Engineering Mechanics
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Module - 05
Lecture - 01
Motion of Particles Planar Polar Co-ordinater

So, for we have been dealing with statics, this is the first lecture in dynamics.

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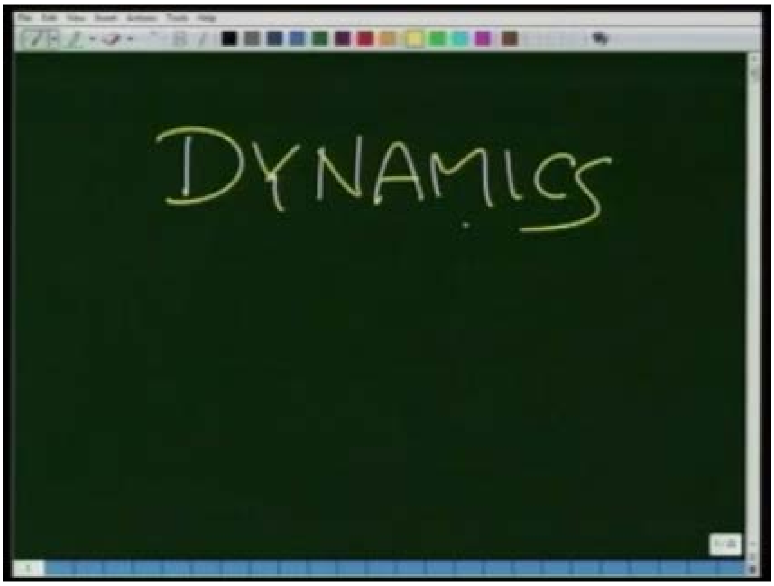

So, we start with how to describe the motion of a particle using different coordinate systems. As you well know in mechanics, if I want to describe the motion of a particle.

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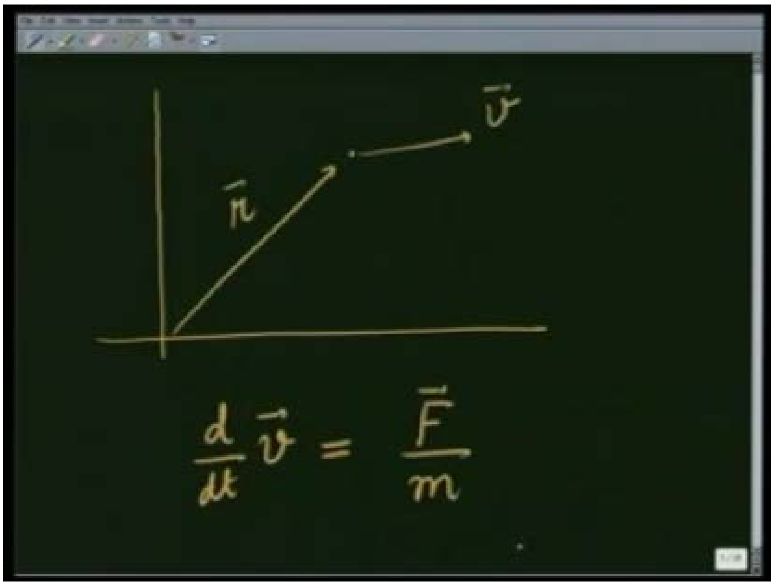

It is described using its position given by vector r, and its velocity. With time the velocity changes, and the velocity changed over dt is related to the force applied F divided by the mass of the particle by Newton second law. So, to describe the vectors I need to fix a coordinate system with respect to which I describe the motions.

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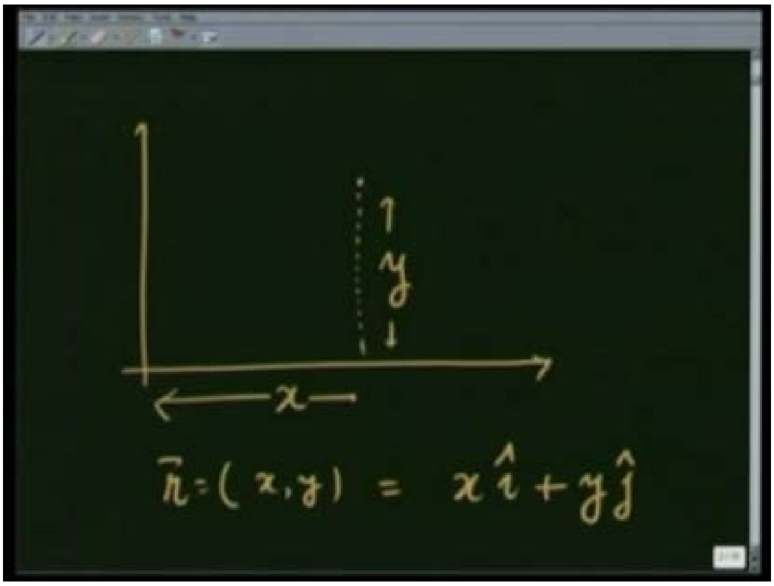

For example, you are well familiar with the Cartesian coordinate system where in a plane, the position of a particle is given by its x coordinate and the y coordinate. So, that

I would write that r in this plane is given by x y or the vector x unit vector I in x direction plus y unit vector j in y direction.

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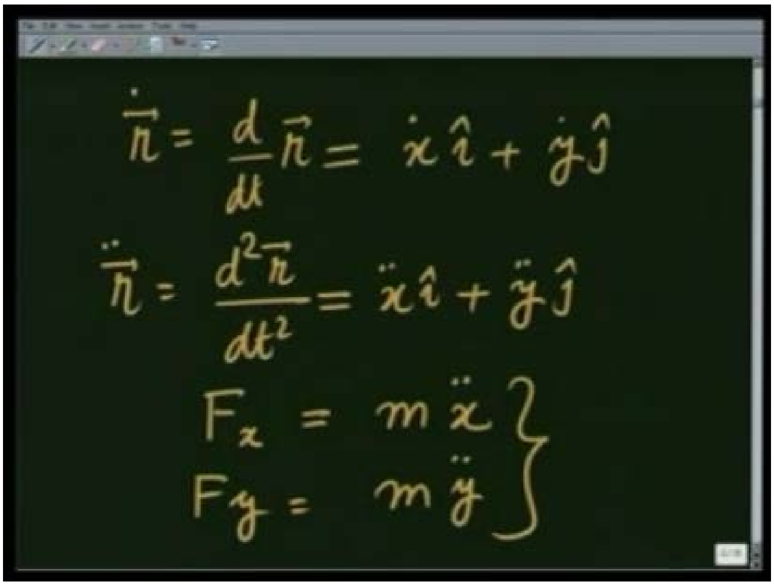

The velocity accordingly is given as r dot, this dot is given to indicate the time derivative with respect, derivative with respect to time, which is equal to x dot i plus y dot j. Similarly, the acceleration r double dot is given as d 2 r over dt square which is equal to x double dot i plus y double dot j. And the Newton second law then takes the form that I equate the components along each direction with respect to forces.

So, that Fx becomes equal to m x double dot, Fy becomes equals to m y double dot. And I can integrate these 2 equations of motion to get how x and y change with time. However, there may be situations where describing the motion in x and y coordinate may not be as easy as we think.

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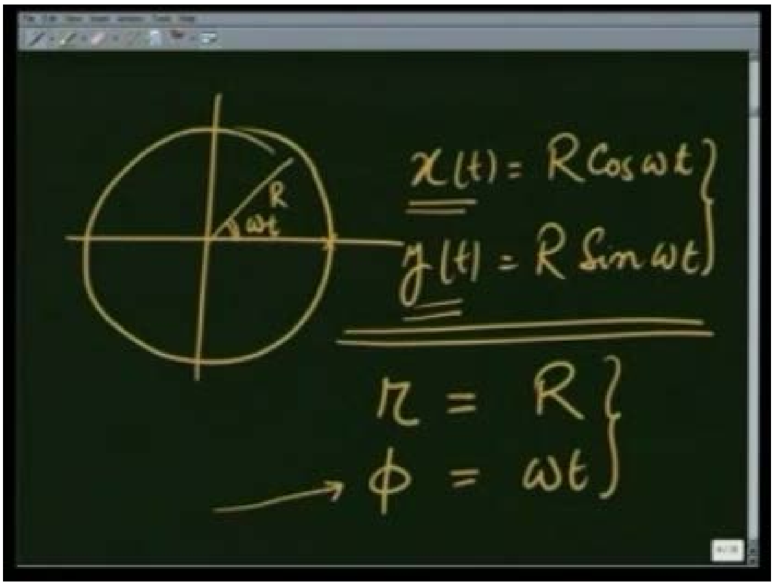

For example, if there is a particle moving in a circle. Let us say of radius R with angle of velocity omega. So, that this angle is omega t that the particle started from here. When its x coordinate with time would be given as R cosine of omega t, and the y coordinate will be given as R sin of omega t, both are functions of time.

On the other hand I could very well write this position as the radius of the circle being constant with time equal to R, and the angle of the radius vector to be equal to omega t. You appreciate the difference between the 2 descriptions, one in terms of x and y coordinates, and the other in terms of r and phi coordinates. In this case both x and y are functions of time whereas, in this case only phi is the function of time.

So, that effectively the change of coordinate with time is in only 1 coordinate. this is known as planar polar coordinate system. and we are going to explore it further. As you can well imagine, this is most useful system when a particle is making rotatory motion or moving around in a circular manner. So, let us start with the description of planar polar coordinate system.

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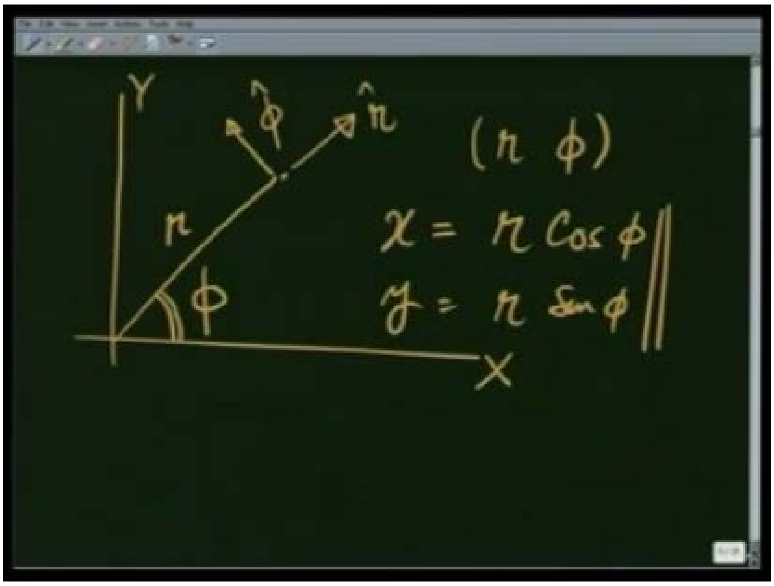

In this system, if a particle is given at a point distance r from the origin, and the radius vector r is making angle phi from the X axis. I am going to describe this as position r and phi. As you can see, I am using two coordinates in a plane. Obviously, x coordinate is given as r cosine of phi, and the y coordinate is given as r sine of phi.

I also need to give the unit vectors in this coordinate system. If I want to describe a vector quantity, the unit vectors are given as r unit vector in this direction. And the other unit vector is in increasing phi direction perpendicular to r so, it is in this direction which I will call the unit vector phi. To make it more clear, let me redraw it and show you these vectors more clearly.

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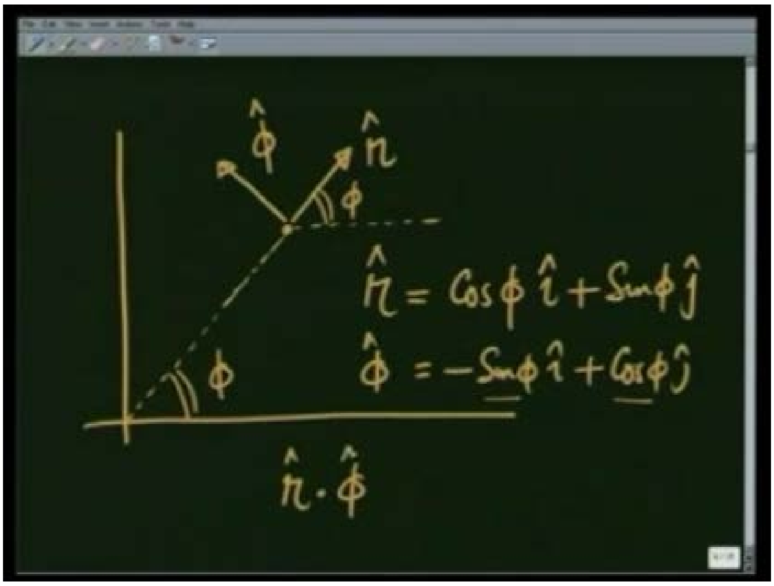

So, for a particle here, I have the unit vector r and unit vector phi, this angle is phi and so is this. You can already see that r unit vector is equal to cosine of phi unit vector i plus sine of phi unit vector j. Since, cosine square phi plus sin square phi is 1, the magnitude of unit vector is obviously 1.

Similarly, unit vector phi is going to be equal to minus sin of phi i plus cosine of phi j. Again, the magnitude of phi unit vector is sin square phi, plus cosine square phi which is 1. You can also see right away, that r dot phi that is there dot product is equal to 0, which implies that the two unit vectors are orthogonal to each other. It is clear from their definition, and let me write them separately.

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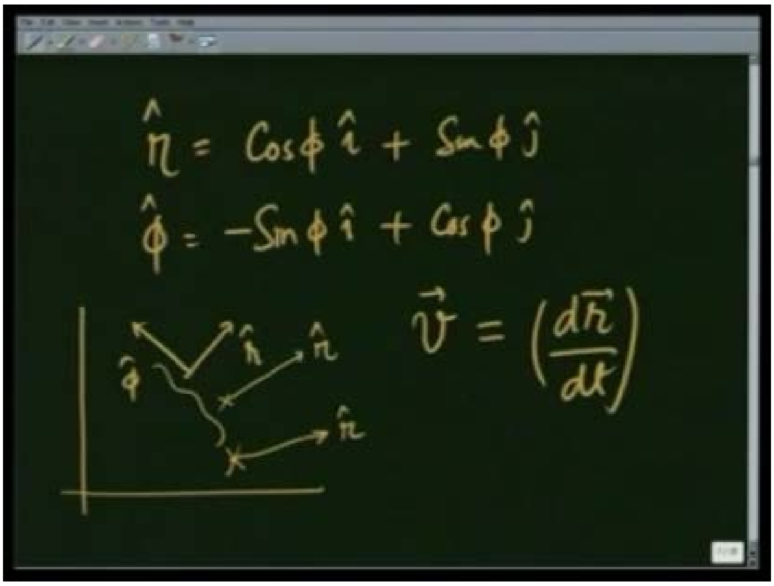

Now, r is equal to cosine of phi I, plus sin of phi j, phi is equal to minus sin of phi i, plus cosine of phi j, just to remind you again. For a given position, this is my r unit vector, and this is my phi unit vector. So, it is clear from their definition that, the unit vectors r and phi depend on what phi is, and what that means, is as I move around depending on the position of the particle, the unit vectors are going to change.

For example, at this position the unit vector r is going to be in this direction, on the other hand at this position the unit vector r is in this direction. So, it changes as the particle moves around, and in calculating the velocities and accelerations in planar polar coordinates, I need to take this change into account.

For example, if I calculate the velocity which is given by dr by dt, as the particle moves around both the magnitude of r and its direction are changing and in the next few minutes we will see how we take that into account. To start with, let us start with the position of the particle.

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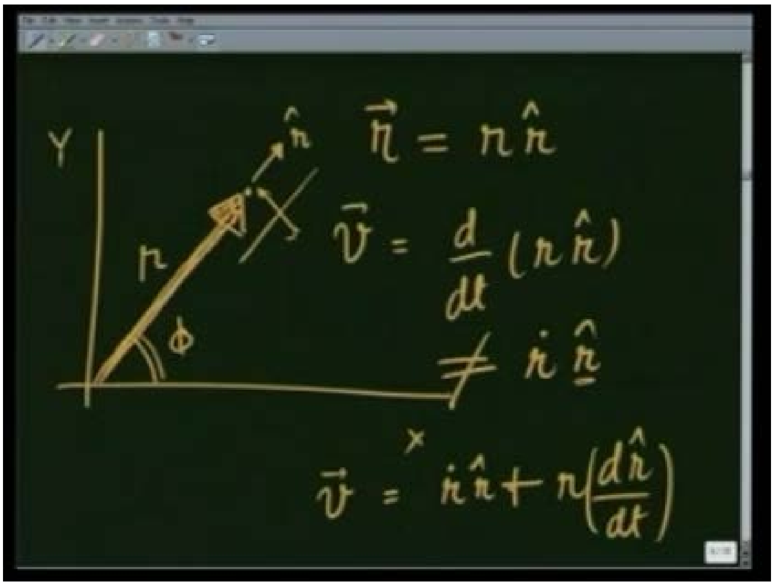

So, a particle is given here at a distance r from the origin, making an angle phi from the X axis. So, position vector r of the particle is going to be given as r, r unit vector. Because this is the displacement vector of the particle, and this is unit vector r. Velocity is going to be d over dt, r, r unit vector, and this is not equal to just r dot r because particle is not just moving in this direction, it is also moving in this direction.

So, and also other way, both r and unit vector r depend on phi, depend on the position of the particle. So, the velocity really should be written as r dot r plus r, dr, dt, that is the rate of change of the unit vector r itself. Let us see how much it is.

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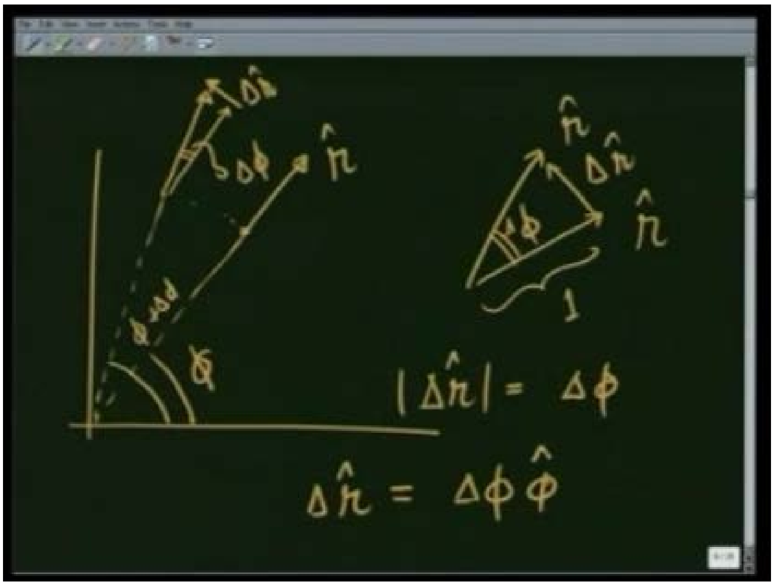

So, if I were to make the unit vector r again. Here is the particle, here is unit vector r, as the particle moves let us say in this direction. So, that phi changes from phi to phi, plus delta phi the unit vector has changed by this much, initially if I move this parallely the unit vector was in this direction.

So, this is the change in unit vector r, how much is this angle? This angle is delta phi. If I redraw it, initially vector was like this, the final unit vector is like this. This is the change and this angle is phi. Since, this magnitude is 1, you can see the delta r magnitude is going to be of, this is delta phi sorry, is going to be delta phi. And the direction, it is in the direction of increasing phi, the way I have made it therefore, delta r is going to be equal to delta phi, phi. Let us see from a mathematical point of view, recall from an earlier slide that.

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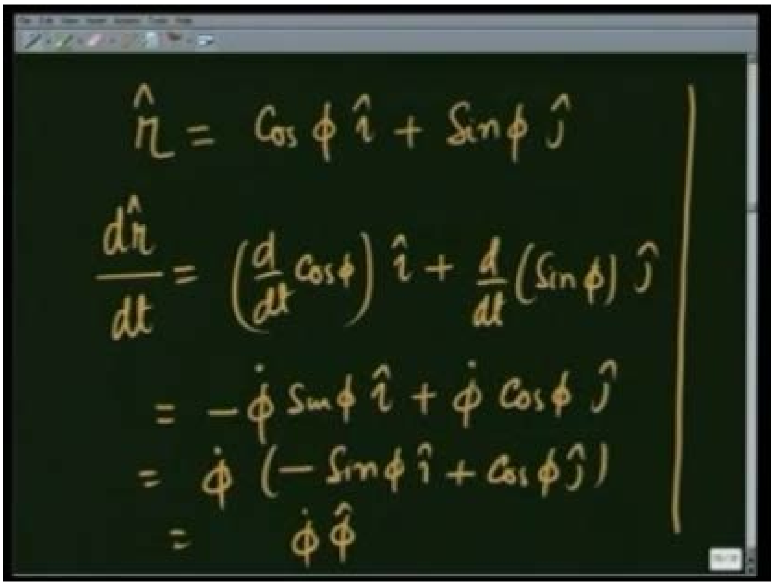

The unit vector r was written as cosine of phi i, plus sin of phi j and therefore, rate of change of the unit vector dr dt is going to be equal to d over dt of cosine of phi, i unit vector is a fixed unit vector so, it is constant, plus d over dt of sin of phi j. And this comes out to be minus phi dot, sin of phi i plus phi dot, cosine of phi j, which is equal to phi dot, minus sin phi i plus, cosine phi j which is phi dot phi. So, we see from the definition also that, the rate of change of unit vector r is phi dot phi. It is good see a derivation mathematically as well as geometrically.

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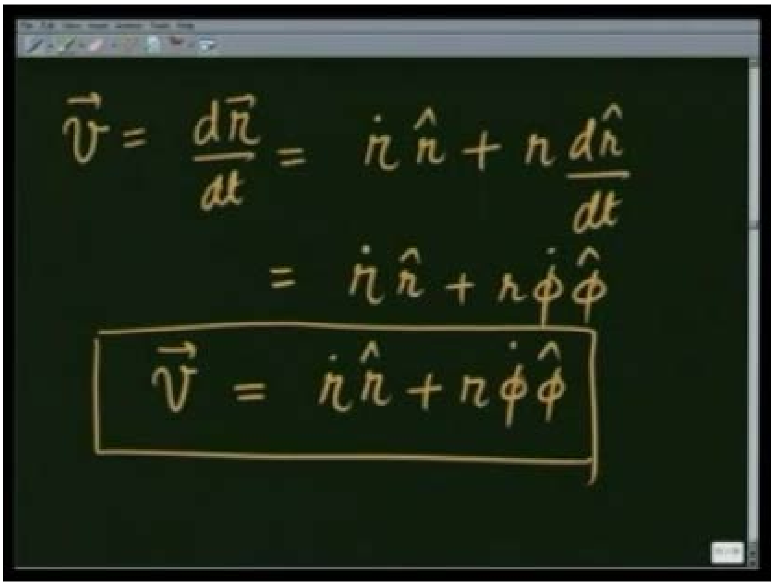

So, if you write the velocity v as dr dt, this is equal to r dot r plus r, dr, dt, which is going to be equal to r dot, r plus r, phi dot phi unit vector. So, the velocity of a particle in planar polar coordinates, I want to remind you again, we are talking about motion in a plane, is equal to r dot r unit vector, plus r phi dot phi unit vector. Let us look at it geometrically.

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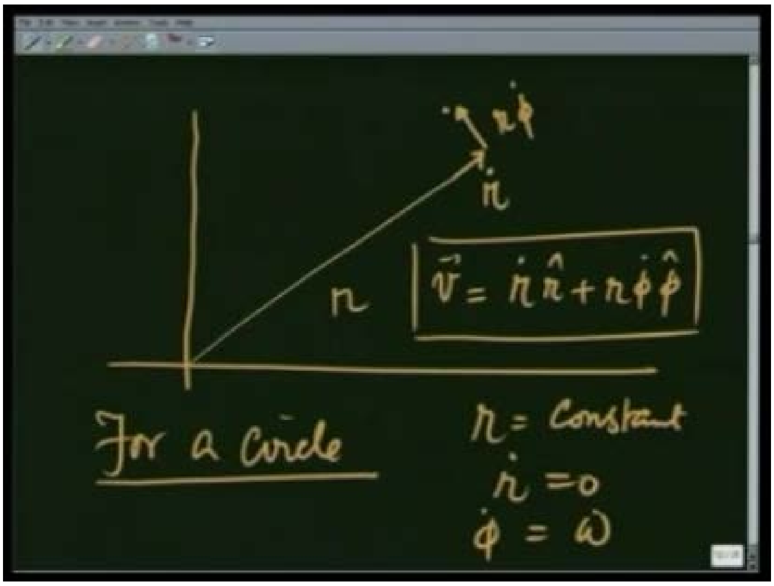

So, a particle is moving around at a distance r, as it moves to its new position, it has moved this way at a rate of r dot, and it has moved this way at the rate of r phi dot. So, that its net velocity is a combination of r dot in r direction, plus r phi dot in phi direction. To make it look more familiar, let us look at a particle which is moving around in a circle. For a circle, that is if a particle is moving in a circle, r is a constant. And therefore, r dot 0 and phi dot is the angular speed and therefore, its velocity is going to be given as.

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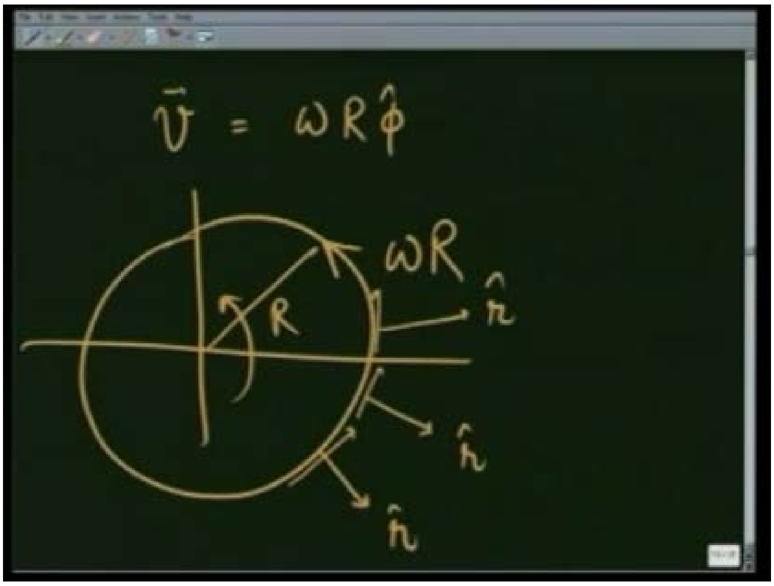

V is equal to omega R phi, that is if a particle is moving around in a circle. It is moving in direction phi with the speed omega R, R is the radius of the circle, omega is at which the angular speed to the particle. So, you see, I am describing the same motion, except now on using planar polar coordinates. And it is important to keep in mind that, when I am considering this motion, the R unit vector keeps on changing depending on the position of the particle. If r unit vector changes so, does the phi unit vector, and that is going to be important when we derive the expression for acceleration in planar polar coordinates.

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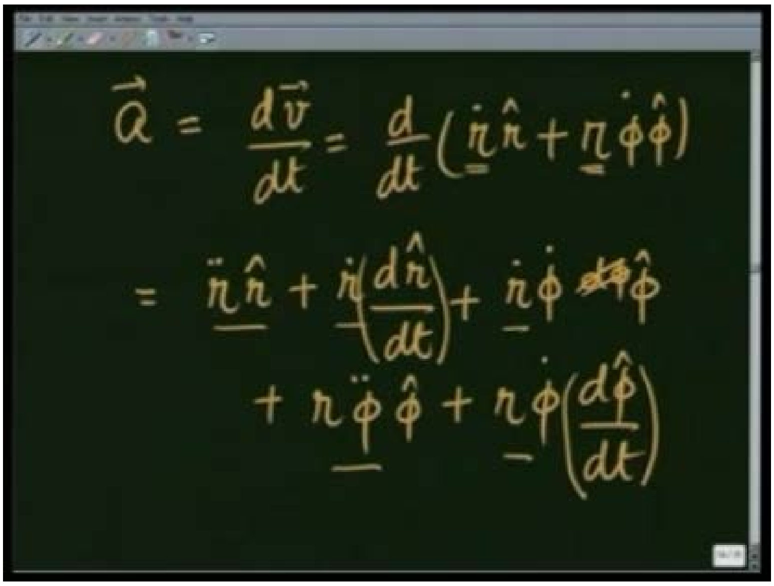

To derive the acceleration I just differentiate the velocity of the particle dv, dt and this is going to be equal to d over dt of the velocity in planar polar coordinates r dot r, plus r phi dot phi, which is equal to r double dot r, that is I differentiated this term, plus r dot dr dt, plus r dot phi dot. I differentiated this term d, sorry this is phi unit vector, plus r phi double dot phi unit vector, plus r phi dot d phi dt. So, I have 1, 2, 3, 4, 5 terms dr, dt I have already calculated, the new term now is d phi dt, and this is what I am going to calculate next. Collect all the terms and see what the expression for acceleration looks like.

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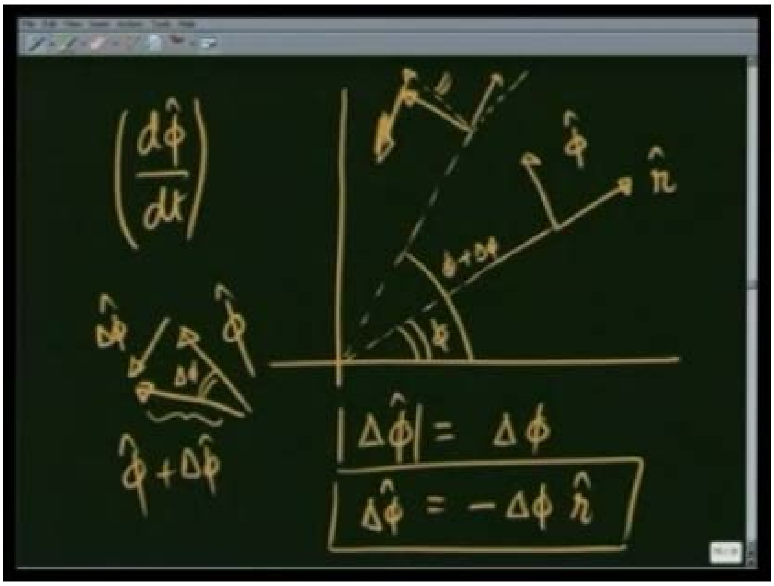

To get d phi by dt, I will first again look at it geometrically and then, derive it from the definition. If you look at it geometrically, this was my r unit vector and this is the phi unit vector. When I move to a new position, this was phi, this is phi plus delta phi, both unit vectors I have also moved from their original direction by angle delta phi.

This was my original phi. So, original phi was something like this, which is this vector, and the new phi is like this, and this is the change delta phi in the unit vector. You can already sense that, this is in the radial direction, but pointing the other way. So, delta phi is going to be equal to, this angle is delta phi, this magnitude is 1. So, this is going to be equal to, the magnitude is going to be equal to delta phi itself, and the direction is in the direction of r, but in the opposite direction so, minus delta phi r.

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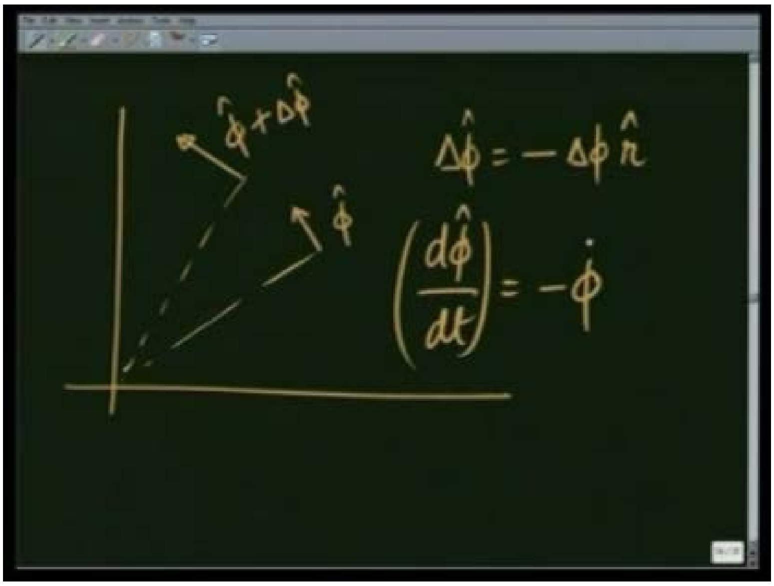

I show it again, this was my original phi, after the particle moved phi has become this. So, let me write this as phi plus delta phi, and delta phi comes out to be minus delta phi r unit vector, and therefore, d phi over dt is nothing but minus phi dot r. I shown it purely on geometric will bases, I will go to the definition of.

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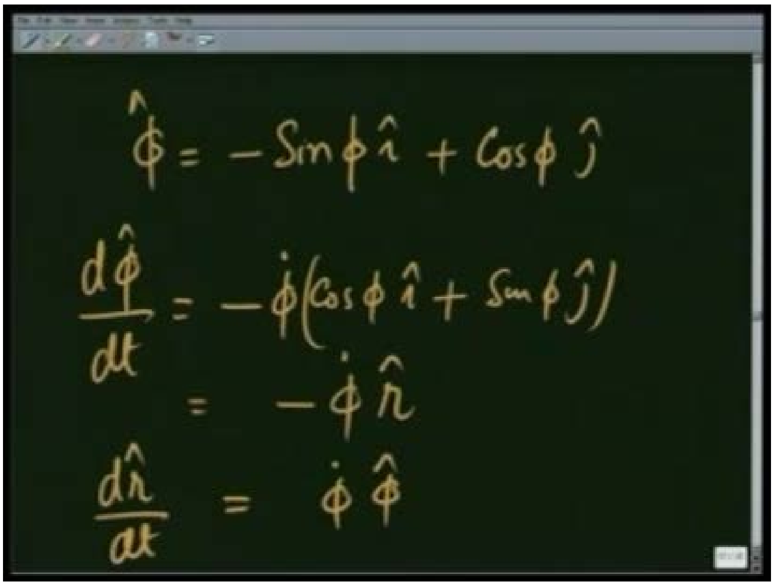

Phi unit vector which was minus sin phi i, plus cosine of phi j, and differentiate to get minus phi dot cosine phi i, plus sin phi j. I have left the steps in between because I have already work them out, and this is minus phi dot r unit vector, gives the same answer as by geometrical argument. So, now we have both dr dt and d phi dt, dr dt, you recall was equal to phi dot in phi direction. And therefore, I can write an expression for the acceleration in planar polar coordinates.

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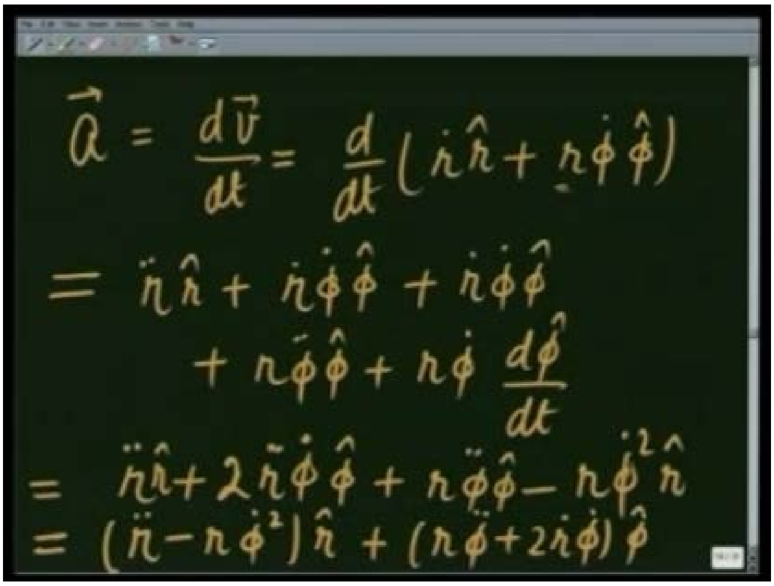

Let me write it again, a was equal to dv, dt which is equal to d over dt of r dot r, plus r phi dot phi which is equal to r double dot r, plus r dot dr dt, which is nothing but phi dot in phi direction, plus r dot from here phi dot in phi direction, plus r phi double dot in phi direction, plus r phi dot d phi dt, which are just calculated. Collecting all terms together you will see, this comes out to be r double dot plus 2 r dot r dot phi dot this is r double dot in r direction r dot phi dot in phi direction plus r phi double dot in phi direction and d phi by dt recall is minus phi dot r. So, this becomes minus r phi dot square in r direction which therefore, can be written as r double dot, minus r phi dot square in r direction, plus r phi double dot, plus 2 r dot phi dot in phi direction.

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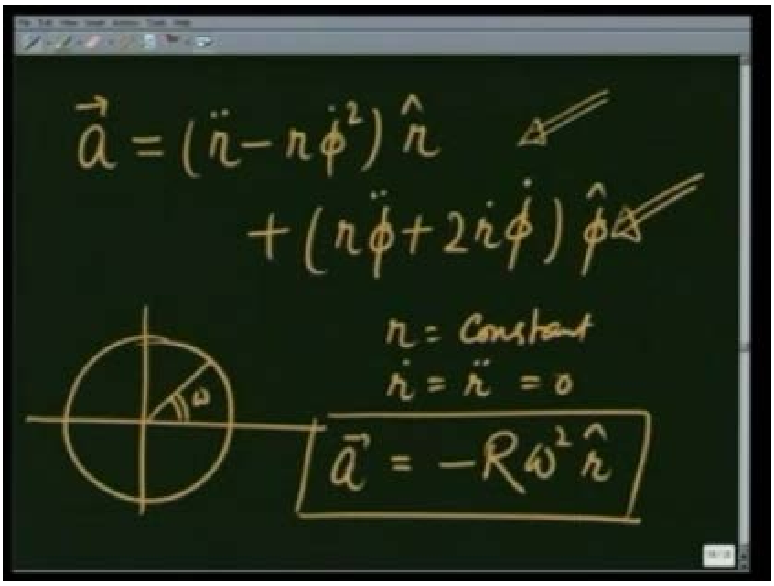

Let me rewrite it for you again this is equal to r double dot, minus r phi dot square in r direction, plus r phi double dot, plus 2 r dot phi dot in phi direction. This relationship looks quite complicated, let me give you again a familiar example of a particle moving in a circle, for which r is constant and therefore, r dot is equal to r double dot is equal to 0. And if it is moving with constant angular velocity omega then, a comes out to be minus the radius of the circle omega square r, which is your familiar centripetal acceleration.

So, you can see when the particle is moving around in a circle, use of planar polar coordinates makes things slightly easier, although the expressions may look complicated. Now, to apply it, you have to practice it. I will solve 1 or 2 examples using planar polar coordinates for particles moving in a plane, which will give you some idea has to how to use these coordinates. As my first example of application of planar polar coordinates.

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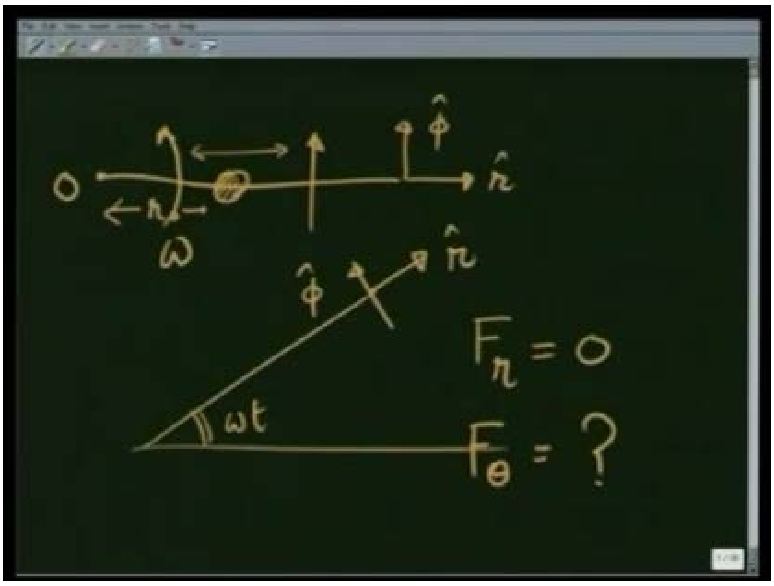

Let me take a wire that is rotating with the constants angular speed omega in a plane, and let me put a bead here at a distance initially r 0 from the center, about which the wire is rotating. And I want to know, how the position of this wire changes with time, how its velocity changes with time, what is the force that the wire applies on the bead? Obviously, if it is frictionless, there is no force applied in this direction, there is force applied in this direction because the wire forces the bead to move in this way. And I am going to describe the motion using planar polar coordinates.

So, right away I see my r direction is going to be this, this is going to be my phi direction. In general if wire is making some angle omega t from its initial position, this is going to be the r direction, and this is going to be the phi direction. The force in the radial direction is 0 say therefore, the component of force in the radial direction I write as 0, and the component of force in theta direction I do not know, but it is non 0, and I wish to calculate this. If I write the acceleration of the particle.

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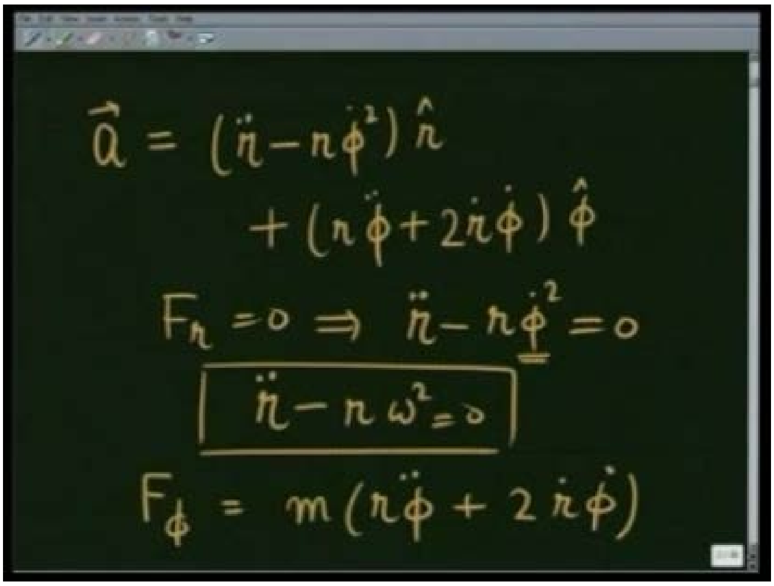

And planar polar coordinates it is going to be r double dot, minus r phi dot square in r direction, plus r phi double dot plus 2 r dot phi dot in phi direction, and I have just told you F r is 0, and this implies r double dot, minus r phi dot square is 0. And since, I know that the particle is moving with the constant angular speed. So, phi dot is the constant which is equal to omega therefore, the equation becomes r double dot, minus r omega square is equal to 0. Similarly, in the theta direction, F theta is going to be equal to m r phi double dot, plus 2 r dot phi dot. Since the wire is moving with the constant angular speed, this term is 0.

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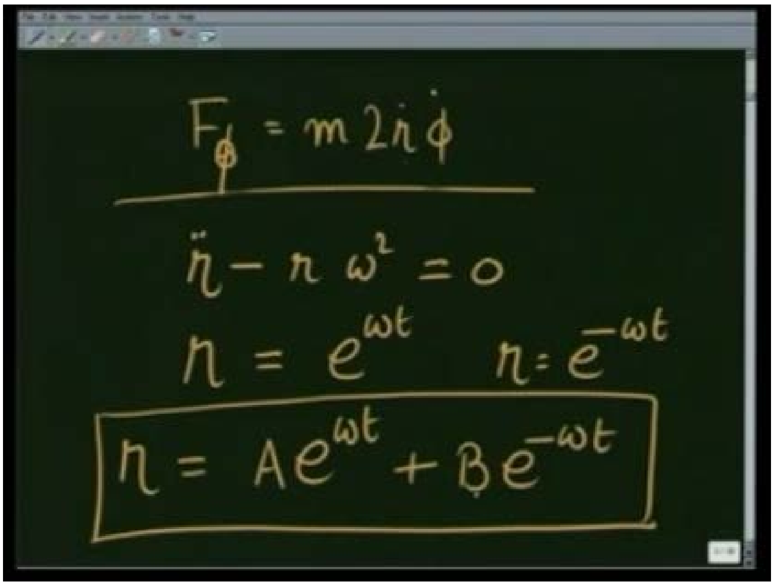

And therefore, I have F theta, that is the force or F phi sorry, that is the force at the wire applies in phi direction to be equal to m 2 r dot phi dot. To start solving the equation, let me go back to the earlier equation r double dot minus r omega square is equal to 0. You can see by inspection that, this has 2 solutions e raise to omega t, or r is equal to e raise to minus omega t.

And therefore, the general solution r is going to be A, where is a constant, e raise to omega t plus B, where B is another constant minus omega t. This is how r is going to change as a function of time. The constants A and B are fixed by the initial condition.

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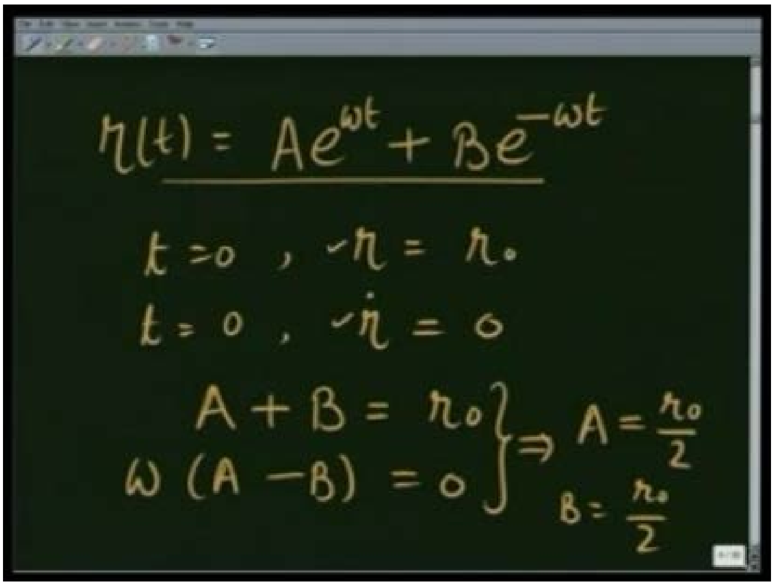

So, the solution r as the function of time is A e raise to omega t, plus B e raise to, minus omega t. And if I am given that at t equal to 0, the particle is at some initial position r 0 and at t equal to 0. Suppose, it was not moving in r direction so, that r dot was equal to 0. Then, I have writing these quantities from this equation at t equal to 0, A plus B is equal to r 0 and if I take the derivative omega A minus B is equal to 0. And these two equations give me A is equal to r 0 over 2, and B is equal to r 0 over 2.

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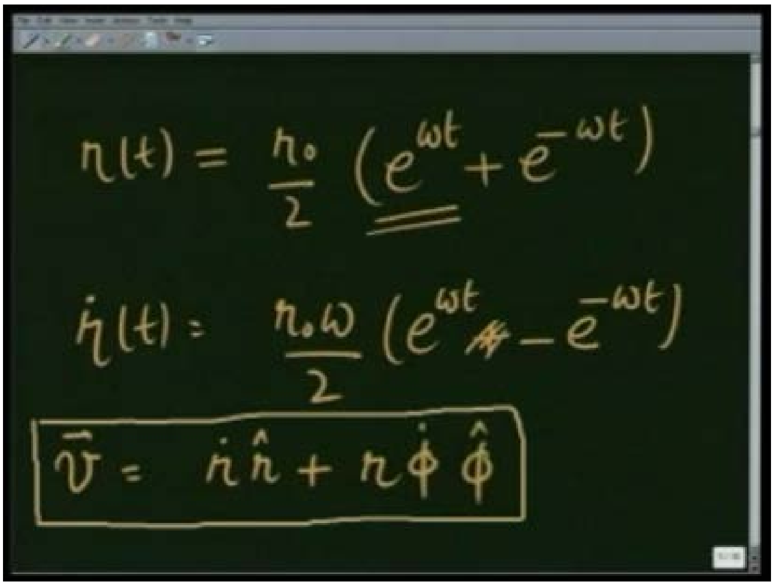

And therefore, the general solution R t for a particle on a wire, that is rotating at a constant angular speed, and the particle is started from a distance r equal to r 0 with no initial radial speed is r 0 divided by 2 e raise to omega t, plus e raise to minus omega t.

You can see r increase a exponentially, r increases very fast with time, how about the velocity? In radial direction is going to be equal to r naught omega, divided by 2, e raise to omega t, plus well, when I differentiate it, it is going to be minus e raise to minus omega t. General velocity v is going to be r dot r, plus r in phi direction, and you can substitute the values and find the general velocity. How about the acceleration of the particle?

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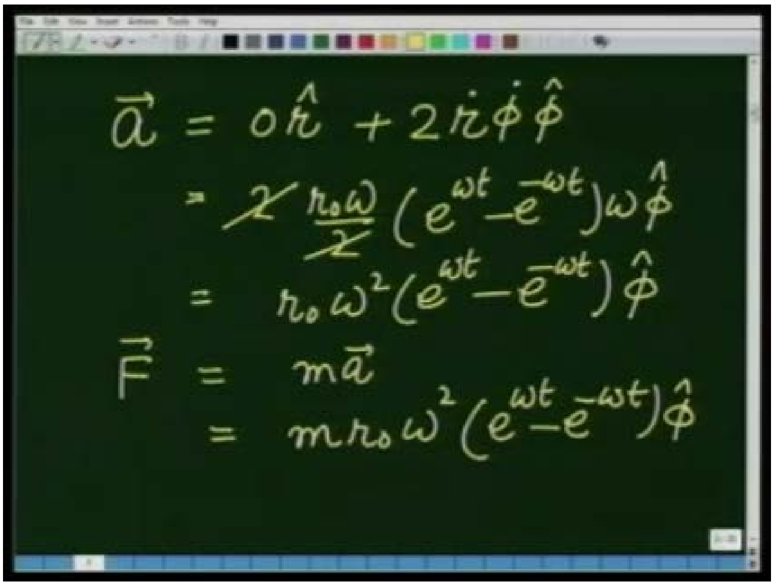

The acceleration as you have already calculated is 0 in r direction, and 2 r dot phi dot in the phi direction. Using the expression for r derived earlier, this comes out to be 2 r 0 omega by 2 e raise to omega t, minus e raise to minus omega t times omega which is the value of phi dot in phi direction. This two cancels, and the acceleration therefore, is r naught omega square e raise to omega t, minus e raise to minus omega t in phi direction.

This is the general expression for the acceleration for this bead moving on the wire. How about the force? The force applied by the wire is given by ma and is therefore, equal to m r naught omega square e raise to omega t, minus e raise to minus omega t in phi direction. So, you see that we have solved this problem in planar polar coordinates in a simple manner. I would urge you that, you try solving the same problem using x and y coordinates.

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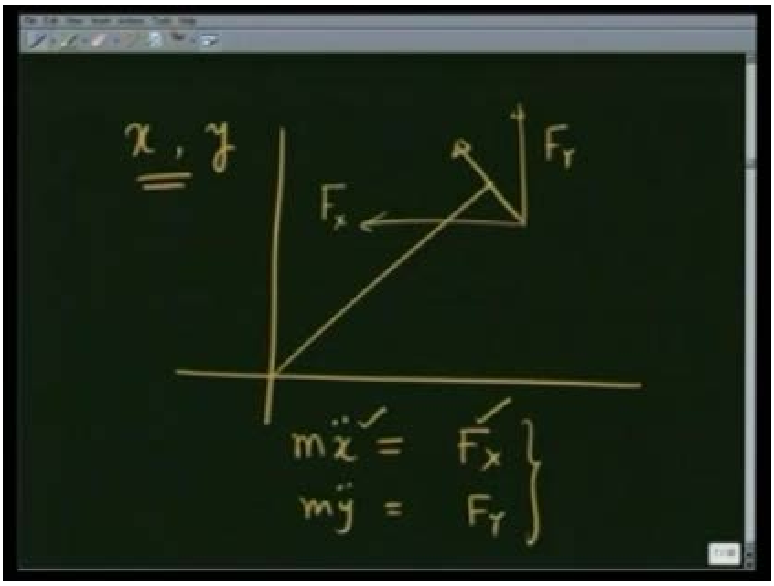

Just to give you an idea when the wire is moving, wire is applying a force like this. which is going to have a component in x direction, which is going to have a component in y direction.

And therefore, the equations you will end up writing would be, m x double dot is equal to Fx, which is going to be the component of the force in this direction m y double dot is going to be equal to Fy which is this component. And solve these with respect to time, F changes with time, x changes with time. This solution is going to be the slightly more complicated. So, in such situations use of planar polar coordinates help us. Also once I get the general solution, an interesting situation is.

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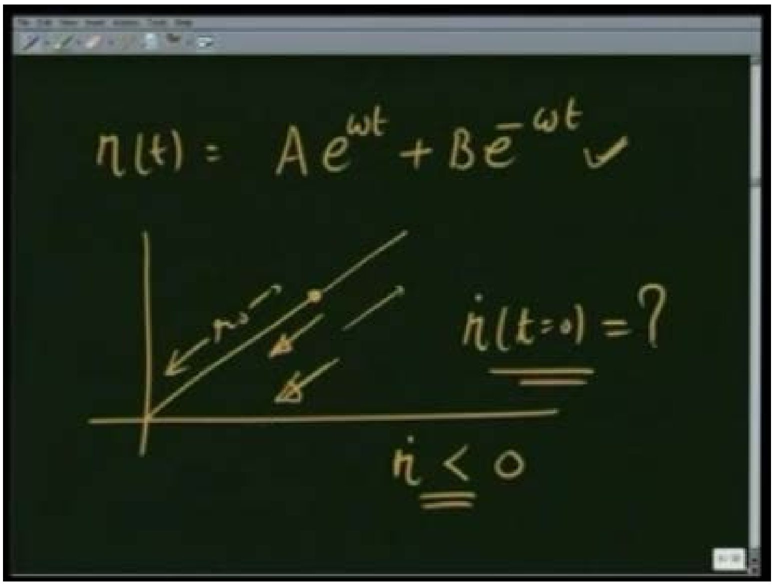

When I ask, which a recall this is e raise to i omega t, plus B e raise to minus omega t. I am going to ask, can I start the bead from a distance r 0 and give it a velocity in this direction so, that the bead never moves out, it keeps on moving in. Look at this, it is just an interesting application of this solution. So, what I am asking is, what should r naught at t is equal to 0 be so, that r dot is always less than 0, that is r keeps on decreasing, let us see that.

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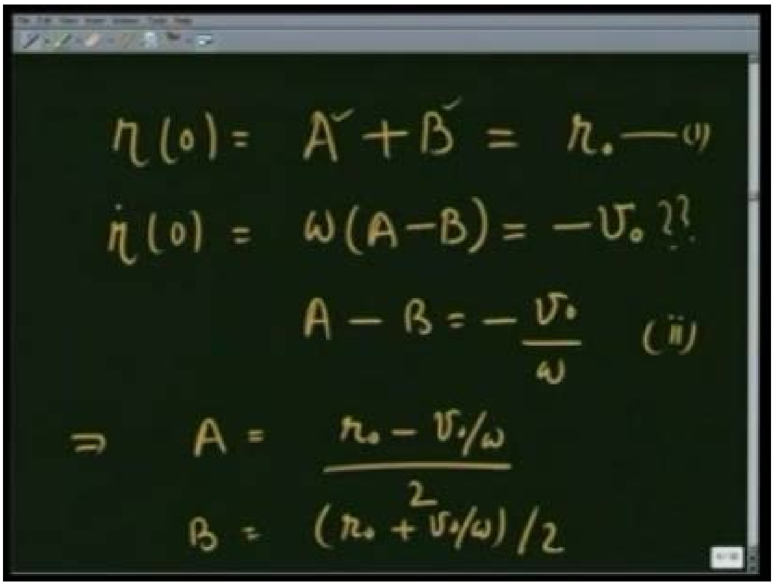

So, I am given at r at 0 which is going to be A, plus B is equal to r 0. I have already calculated r dot and at 0, this is going to be omega A minus B, and I want to have velocity going in. So, minus v 0, I want to find that v 0 with which I push the particle in so, that it keeps on moving in. So, I want to find that velocity, alright.

So, what I want is that, such A and B so, that the particle keeps moving in. This is my equation number 1, equation number 2 becomes A minus B is equal to minus v 0 over omega, this is my equation number 2. And this gives me A is equal to r 0, minus v 0 over omega divided by 2, and B is equal to r 0, plus v 0 over omega divided by 2.

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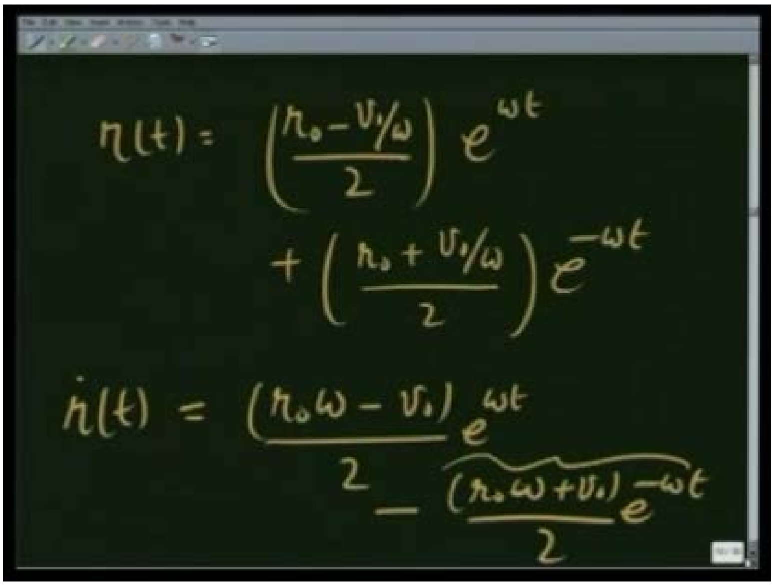

So, my solution is going to be r 0 minus v 0 over omega divided by 2 e raise to omega t, plus r 0, plus v 0 over omega divided by 2, e raise to minus omega t. And therefore, r dot as the function of time is going to be r 0 omega, minus v 0 divided by 2, e raise to omega t, minus r 0 omega, plus v 0 divided by 2, e raise to minus omega t. This term is already negative, and if I want the velocity to remain negative.

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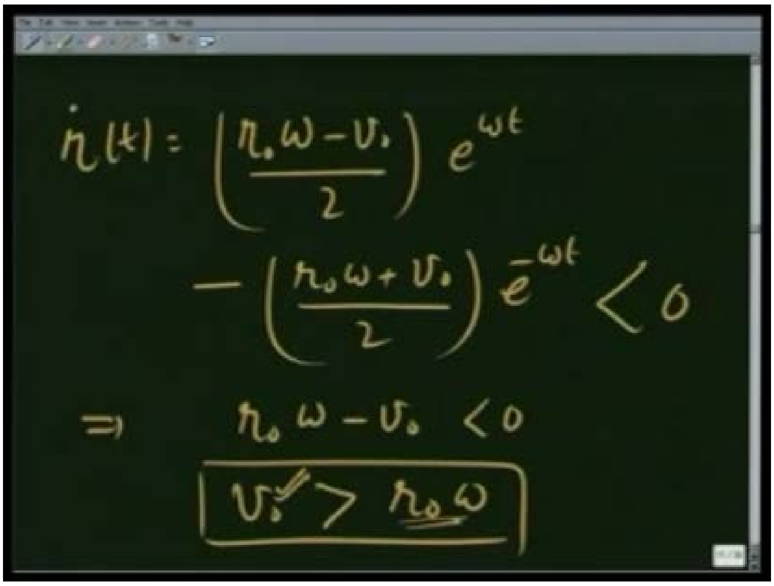

That means, I want r dot t which is equal to r 0 omega, minus v 0 over 2 e raise to omega t, minus r 0 omega, plus v 0 divided by 2, e raise to minus omega t, less than 0. This implies r naught omega, minus v 0 should always remain less than 0, or v 0 should remain r 0 omega.

So, as another example of this, what we have shown is that, depending on the initial condition, if I choose my velocity of pushing the particle in so, that it is greater than r 0 omega, the particle would keep on moving in. As another example of the application of planar polar coordinates.

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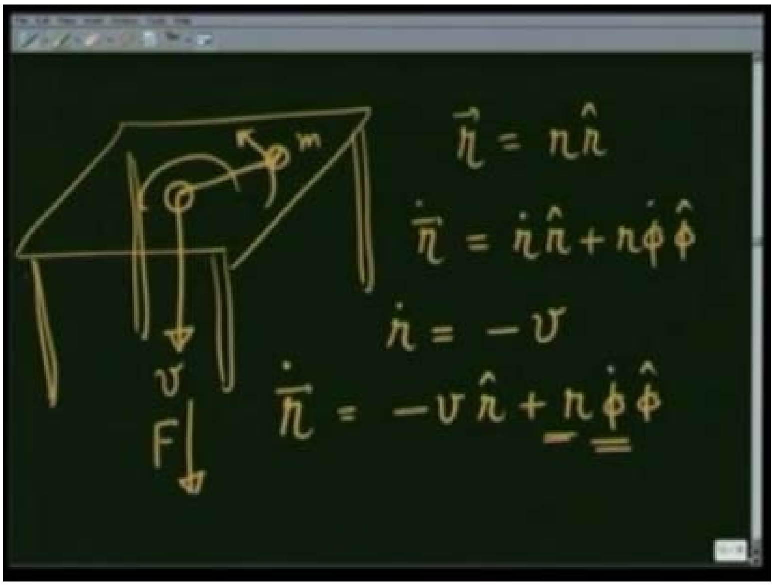

Let me take a particle which is moving circularly on a table. Here is the particle of mass m, it is tied with the thread and thread is passing through a hole in the middle of the table. And I am pulling this thread or string in with the constant speed v. Again, since the motion is circular, it helps to use planar polar coordinates.

So, the position of the particle at any time is given as r is equal to r unit vector r is velocity, r dot is equal to r dot r, plus r phi dot phi, r dot is given to be constant and r is decreasing so, it is some minus v. And therefore, the velocity r dot is equal to minus v r, plus r phi dot phi. I want to find the subsequent motion. How phi dot changes with time, how r changes with the time, what is the force F needed to pull this thread down? Obviously the particle is moving in the circle and therefore, there is under the table is frictionless there is no force in phi direction.

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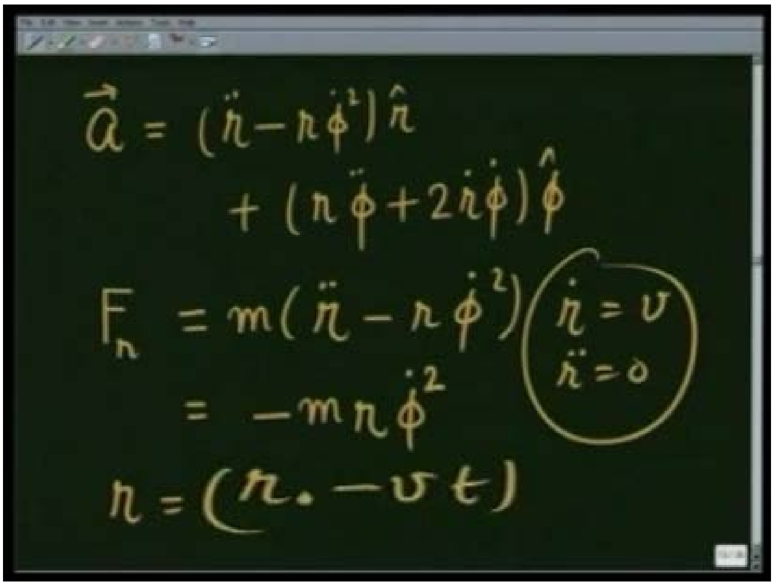

So, let us go back to what the acceleration of the particle is? It is r double dot, minus r phi dot square in r direction, plus r phi double dot, plus 2 r dot phi dot in phi direction. The force that I am pulling it in with there is a radial direction therefore, that is equal to m r double dot, minus r phi dot square.

And since, it is being pulled in with constant speed, that is r dot is equal to v, and r double dot therefore, is equal to 0, I have Fr is equal to minus m r phi dot square. Also r is going to be its initial r 0, minus vt because r just keeps on decreasing at constant rate. On the other hand, since there is no frictional force, there is no force in phi direction. And therefore, I am going to have F phi, which is equal to m a phi which is equal to m r phi double dot plus 2 r dot phi dot is equal to 0..

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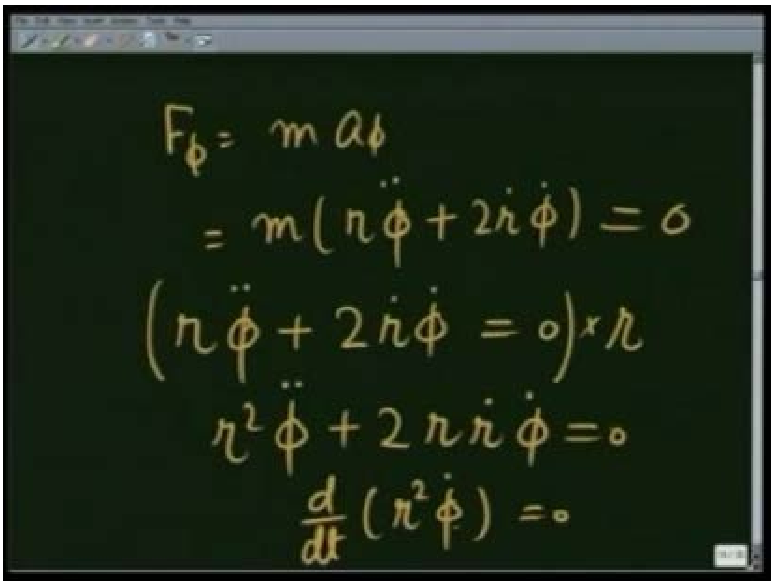

So, r phi dot double dot, plus 2 r dot phi dot is equal to 0. I can easily integrate this equation if I multiply this whole thing by r so, that I get r square phi dot, plus 2 r r dot phi dot is equal to 0, and you can easily see this is nothing but derivative of sorry, this is phi double dot r square phi dot. First term gives you r square phi double dot, second term gives you 2 r r dot, and this is equal to 0. And therefore, r square phi dot is constant. Since, the derivative of r square phi dot 0.

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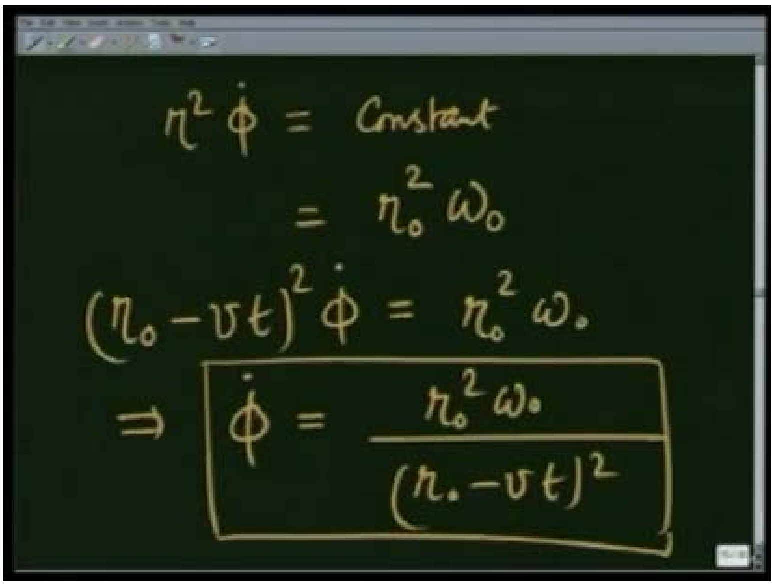

And therefore, r square phi dot is a constant. So, if the initial position was r 0 and its initial angular speed was omega 0, this remains fixed. As I already told you, r changes according to r 0 minus vt square phi dot therefore, is going to be equal to r 0 square omega 0.

And therefore, the angular speed if I am pulling the wire end with the constant speed is going to change as r 0 square omega 0 over r 0 minus vt square. So, we have obtained how the angular speed of the wire changes. How about the force needed to pull the wire in?

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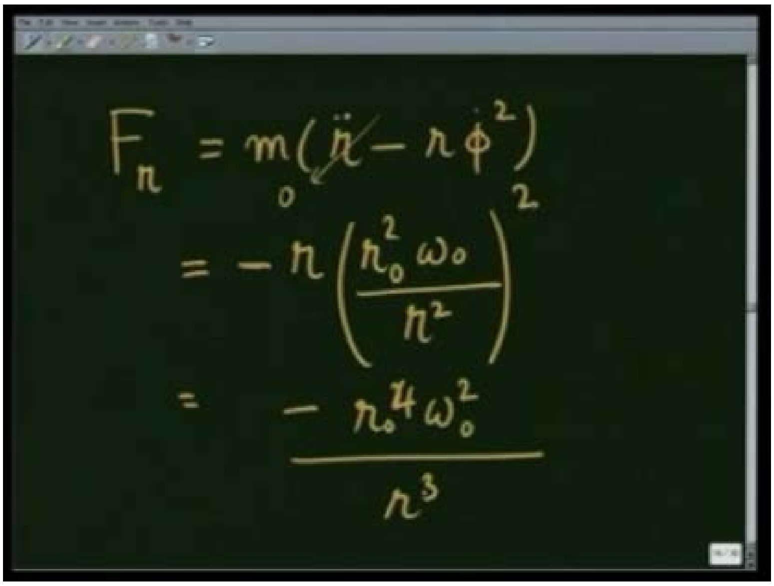

That as I told you earlier is F r which is equal to m r double dot, minus r phi dot square, r double dot is 0 because I am pulling the wire in with constant speed. And therefore, this is going to be equal to minus r phi dot square is nothing but r 0 square omega 0 over r square, and I square it which is going to be equal to minus r 0 square omega 0, r 0 raise to 4, omega 0 square over r cubed.

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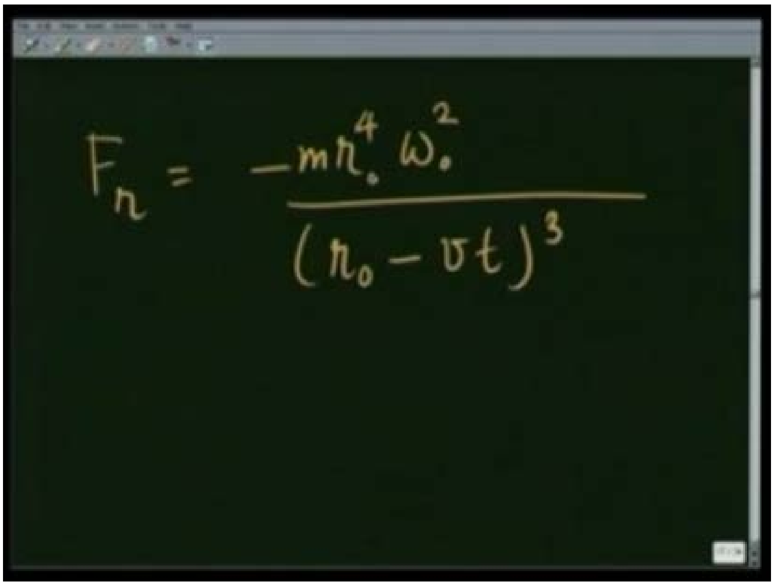

And therefore, the force needed to pull it in, is equal to m minus r 0 raise to 4, omega square omega 0 square over r 0, minus vt cubed. So, as time increases, this quantity becomes small and smaller, and force that you need to pull the wire in with constant speed keeps on going up. So, these two examples, I have shown you how to use planar polar coordinates.

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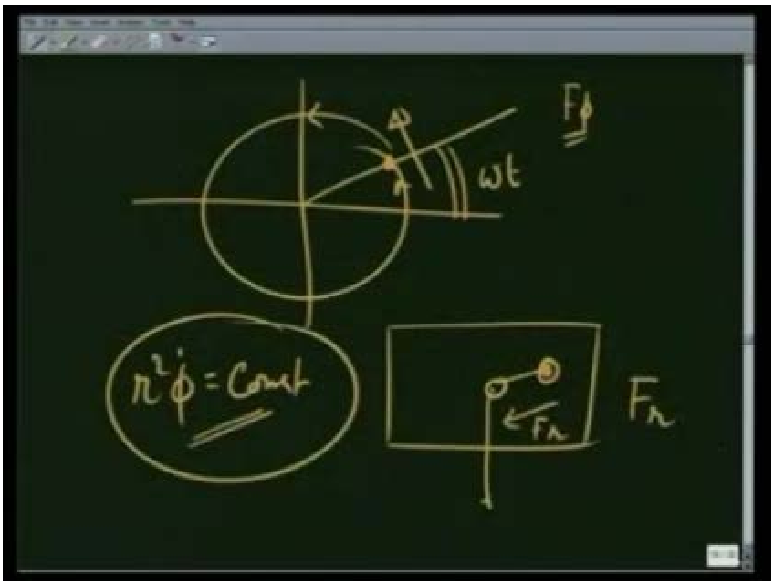

In one case where there was a bead on a wire, and the wire was moving with some speed omega so, this was omega t. The force was in phi direction, in the other case where we are pulling a mass in, the forces in radial direction. In this case, we can solve for F phi, in this case we solve for Fr, in this case I also got r square phi dot to be a constant. And you would recall from your previous courses in your 12th grade that, this is nothing but a statement of conservation of angular momentum. In this case, the forces radial and therefore, the conservation of angular momentum takes place.

Having done the planar polar coordinates, the obvious question to ask is, what is the corresponding system of coordinates if I want to describe a 3 dimensional motion? A very straight forward application of this, extension of this to 3 dimensions is.

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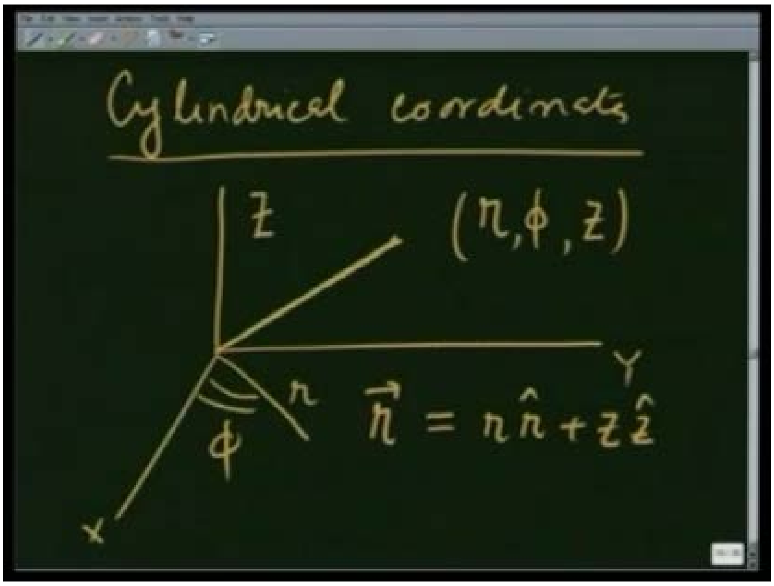

The cylindrical coordinates in which I describe the motion in the X Y plane in terms of this r and phi. Notice, how I have changed the orientation of X and Y and will require some getting used to. And the third dimension is treated as z itself.

So, that the position of a particle is going to be given by r, which is in the plane X and Y phi and z. z giving you the third dimension. Obviously, the vector r which is this direction, is going to be given as r, unit vector r, plus z, unit vector z. Let me show it more clearly in the next picture.

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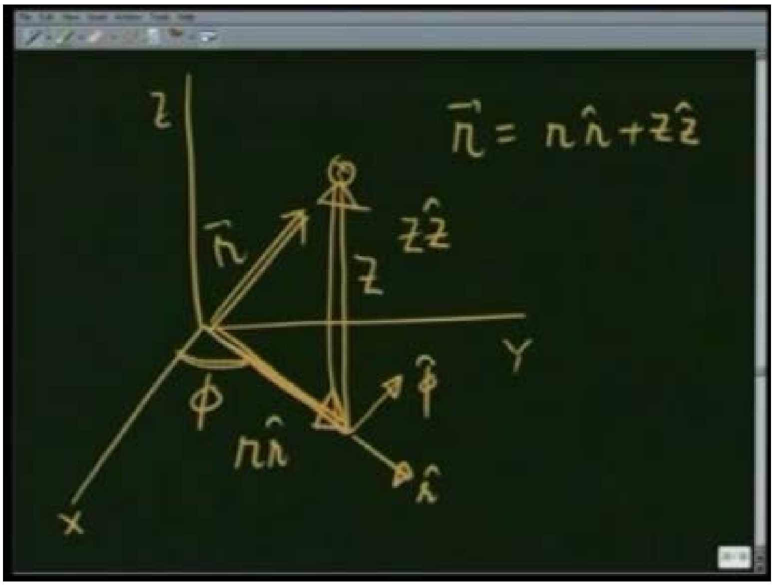

So, I have X, Y and Z, the position of the particle is here, which is at the height z from the X Y plane. In the X Y plane the radius vector makes angle phi from the X axis. So, this vector here is nothing but r, r, the unit vector r is in this direction in the X Y plane, and this vector is nothing but z, z unit vector and therefore, the position vector r is given as r, r plus z, z. The phi unit vector is again in the X Y plane perpendicular to r, and perpendicular to z like this. How about the velocity of the particle? Again it is very simple in this case because the motion in the X Y plane is being given by the planar polar coordinates, and the y direction, z direction is fixed.

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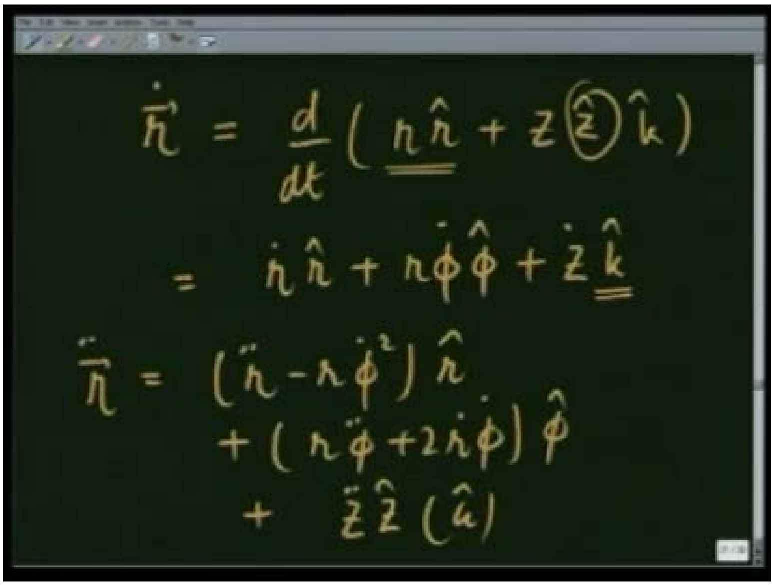

So, therefore, r dot, the velocity is going to be equal to d by dt, r, r plus z, z, you are more use to calling this k so, I am using them interchangeably. This is going to be equal to r dot r, plus, I have already derived this, I am going to write the result right away without any derivation, r phi dot phi, plus z dot k z dot z, there is no derivative of this because z unit vector is fixed.

Rest derivation of the acceleration, I leave for you as an exercise, I will just give you the answer which is going to be same as in the previous case. For the plane in a XY plane, which is going to be r double dot, minus r phi dot square in r direction, plus r phi double dot, plus 2 r dot phi dot in phi direction, plus z double dot and z are equivalently k direction.

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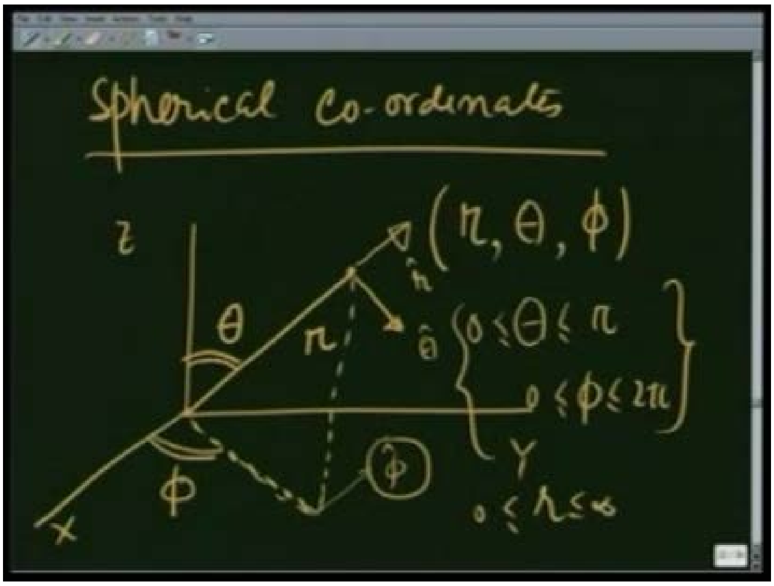

A more interesting extension is this spherical coordinates to describe 3 dimensional motion. And in this case, again if I my, make my X, Y and Z coordinate system, the position of a particle is given by its distance from the origin. The angle theta that this radius vector makes with the z axis, and angle phi that the projection of radius vector, this is the projection I am showing again, makes with the X axis.

So, now I am using r, the distance from the origin, the angle theta that is made from the z axis and angle phi that the projection of r makes, form the x axis. The unit vector r is going to be in direction of the radius, the unit vector theta is going to be in the direction of increasing theta, and unit vector phi is going to be in direction of increasing phi. You can see that the phi unit vector is confined 2 XY plane whereas, r and theta are going to have component in all X Y and Z direction.

And in the next few minutes, we are going to see how these unit vectors are represented. I just want to mention here, the theta in this case varies from 0 to pi, and phi varies from 0 to 2 pi, and I leave it for you to sort of work out that, this covers the entire space, and r obviously, varies from 0 to infinity.

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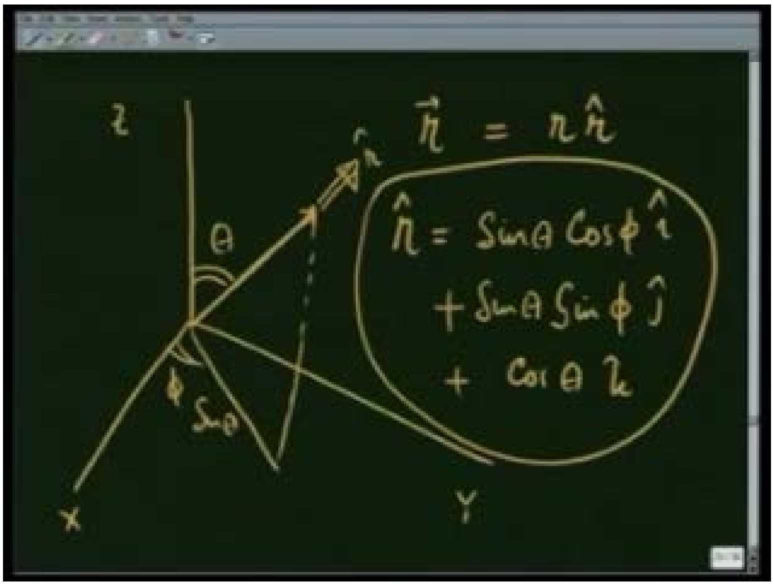

Let us and see that, I am representing a particle by vector r, this vector r which is r, r again, where unit vector r is in this direction. The components of unit vector r, r therefore, going to be, it is very easy to see that the z coordinate is going to be cosine theta.

The projection on the XY plane of this is nothing but sin theta and therefore, the x component is sin theta, this is phi cosine of phi I, plus sin theta, and the y component of this projection sin of phi j, plus cosine of theta k. This is the unit vector in r direction. How about the unit vector in theta direction?

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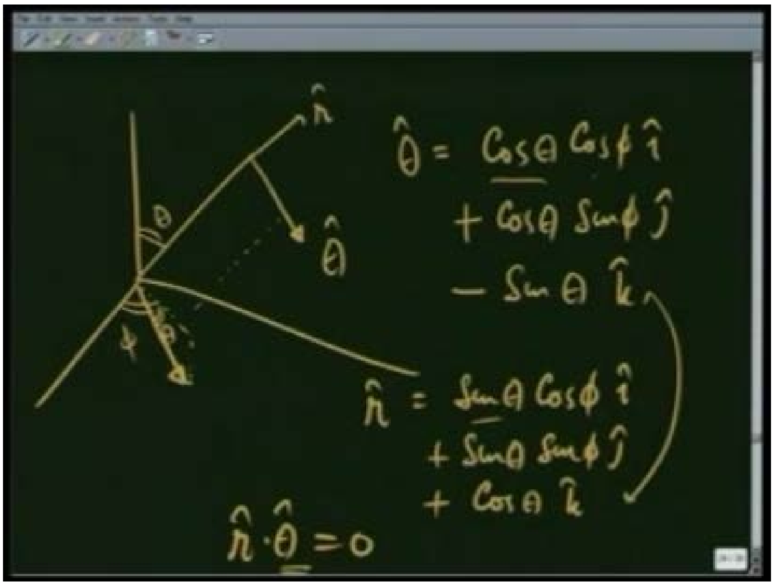

Unit vector in theta direction is going to be like this, this is r, this is theta. If I transport it down just to make it look easy, this is pointing down from the plane of X and Y axis by an angle theta because this angle is theta. And therefore, you can see the components of theta in X and Y direction are going to be cosine of theta cosine of phi because this angle is phi I, plus cosine of theta sin of phi j and it is z component is this in the opposite direction.

So, minus sin of theta k, recall that r was sin of theta cosine phi I, plus sin theta sin of phi j, plus cosine of theta k. You can see right away, that r dot theta is equal to 0. These two together give you cosine theta, cosine theta time sin theta, cosine square phi plus cosine theta sin square sin square phi, minus these two give you minus sin theta cosine theta giving you this. Similarly, if I were to write the components of phi vector.

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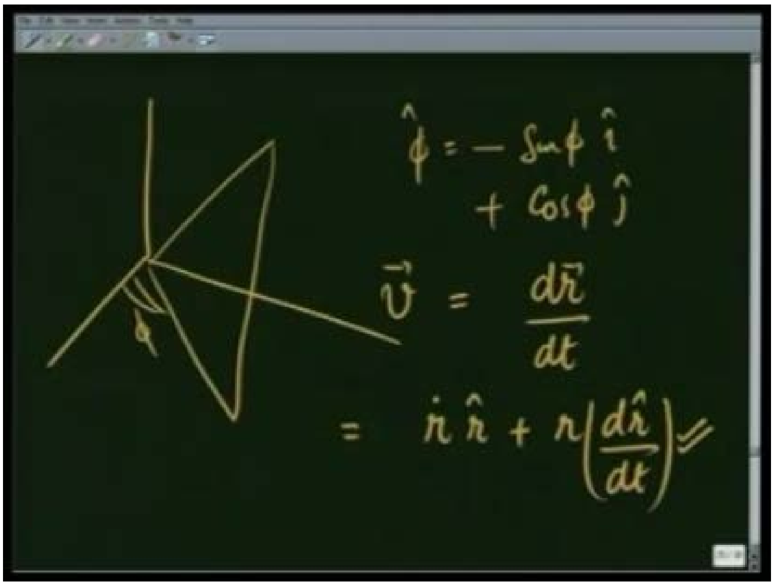

Phi vector as I told you earlier is confined to XY plane and therefore, phi unit vector is just like in planar polar coordinates, which is equal to minus sin phi i, plus cosine phi j. So, I have all three unit vectors. Now, how about the velocity? Velocity is equal to dr dt which is equal to r dot r, plus r dr dt, expression for dr dt, rate of change of the unit vector in this case is going to be slightly more complicated. Because now it has all three components, but we can geometrically see how this is going to be.

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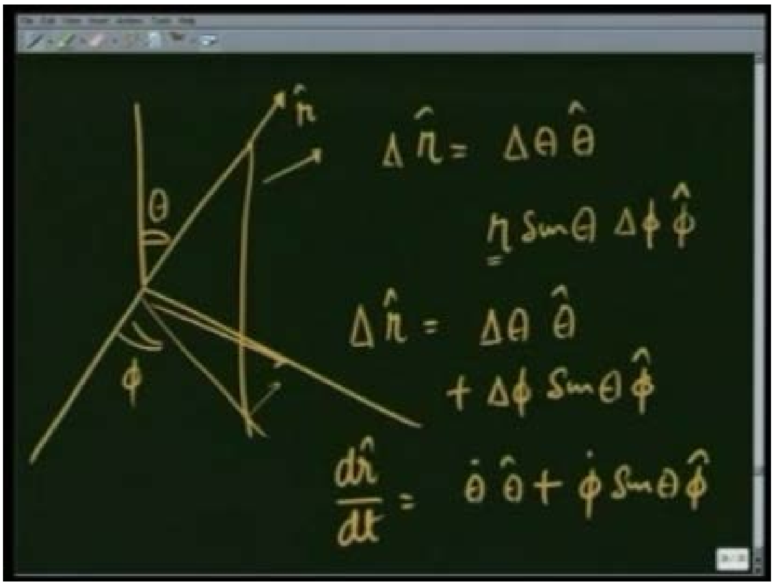

Here is vector r, if I were to change only theta and keep phi fixed then, you can see the r unit vector is going to change from here to like this. And that change is going to be equal to delta theta in theta direction. On the other hand, if I keep theta fixed and move, change phi then, the r unit vector, this component is going to change.

And that component is nothing but r sin theta since, this is a unit vector this r is 1 and it is going to change by delta phi in phi direction. And therefore, the net change in delta r in r unit vector is going to be delta theta in theta direction, plus delta phi sin theta in phi direction. And therefore, dr by dt is going to be theta dot theta, plus phi dot sin theta phi.

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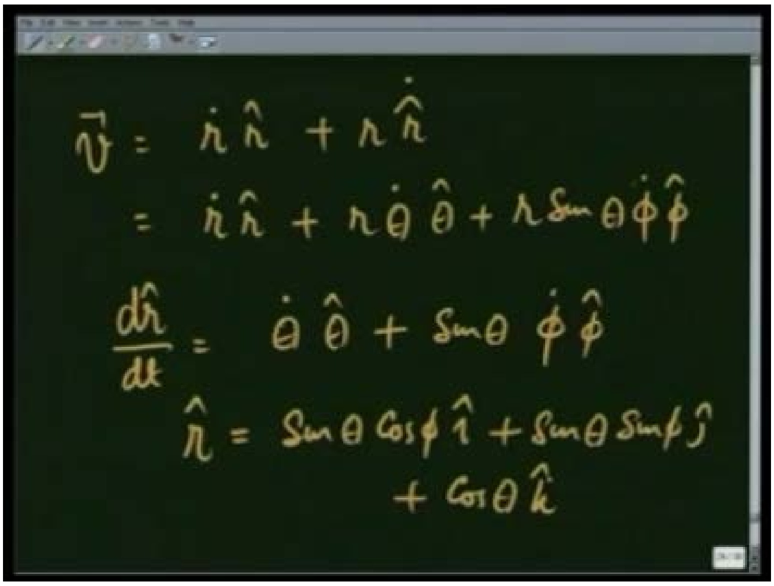

Therefore, the velocity which is r dot r, plus r, r unit vector rate of change, is going to be equal to r dot r, plus r theta dot in theta direction, plus r sin theta in phi direction. Let us see whether this, which we derive to be theta dot theta, plus sin theta phi dot phi can be derived directly from the definition. r was equal to sin theta cosine phi i, plus sin theta sin phi j, plus cosine of theta k.

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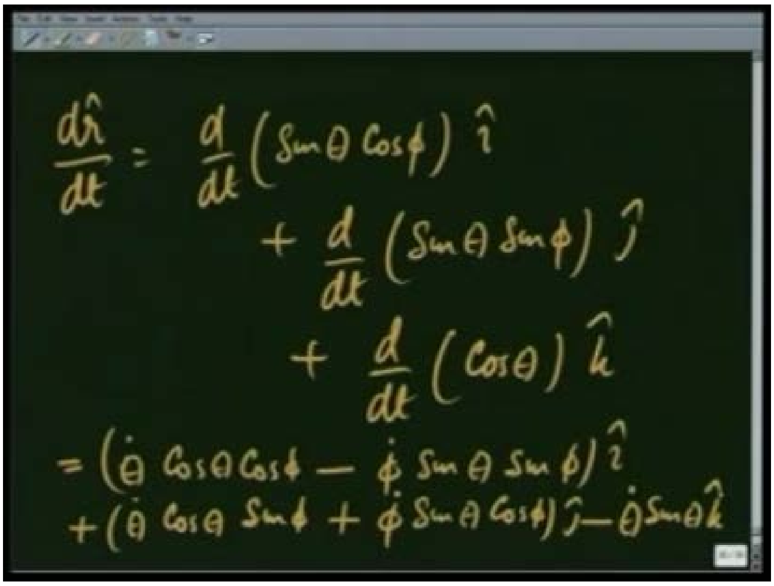

If I take the derivative, I have dr over dt which is equal to d by dt of sin theta cosine phi i, plus d over dt of sin theta sin phi j, plus d over dt of cosine theta k. It turns out to be equal to theta dot cosine theta cosine phi, minus phi dot sin theta sin phi i, plus theta dot cosine theta sin phi, plus phi dot sin theta cosine phi j, minus theta dot sin theta k. If you collect all the terms together, you can see easily.

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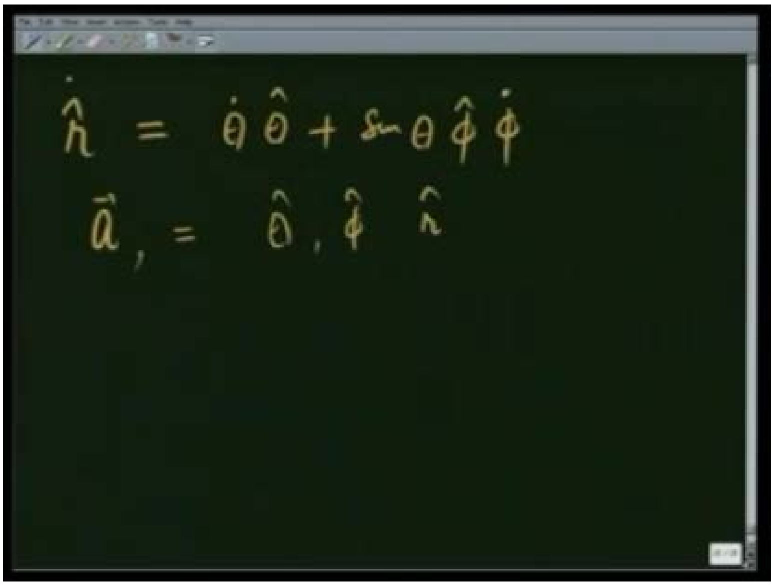

Let this comes out to be equal to theta dot theta, plus sin theta phi sin theta phi phi dot. Rest of the exercises, that is calculating the acceleration and finding it in terms of theta phi and r, I leave as exercise for you. To some the lecture, I would like to value that we covered planar polar coordinates, in which I described the motion in a plane, using the radial distance and the angle from x axis.

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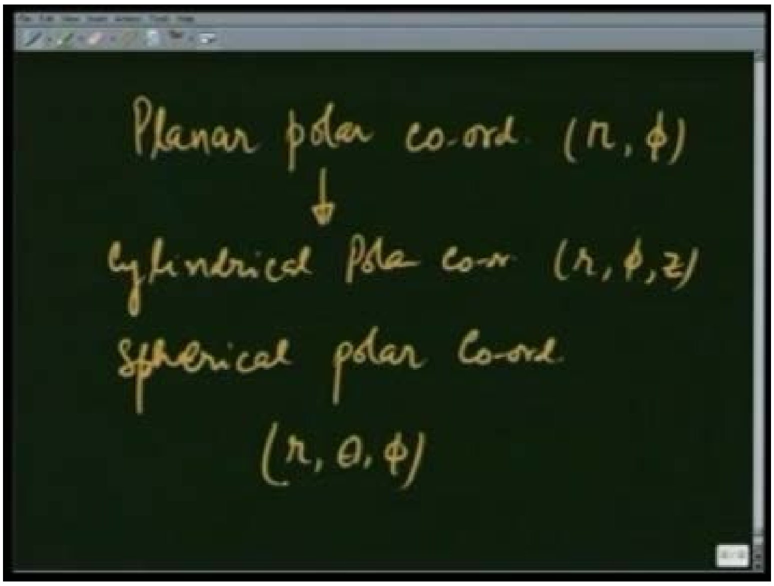

We extended this to cylindrical polar coordinates, in which I use r phi and z as my coordinate system. And we also use spherical polar coordinates in which I use r theta and phi as coordinates to describe the distance or the displacement of a particle.

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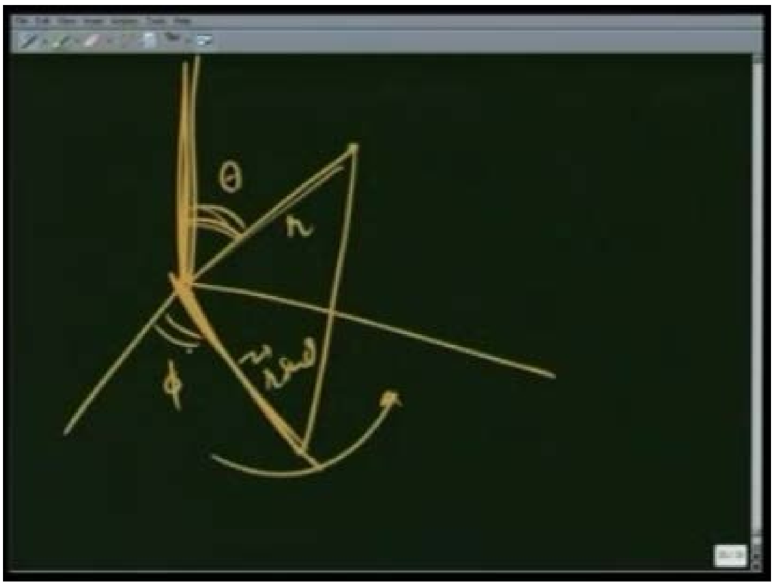

Before we just the end the lecture, I just would like to tell you how to thing about spherical polar coordinates. You can think of these as two sets of planar polar coordinates.

One set being r and theta, that is the particle moves in z and this plane, z and the this plane of this line, z axis and this line. With this angle being theta and this r, this is describes one set of planar polar coordinate. And the other set being the XY plane with this being r sin theta and this angle being phi.