 Mg cosine theta sin theta 0 plus 3 over 4 plus 3 over 2 is 9 fourth Mg cosine theta sin theta. The point where the ladder leaves the wall N 2 is 0 and this gives me 3 over 2 sin theta 0 is equal to 9 over 4, sin theta or sin theta equals 2 third sin theta 0. So, if we start with an angle theta 0 at sin theta equals 2 third sin theta 0 a ladder would leave the wall. So, let us see the picture again.

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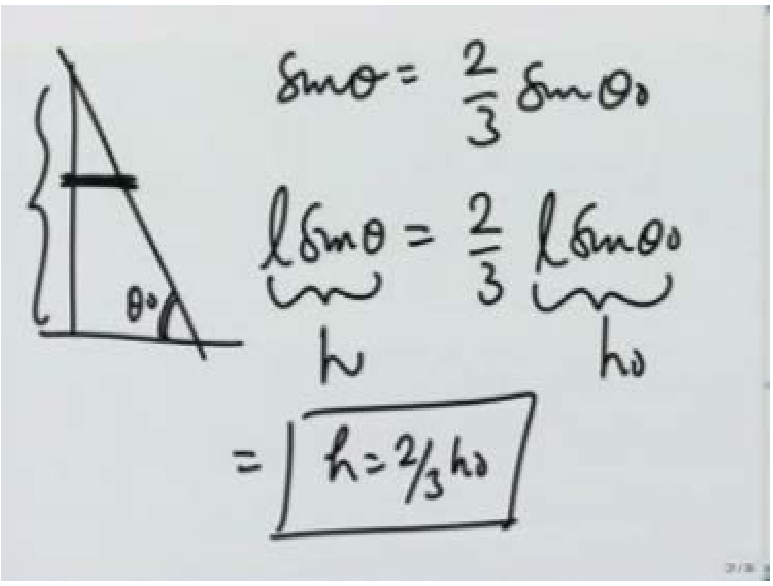

This is theta 0 at 2 thirds of sin theta 0. That means, with a height 2 thirds of the original height the wall and ladder would separate. So, sin theta equals 2 thirds sin theta 0 or l sin theta is equal to 2 thirds l sin theta 0, L sin theta 0 is the initial height. This is the height at which the ladder leaves the wall. So, h equals at 2 thirds the original height a wall leaves. A wall and ladder separate. So, this is another example of how the combination of translational and rotational motion can be solved.

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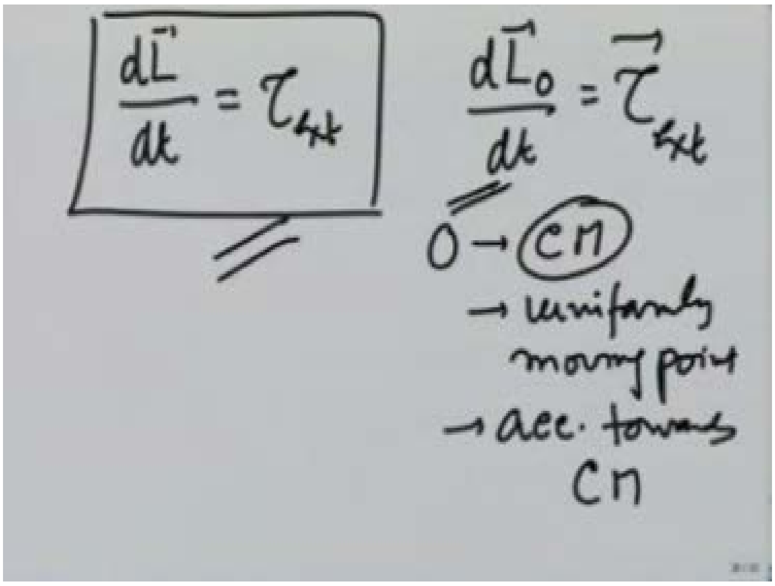

So, what we have done in this lecture so far, is considered various applications of the equation d L over d t equals tau external. In those problems where the axis of rotation remains fixed or it translates parallel to itself. In that case we saw that we apply d L o over d t is equal to tau external, where O is either the CM or a uniformly moving point or accelerating towards CM.

The safest point to take always a CM because that you are sure off. With these we got certain solve certain interesting problems. You can keep making problems more complicated, but the application remains basically of these equations. What I would urge you now is apply these to as many difficult problems as you can. Good reference books to find such problems are Merriam dynamics and Kleppner and Kolenkow's of introduction to mechanics.