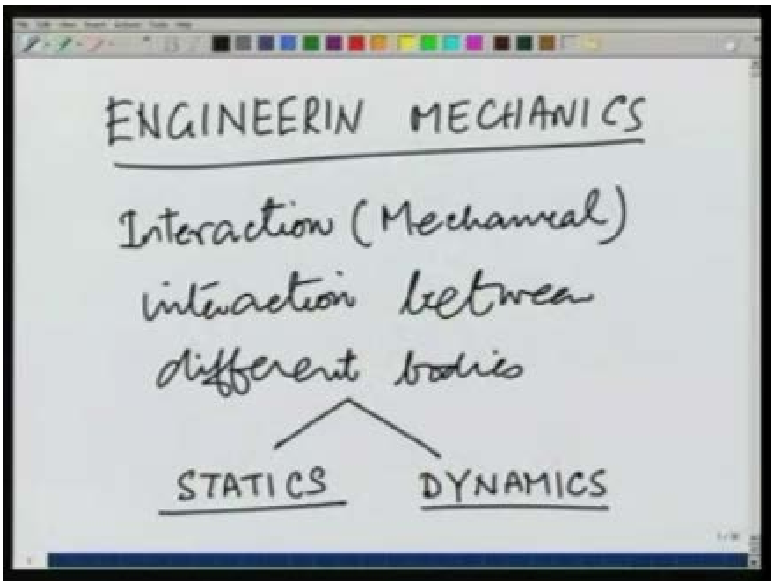
Engineering Mechanics

Prof. Manoj Harbola
Indian Institute of Technology, Kanpur

Module – 01
Lecture - 01
Review Vector and Laws of Motion

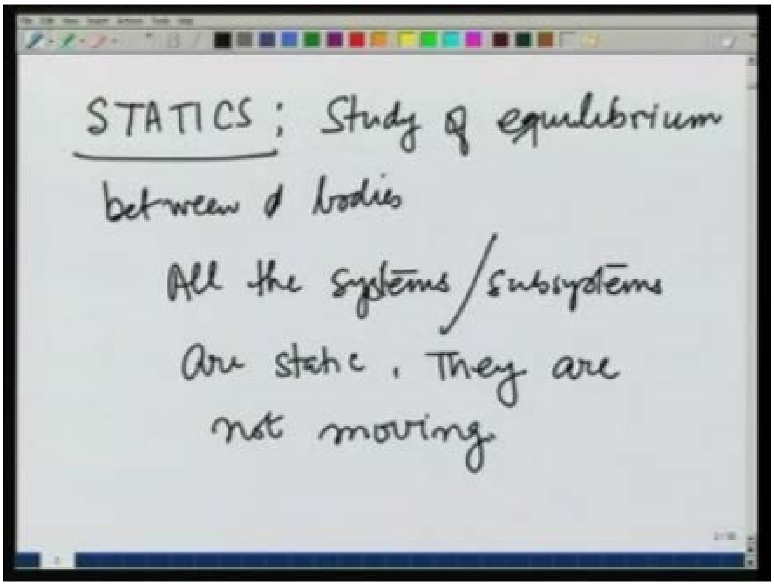
(Refer Slide Time: 00:24)



This is the course on engineering mechanics in which, we would be studying interaction or let me be more precise, mechanical interaction between different bodies when they interact through the forces applied on each other. This would consist of 2 parts: statics and dynamics.

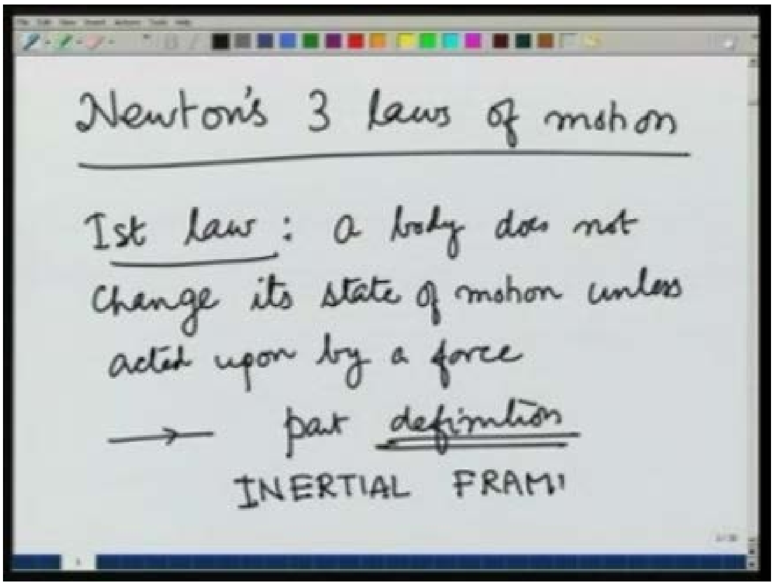
In a statics we would mainly, be concern with equilibrium between different bodies. I would specify later what we mean by equilibrium and in dynamics we would be concerned with, how different bodies move under the influence of different forces they apply on each other or when the forces applied from outside. And the first part we are going to focus on a statics; which is the study of equilibrium between different bodies.

(Refer Slide Time: 01:57)



So, of equilibrium between bodies. When we say equilibrium, in general it means that there is no acceleration on any part of the system. In the statics we specifically we concern with, when all the sub systems, all the systems or sub systems are, not only not accelerating are static that is, they are not moving. The study of statics or dynamics is based on Newton's 3 laws of motion.

(Refer Slide Time: 03:02)

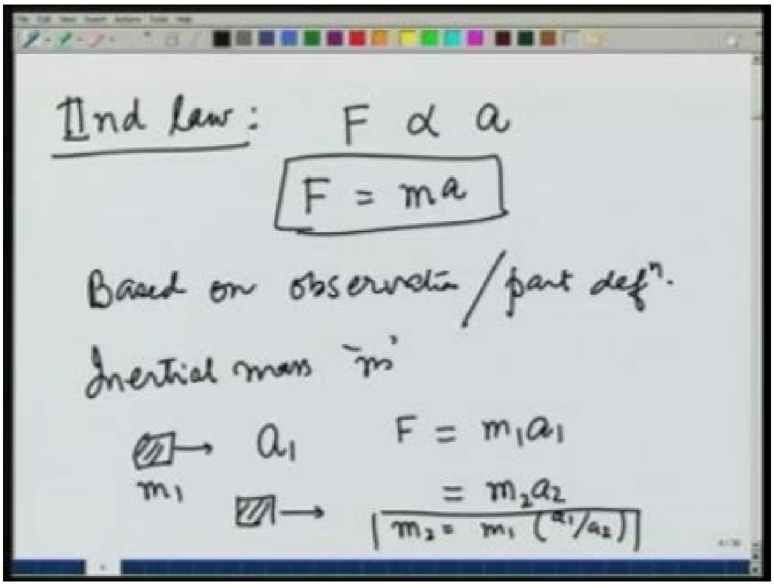


So, let us start by their description: The first law states that, a body does not change its state of motion unless acted upon by a force. So, if a body is moving on a straight line, it will keep moving in this that straight line until a force is applied. Similarly, if a body is static it sitting somewhere, unless of force is applied it will not start moving spontaneously.

The first law is part is based on observation and it is part definition. You may ask definition of what? It gives you the definition of an inertial frame and we do most of our calculations in an inertial frame. An inertial frame by definition then, is the 1 in which the body does not change its state of motion unless a force is applied. For example, in this room, for all practical purposes this room is a good inertial frame because, if I see somebody or somebody is somewhere, it is not going to change its state of motion without a force being applied to it.

On the other hand, suppose I am on a train which suddenly starts moving; as soon as it starts moving you see objects outside which are accelerating in the opposite direction by the same acceleration. So, without any apparent force. So, that accelerating train is not good inertial frame, is not in inertial frame at all.

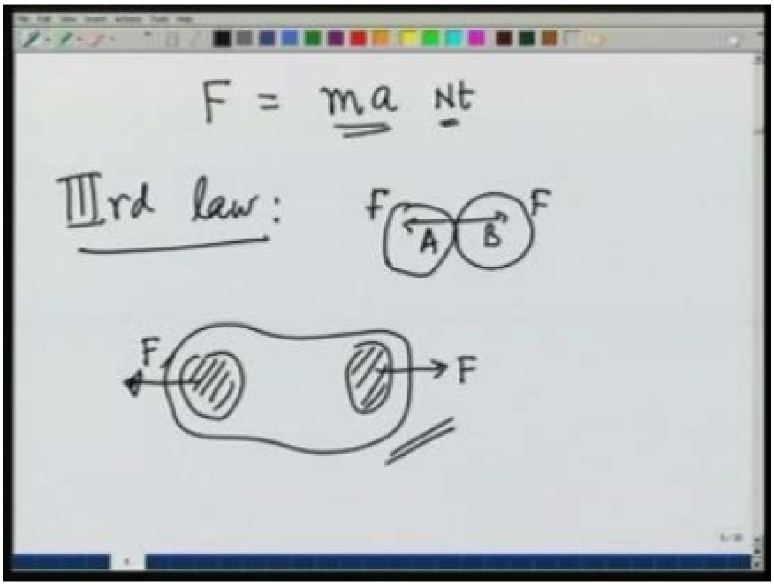
(Refer Slide Time: 05:24)



Then, there is second law that states that, force applied on a body is proportional to the acceleration that it produces. Then we write F equals ma which defines for us, a mass as well as the force. So, this is also based on observation and part definition. It defines for us and something, call the inertial mass.

Suppose I take a standard body, apply a force on it and produce an acceleration a1 so that, F equals m1a1. And I take another body, apply the same force on it may be by a spring, may be heating it or something and find that acceleration is a2. Then, the mass of the second body is going to be, m1 times a1 over a2. This becomes the operational definition of inertial mass. So, this is part definition.

(Refer Slide Time: 06:55)

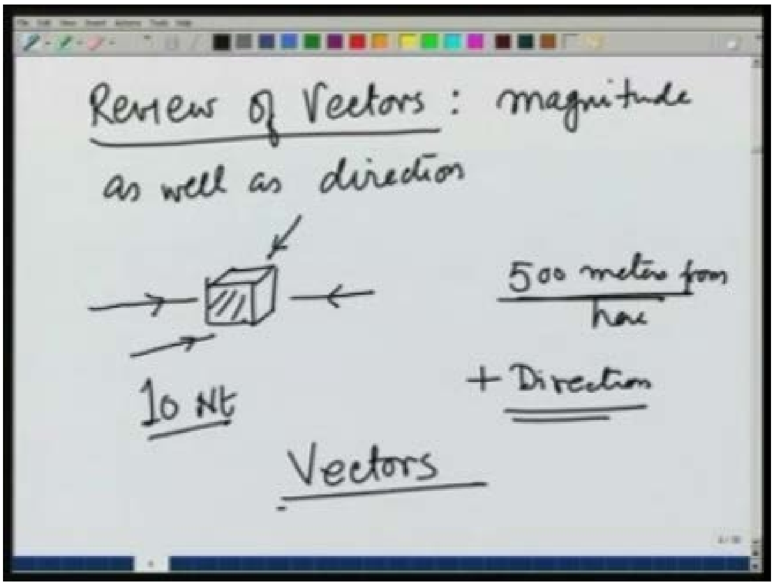


Then it also tells us given a mass, I can also measure the force in Newton's as m times a. Mind you this is operational definition, I cannot always use it though because, suppose I am pushing a wall the wall does not accelerate. So, I cannot really determine this mass by measuring the acceleration when I am pushing it.

Then, there is Newton's third law that states that, for an action there is always a reaction; that means, if there is a body A, it is pushing another body B by a force F then, there is going to be reaction on A by B in the opposite direction. A very confusing situation arises in this when most of the students ask, if the forces are in opposite directions, why do not they cancel each other? They do not because, you see force A is applying a force on B it produces something on B, on the other hand A is being pushed by a force by B in the opposite direction. So, it is acting on a different body therefore, they would not cancel. However, if I take the entire system as 1, then there being internal forces, they do cancel. But be very careful in applying it.

Having given this preliminary discussion of Newton's laws, of which will mostly be using the third law in the statics part. And will be using the second and third law in the dynamics part. Let us now, start with a review of vectors.

(Refer Slide Time: 08:40)



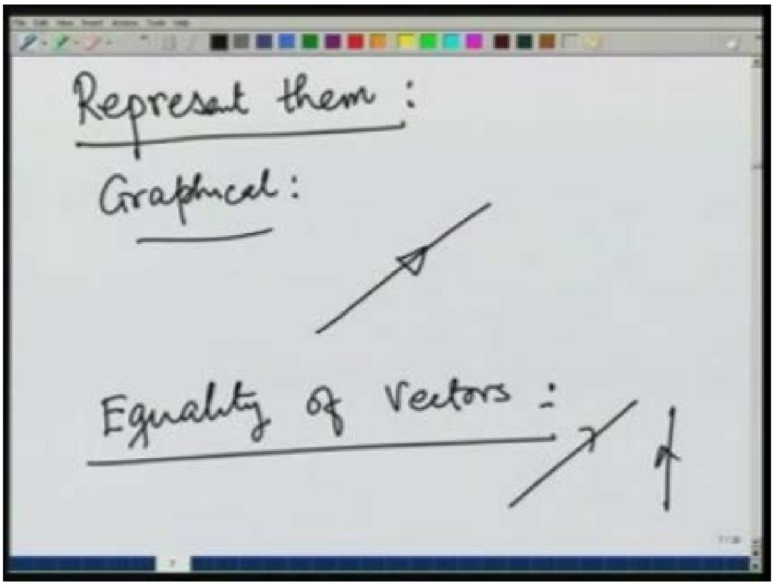
Because we will be using vectors extensively to represent forces, velocities and things like those. So, it is a good way to start this course by reviewing what we know about vectors I am sure most of you have learnt about it in your twelfth grade. But, we now make it slightly more sophisticated.

Why do we need vectors? It is because there are certain quantities, which have magnitude as well as the direction. For example, if somebody tells you that, I am pushing a box by a force of 10 Newton's, it does not convey the full meaning until I say that, I am pushing it to the right, to the left, in this direction or in this direction. Until a direction is specified the complete description is not there. So, to specify a force I need both its magnitude as well as this direction.

Similarly, suppose somebody comes and ask you, where is your friend's house? And you say it is 500 meters from here. Again it will be a meaningless statement unless you tell him that, it is 500 to the east, to the west to the north, to the south, southeast. So, plus the direction is also needed. So, there are certain quantities for which you need the magnitude as well as the direction and these quantities we want to call vectors.

Having defined vectors, how do we represent them?

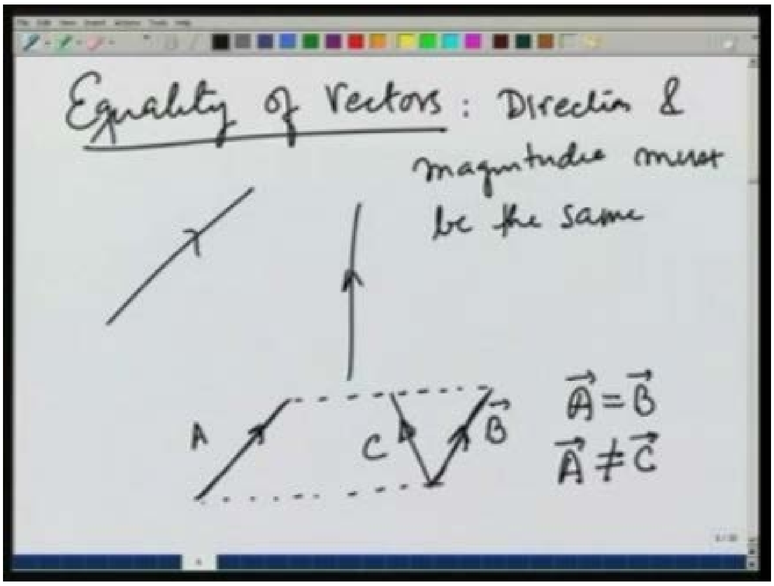
(Refer Slide Time: 10:35)



Let us ask that question. There are 2 ways: 1 is graphical and 1 is algebraic. We will first do graphical method and see that it is, it gets little complicated when we go into many many vectors and do many operations. Then, we will do a algebraic way of writing vectors. So, graphical method of representing a vector is that; you make an arrow with the arrow showing the direction and the length of the arrow showing the magnitude of the vector.

In this manner, if we now have 2 vectors, how do we decide whether the 2 vectors are equal or not?

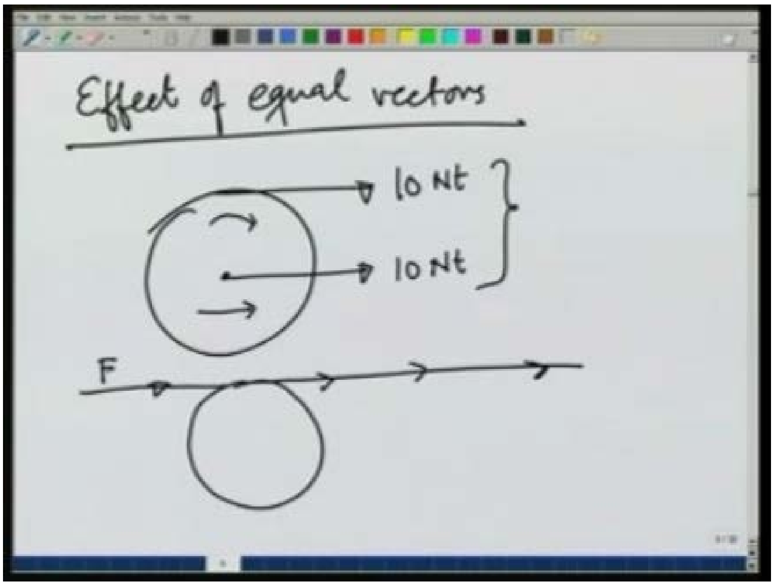
(Refer Slide Time: 11:37)



If there are 2 vectors given there will be equal if they produce the same effect. So, there direction and magnitude must be the same. Graphically that means that, if there is a vector A, another vector which is parallel to it and has the same magnitude B is equal to A. So, I can have a vector parallely shifted compare to A, but still it can be equal to A. So, in this case A is equal to B because, both of them have the same direction and the same magnitude.

On the other hand, if I make a vector here C its magnitude may be the same as A, but it is not, it does not have the same direction as A and therefore, A is not equal to C.

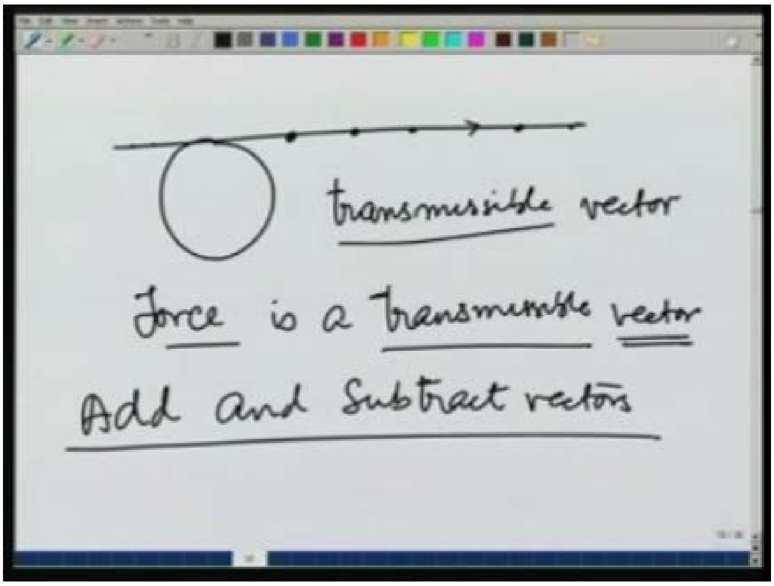
(Refer Slide Time: 13:02)



1 has to be very careful in the effect of equal vectors. 2 vectors being equal, does not mean that their effect will always be the same. For example, if I take a wheel, I apply a force on its top say 10 Newton's. If I apply the same force on the axial, in the same direction although the 2 vectors are equal, their effect would not be the same. In the first case, the wheel will start rolling, in the second case is it will just move forward without rolling. So, although the vectors are equal, their effect is not the same.

On the other hand think of, of the same wheel, if there is a rope pulling it which is tied to its end; whether I apply a force here, here or here along the line of action, its effect could be the same. I can also hit the disc from this side by the same force and that would also produce the same effect. So, equality of vector has to be further specified by something else and that is;

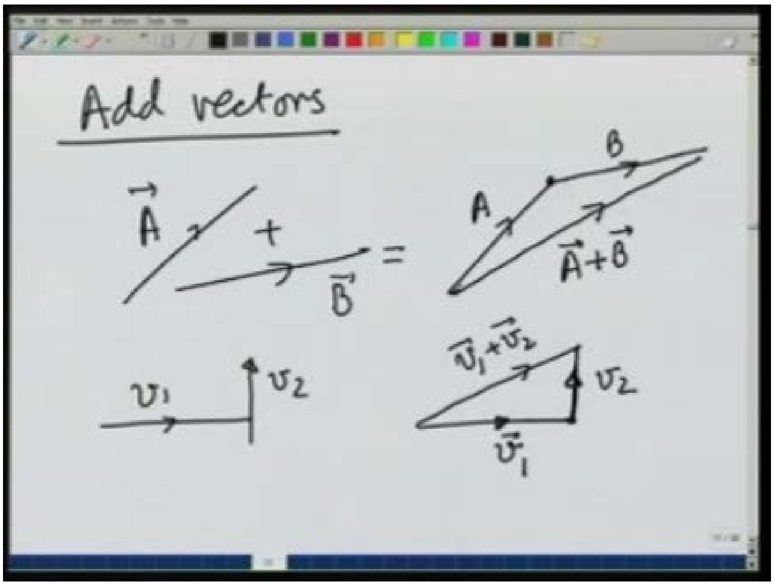
(Refer Slide Time: 14:25)



If, there is a vector and no matter where I applied along the line of its application if the effect is the same, then it is known as transmissible vector. So, not only equal, but if I can apply at any point along the line of its action and it produces the same effect, then it is transmissible vector. For example, force is a transmissible vector.

So, force has this quality called the transmissibility. Having talked about equality of vectors and transmissibility of vectors; let us now, see how do we add and subtract vectors using our graphical method. This is more or less a review because, I am sure you have learnt this in twelfth grade.

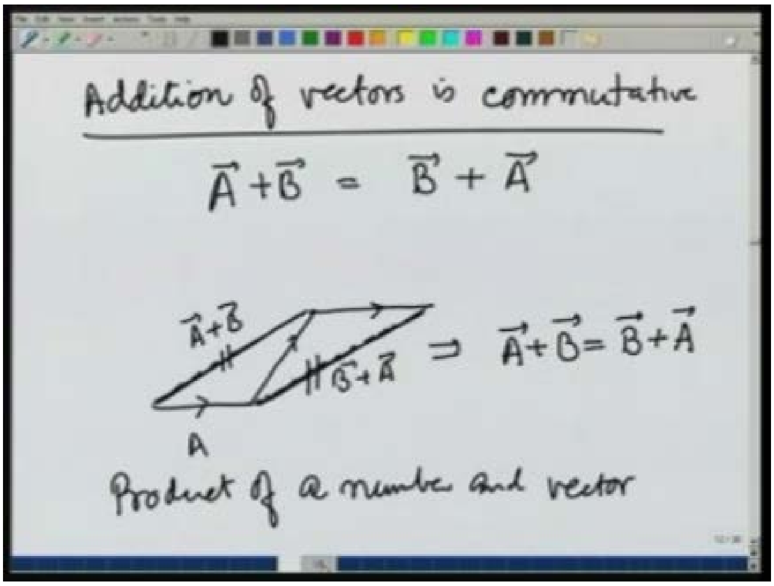
(Refer Slide Time: 15:39)



So, if I want to add, 2 vectors A and B, what we do is; so we are doing A plus B, what we do is, take A on its head put the tail of B and then draw a vector starting from the tail of A to the head of B and that gives you A plus B. You may ask, why do I add this way, does it make sense? Exactly it does. Suppose there is a ball moving like this, with velocity v1 and I hit it so that, it acquires a velocity v2 in this direction. Experience tells me, that it would be moving in a direction like this, which would be a sum of v1 plus v2. You see I have added v1 and v2, using the prescription I just gave you, I draw v1 and put the tail of v2 at the on the head of v1, draw this vector and this gives me v1 plus v2.

So, this is how we define the addition or we obtain the addition of 2 vectors.

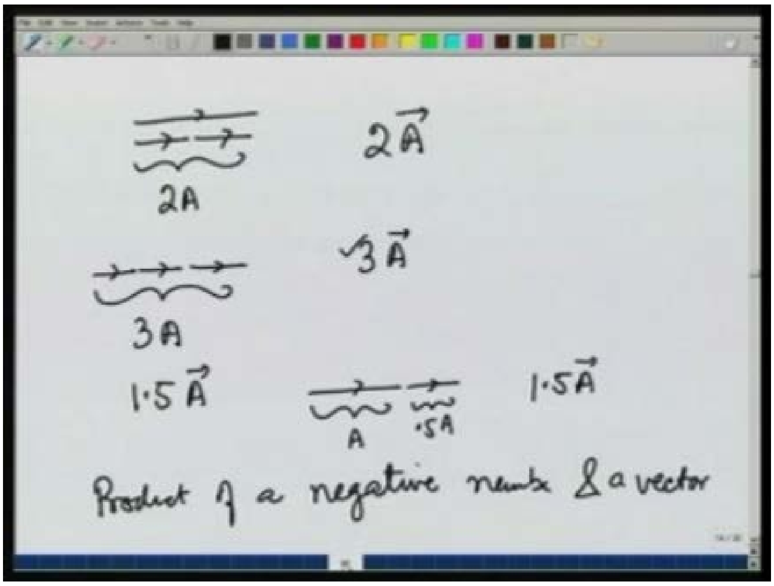
(Refer Slide Time: 17:08)



It is also easy to see that addition of vectors is commutative, that is, A plus B is same as B plus A. Let us see that. So, let there be a vector A and let there be a vector B, then this is A plus B. On the other hand, if I draw the vector A with this tail at the head of B then, this would give me B plus A. But, this arm is parallel to this because, this is the parallelogram. So, these 2 vectors are parallel and because this is the parallelogram this magnitude is same as this and this implies A plus B is the same as B plus A. So, summation of 2 vectors is commutative. You see graphical method is giving you a very visual way of looking at vectors.

Next, we can use addition of vectors to define the product of a number and vector.

(Refer Slide Time: 18:50)

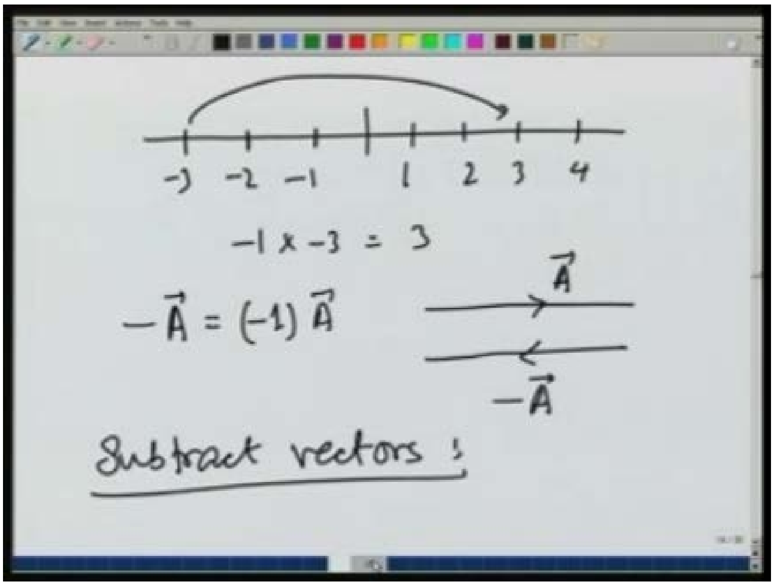


For example, if I take A and add A to it, you can easily see that the length becomes doubles. And therefore, its magnitude is 2A and since this is the same direction as A, I can write this as 2times A vector. Similarly, if I add 3 vectors the magnitude would become 3 times A and it is in the same direction as A so therefore, I can write this as 3A.

So, this defines the multiplication of a number with the vector. It is nothing but, adding that vector that many times. How about 1.5A? 1.5A would be A plus half the magnitude, but in the same direction. So, this would be A, this would be 0.5A and their addition gives me 1.5A. Can I multiply a vector by a negative number? The answer is “yes”.

So, product of a negative number and a vector, recall from your school days what does, multiplying by negative number mean?

(Refer Slide Time: 20:15)

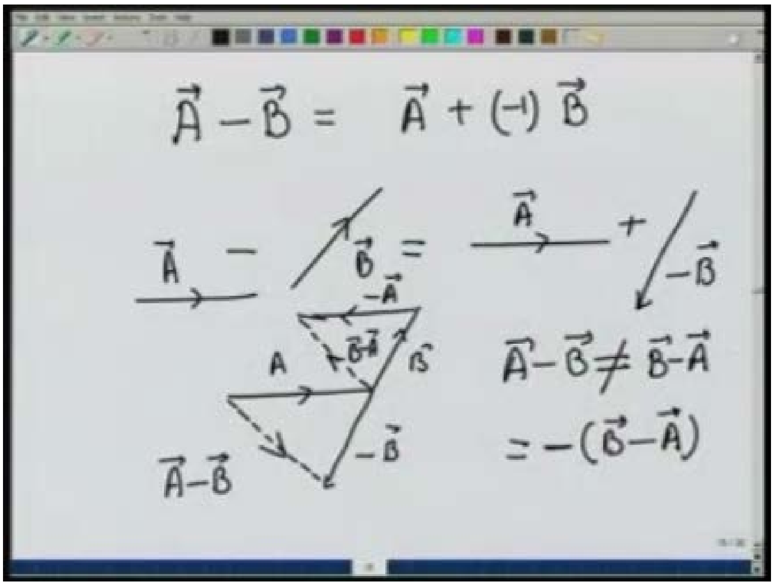


If, I go to number line, this is 1 2 3 4; minus 1 is in the other direction, minus 2 is also in the other direction, minus 3 is in the other direction and so on. If I multiply minus 3 by minus 1 it becomes 3. So, minus 1 times minus 3 becomes 3. What the multiplication by a negative number is doing therefore is, changing the direction. Similarly, if I take negative A, it will be minus 1 times A. This would be nothing but, change in the direction of A.

So, if A is this; minus A would be point A in the other direction. That is the meaning of multiplying by a negative number. Of course, once you define multiplication by minus 1; you can define multiplication by minus any number. You just make it that many times more. Having obtained minus A, it is now very straight forward to subtract vectors.

So, how do we do subtraction of vectors?

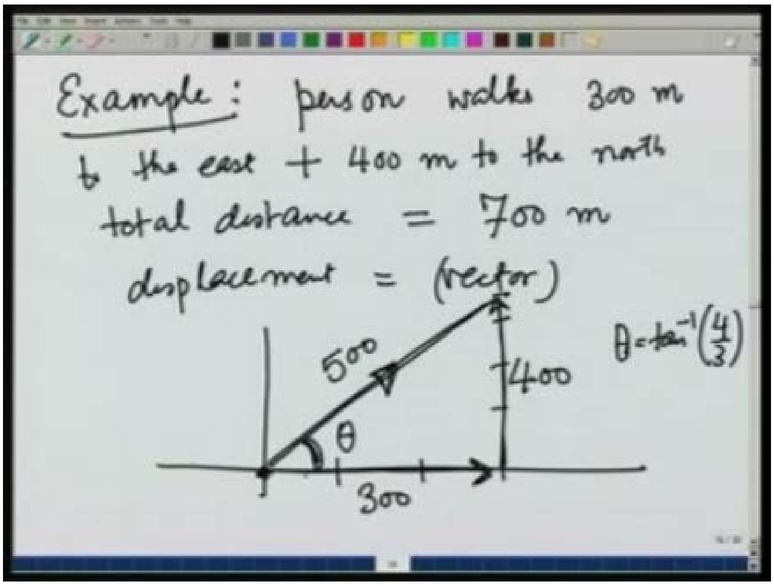
(Refer Slide Time: 21:32)



Suppose, I have to subtract B from A, this would be same as A plus minus 1 times B. So, what do we do? We take vector A and I want to subtract B from it, this would be the same as taking A and adding minus B to it. So, A minus B would be; A minus B would be this, this would give me A minus B. And you can see right away, that A minus B is not same as B minus A. Because B minus A would be, B minus A; minus A would be opposite to A and therefore, this would be like this, this is B minus A.

So, you can see that A minus B is not the same as B minus A. But, through graph you can see very easily that A minus B is equal to minus of B minus A. So, what we have done is addition and subtraction of 2 vectors. Similarly, if we have to add 3 vectors, you can do A plus B then add the third one to it. If we have to do 4 vectors you can do A plus B, C plus D and the 2 resultants again and so on. Let us do 2 examples, showing the summation of 2 vectors and subtraction of 1 vector from the other.

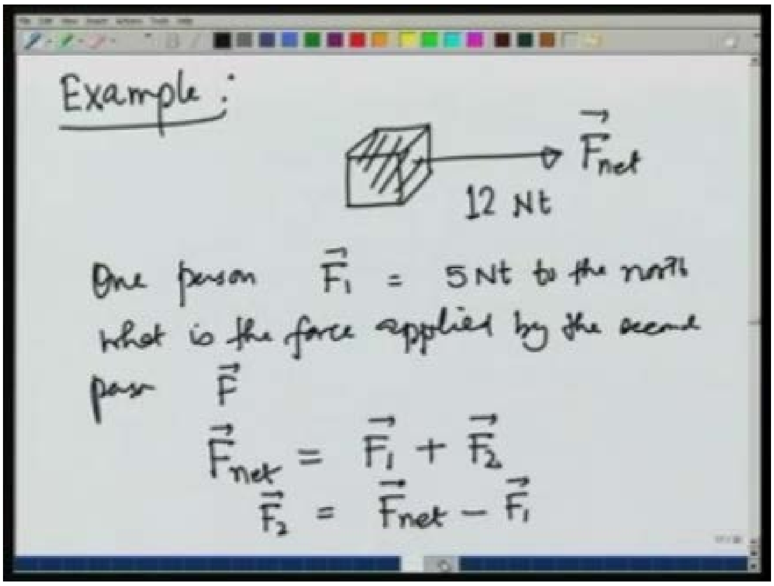
(Refer Slide Time: 23:39)



So example 1: suppose there is the person, walks 300 meters to the east and then 400 meters to the north. We want to ask, what is the total distance he covered? This is obviously, 700 meters. And then we ask what the vector displacement; which is the vector quantity of the person from the original position? To get that, we have to add the 2 displacements vectorially.

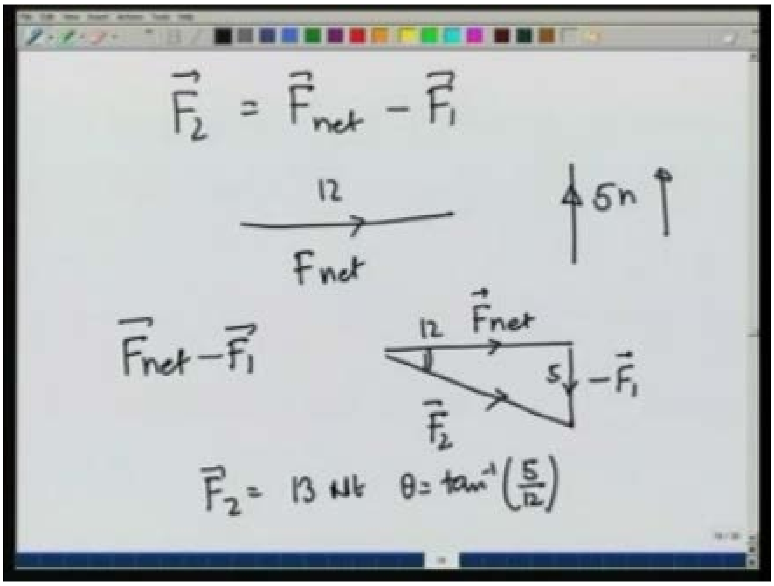
So, suppose the person is started from here, he moved 1 2 and 300 meters to the east and then 1 2 3 400 meters to the north. So, this is his first displacement up to this point and then second displacement from here to here and therefore, the net displacement would be in this direction; this is 400 hundred, this is 300 hundred and therefore, this would be 500 hundred. So, net displacement of the person is 500 hundred meters and in which direction, at an angle theta where theta is going to be tangent inverse of 4 over 3 from the east towards north. So, this is his net displacement. This is how you would find the addition of 2 vectors and which gives you the magnitude and its direction.

(Refer Slide Time: 25:42)



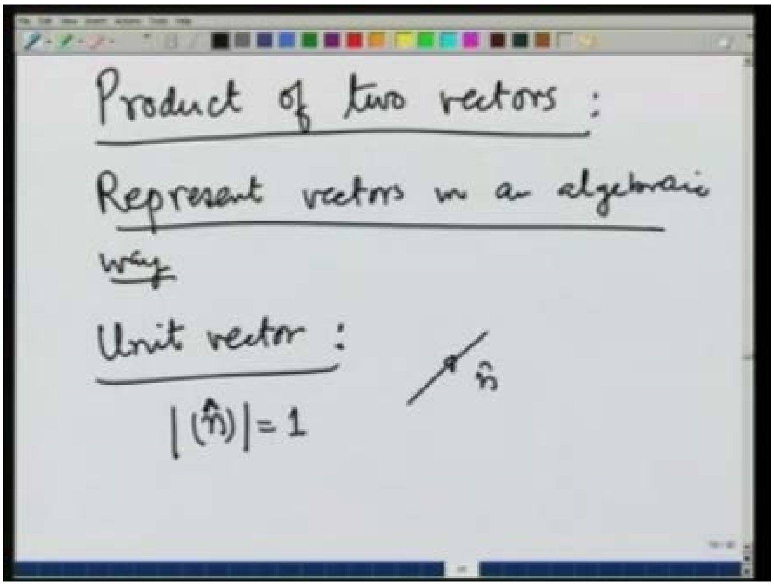
In the second example let us take, the case of a 2 persons pushing a box so that, the net force F net is, let us say 12 Newton's to the right or to the east. 1 person, apply the force F1 which is 5 Newton's to the north. We ask what is the force applied by the second person? Let it be F2. Then we know that F net is going to be equal to F1 plus F2 and therefore, F2 is equal to F net minus F1.

(Refer Slide Time: 27:07)



F2 is equal to F net minus F1. We have been given that, F net is 12 Newton's to the east, F1 this is F net; F1 is 5 Newton's to the north and therefore, F net minus F1 is going to be, this is F net minus F1 will make going to the south and this would give me F2. You can see that F2 is going to be 13 Newton's because, this is 12 and this is 5. At an angle, theta equals tangent inverse 5 over 12 towards south from the east. So, I have given you 2 examples, 1 in which we added 2 vectors and 1 in which we subtracted 1 vector from the other.

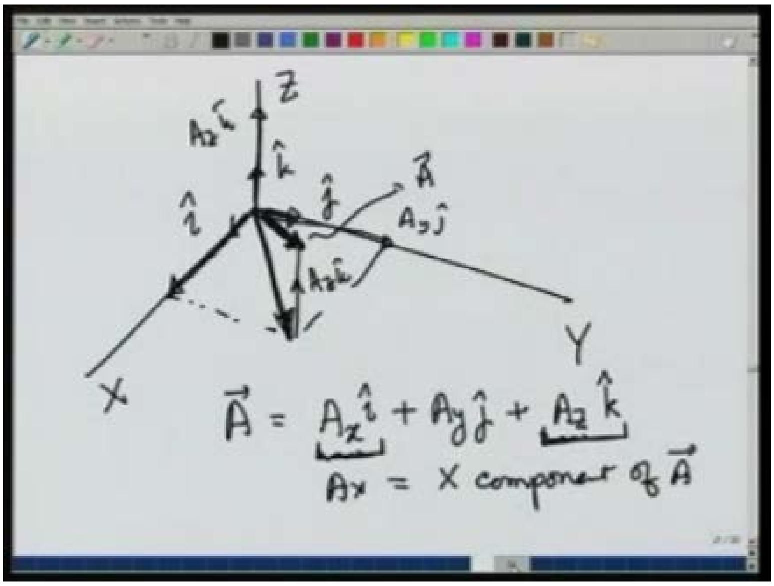
(Refer Slide Time: 28:30)



Next we ask, can we define product of 2 vectors? The answer is "yes", but before doing that, I will take a slide digression and now we will go into vector algebra representing them by numbers and in a different way algebraically. So first, let us represent vectors in an algebraic way and then we will be able to define the product of 2 vectors properly. Not only that representing vectors in algebraic way, would make addition and subtraction of 2 vectors easy when we have many vectors.

So, for that first I introduce the concept of unit vector. A unit vector is a vector of magnitude 1. So, let us say unit vector n and to show that it is a unit vector we make a hat on top of the vector, this magnitude is going to be 1 and it could be any direction representing the vector in that particular direction.

(Refer Slide Time: 30:02)



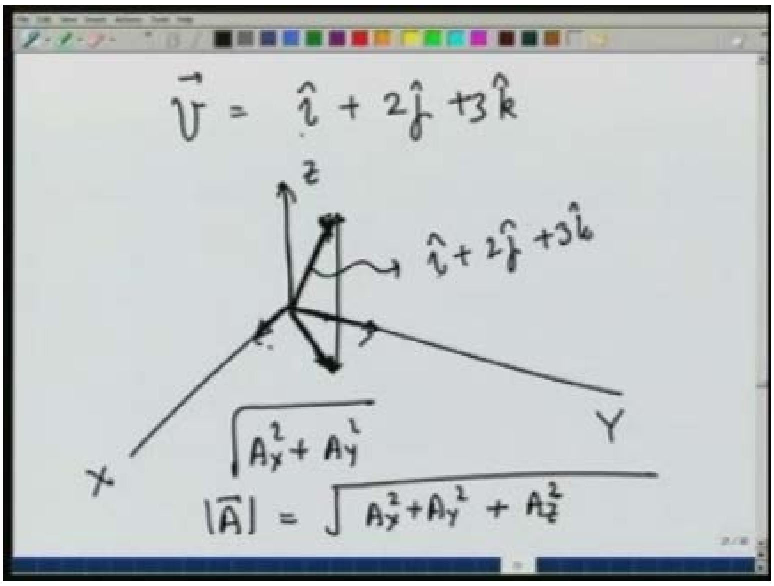
In particular now, we will define take the X Y and Z axis, a Cartesian coordinate system and defined; unit vectors in X direction call it i, unit vector in y direction call it j and unit vector in the z direction call it k. Now, any vector can be written as, A what I call the x component of this vector times i, plus A the y component of this vector j, plus Az and z component of this vector k. So, this is known as the z component, y component, x component.

You can see that, what we are doing is multiplying this unit vector by Ax. So, this will give me Ax times this a vector in this direction, let us say this Axi and I add this to Ayj. So, I will draw it parallely here, this will give me Axi plus Ayj and on top of it we will add say Azk. I move it here parallely. So, the net vector A would be something like this, this is Azk.

Although, in this case I added Ax and Ay force and then Az and got this vector. It does not matter in which order you add these vectors because, we have already proved, that addition of vectors is commutative. So, I could have added Ay and Az first and then added Ax, I will still end up with the same vector A.

So, now the meaning is quite clear, when I write A equals Axi plus Ayj plus Azk I am writing a vector; Ax times I pointing in x direction. Aa vector Ay time j of magnitude Ay pointing in y direction and a vector of magnitude Az pointing in k direction and adding them all up, and that is the vector A. Ax A is called the x component of A and Ay the y component, Az the z component.

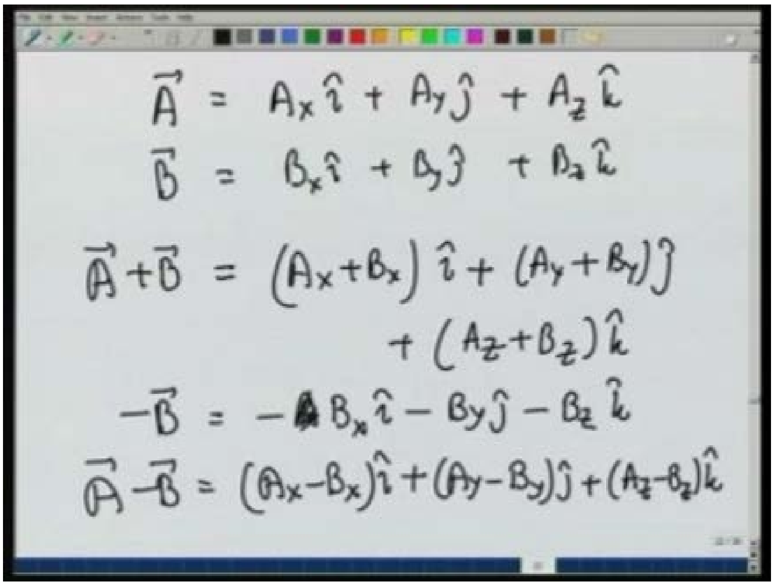
(Refer Slide Time: 32:46)



So, let us see suppose, I have written a vector V is equal to I plus 2j plus let us say a 3k, how would it look? X Y Z. So, I would make vector i unit vector in this direction, plus 2j. 2j would be a vector of magnitude 2 in this direction. So, this would be i plus 2j and 3k would be of magnitude 3 in this direction so, up to this point. Let us make this vector here, this, then summation of all 3 this vector would be i plus 2j plus 3k.

How about the magnitude of this vector? You can see that, I can first add these 2 get the magnitude of this; that would be Ax square plus Ay square square root and then again add a perpendicular vector to it which will be Az. So, the net magnitude would be square root of square of this component which is Ax square plus Ay square and then perpendicular component Az, add them all up and take the square root. This is the magnitude of the vector and you can see the direction is easily seen and visualized by making the vector using these unit vectors.

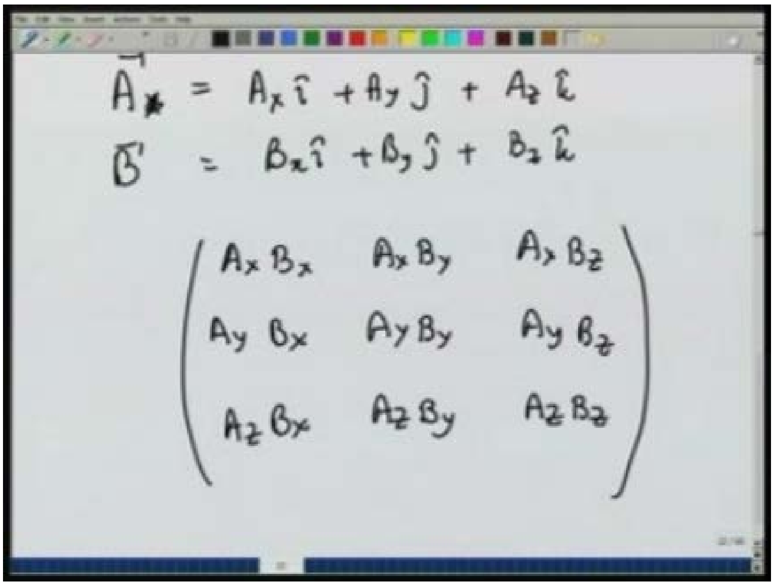
(Refer Slide Time: 34:31)



Having written A vector in its algebraic form, let us ask, how do I add 2 vectors? Suppose, there is another vector B which is Bxi plus Byj plus Bzk, when I add the 2 vectors A plus B, you can see graphically and carry it out that. The net vector would have a X component which is a summation of Ax components of individual vectors, then I add the Y components of individual vectors and then, Z components of the individual vectors.

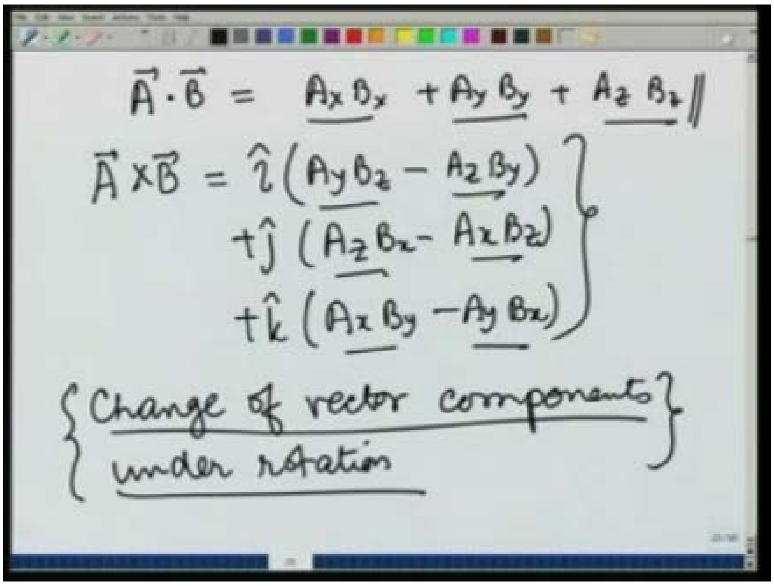
Similarly, minus B would be nothing, but each component of the B vector would become negative and therefore, A minus B which is nothing, but adding A and minus B would be same as Ax minus Bx i plus Ay minus Byj plus Az minus Bz.

(Refer Slide Time: 36:06)



Having written the vectors Ax A Ayj plus Azk and B as Bxi plus Byj plus Bzk, we are now ready to write their product. All possible combinations of the products are: Ax times Bx, Ax times By, Ax times Bz. Similarly, Ay times Bx, Ay times By, Ay times Bz and Az times Bx, Az times By and Az time Bz. These are all possible products of different components of the 2 vectors. Out of which, which ones do we take? Which one’s do we define as vectors and scalars, is the next task that we are going to address. You already know from your intermediate twelfth grade that, they are 2 kinds of products:

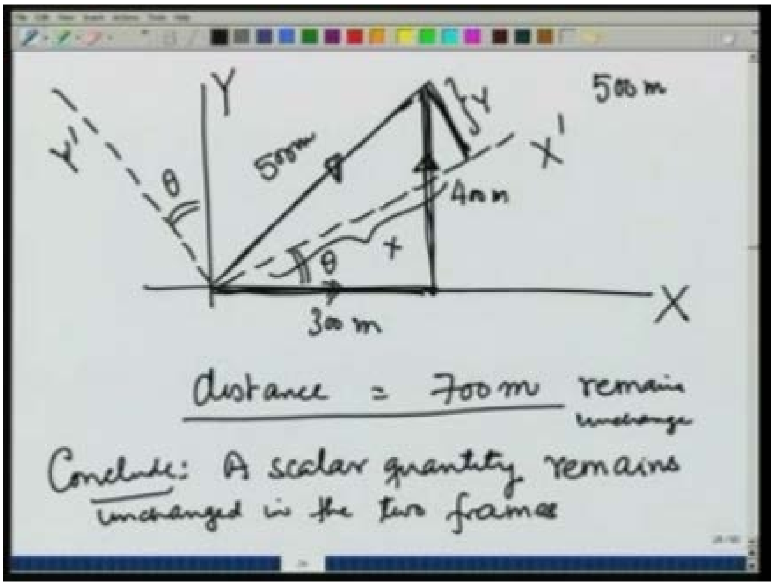
(Refer Slide Time: 37:26)



A dot product; which is the scalar product, which is given as the sum of Ax Bx plus Ay By plus Az Bz and the cross product A cross B which is a vector quantity which is given as, a component Ay Bz minus Az By in X direction plus j Az Bx minus Ax Bz in Y direction plus k the component in Z direction is Ax By minus Ay Bx. But, as I showed you in the last slide, there are these all possible products which are being taken care here or why do we take them in this particular combination? Why do I define this to be a scalar product? Why do I define this to be a vector product? Is the question that I want to address now, so that we get a better understanding as to why, the products of these 2 vectors are defined in such a manner. And for that, I want to look at another property of vector quantity and that is the change of vector components under rotation.

So, far we have just looked at a vector quantity as something, that has the direction and that has a magnitude. But now, we are going to look at some more properties and one property that we especially look at is a change of vector components under rotation. Let us see that.

(Refer Slide Time: 39:35)

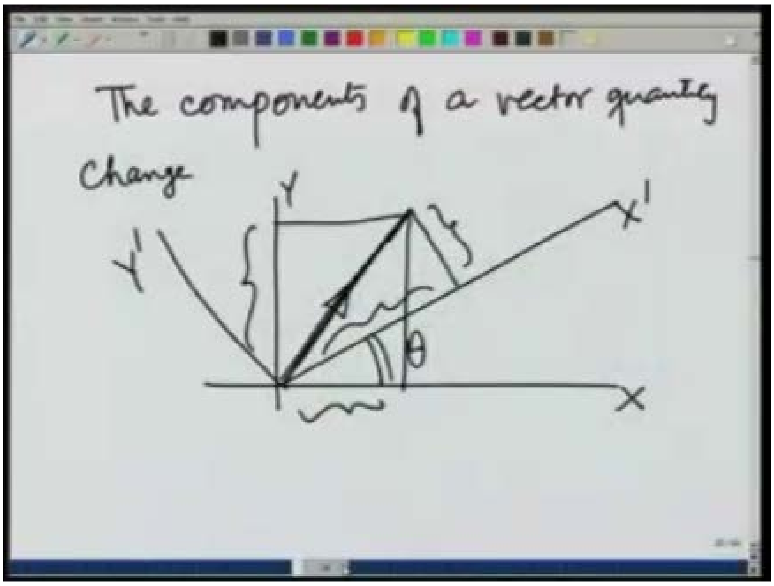


Suppose, you have a friend who asks you, how far is the house of another friend? And you say it is 500 meters or you say you walk 300 meters to the east, I am just taking the previous example. And 400 meters to the north so that, the direct distance of that house is 500 meters. But, you have to walk in a zigzag manner, in that you have to go east first and then north. Total distance you cover is 700 meters.

Now, suppose you have another friend who is looking at it from a different frame. So, for the previous friend you had the X axis along the east and Y axis along the north. But, another friend has his axis X prime in this, add some angle theta from the east and Y prime add some angle theta from the north. Although, the distance traveled by this person in going to the other house is going to be 700 meters in both the frames. However, this vector you can see is going to have components which are not going to be 300 and 400 meters, but this is going to be its X component and this is going to be its Y component.

So, although the vector remains the same it has 500 hundred meters in this north east direction, but its components along different frames are different. On the other hand for the scalar quantity distance, that remains the same unchanged. So, what we conclude: a scalar quantity which is nothing, but a number remains unchanged in the 2 frames.

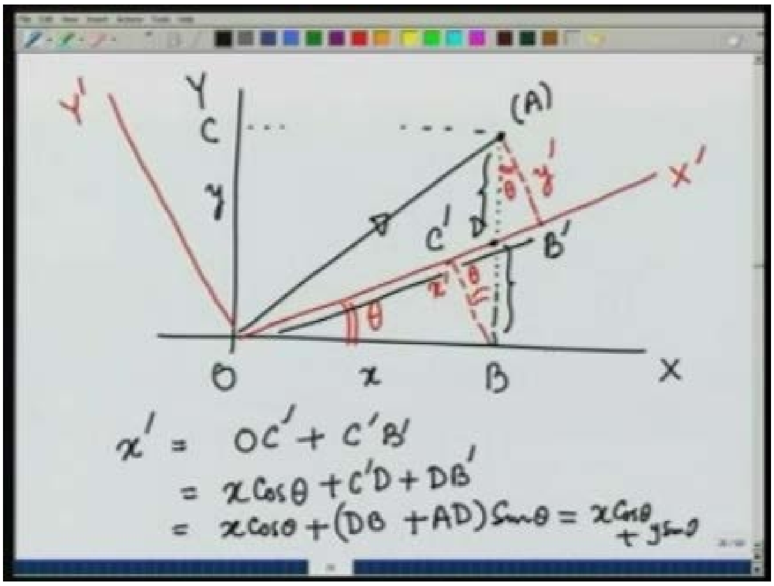
(Refer Slide Time: 42:09)



On the other hand you also see that, the components of a vector quantity change, when we go from one frame say X Y to another frame which is rotated with respected to the first frame: X prime, Y prime which is rotated by an angle theta with respect to the first frame. For the same vector quantity which in space is still pointing the same direction has the same magnitude, this is going to be X component, this is going to be Y component. On the other hand the X prime component is going to be this and Y prime component is going to be this. And the relationship between X prime and Y prime component and X and Y component is well known and that is what we have going to derive now.

So, a vector quantity must follow that relationship, its components should change according to that relationship, when we go from frame to another frame which is rotated with respect to the first thing

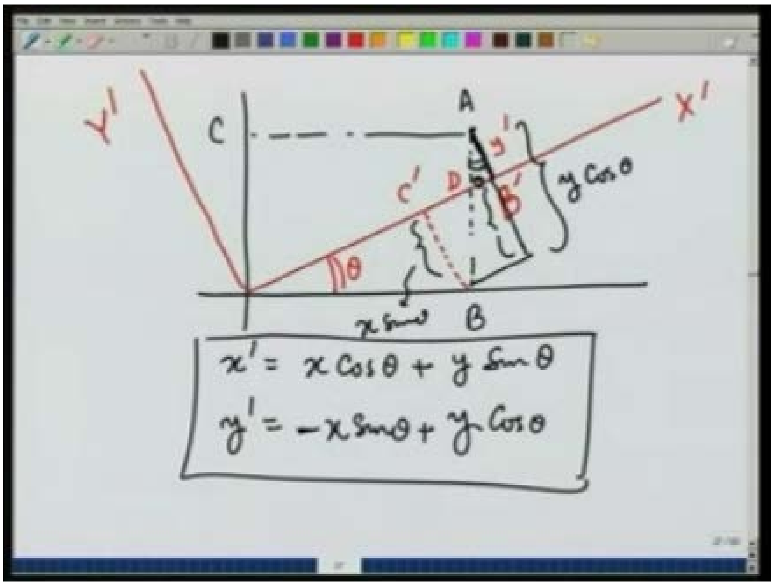
(Refer Slide Time: 43:22)



So, let us derive the relationship. So, let us say there is a vector pointing in a certain direction in the original frame X and Y so that is, this is X component and this it is Y component. I am looking at the same vector from anther frame and let me now use the different color. X prime and Y prime frame which is rotated with respected to the first frame by an angle theta. The component X prime is going to be given here, this is going to be X prime and this is going to be Y prime. You can also see that this angle is also theta.

So, now we want to find let me also draw a perpendicular from here to here, this angle is also theta. Let this point be A B C, this is the origin. Let this point be B prime. Let this point be C prime. So, we see that X prime component is going to be equal to OC prime plus C prime B prime that is, this plus this and let me also call this point where it intercepts, as D. OC prime is nothing but, X cosine theta. So, this is going to be x cosine of theta plus C prime. B prime is nothing but, C prime D plus DB prime. So, this is nothing but, C prime D plus D B prime, which is equal to X cosine of theta C prime. D is nothing but, DB sin of theta. I am deliberating writing it like this plus DB prime is nothing but, AD sin of theta, sine of theta I have taken out.

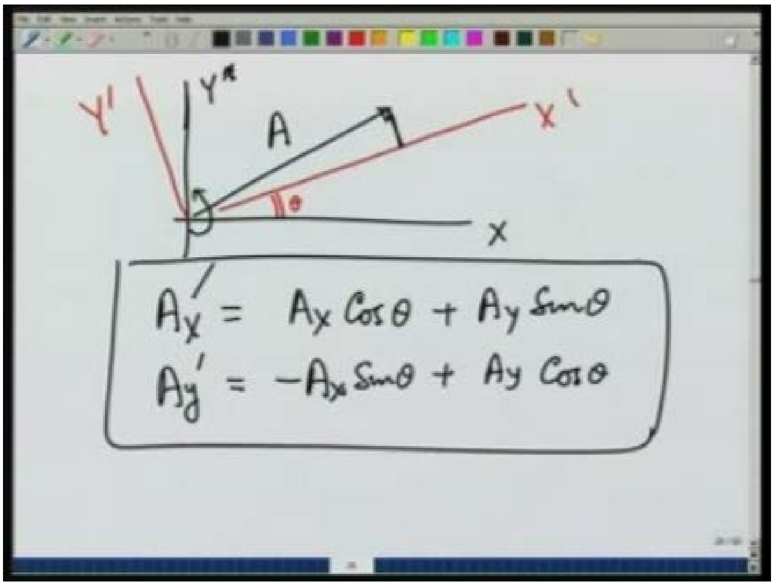
(Refer Slide Time: 46:31)



Now, you see DB plus AD is nothing but, Y. So, I can write this as X cosine of theta plus Y sin of theta and what I have shown you in the previous page is that, X prime is x cosine of theta plus y sin of theta.

Similarly, if I want to calculate Y prime; this length, this, let me extend this here. This distance is nothing but, this angle is theta, y cosine of theta minus this distance which is the same as this distance which is nothing but, x sin theta. So, y prime therefore, is going to be y cosine of theta minus x sin of theta and this how the x and y components of vector are going to change I wrote this for a displacement vector x and y. But, in general you can do the same exercise and you would find that, if I look at a vector

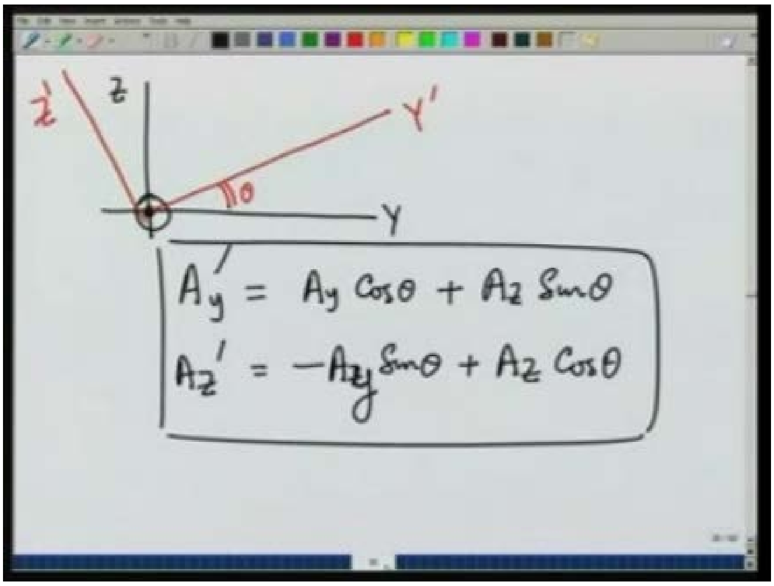
(Refer Slide Time: 47:47)



A in the 2 frames, which are rotated with respect to the each other then, Ax prime that is the component of A in the rotated frame is going to be equal to Ax cosine of theta plus Ay sin of theta and Ay prime is going to be equal to minus Ax sin of theta plus Ay cosine of theta. This is how the components of a vector transform. On the other hand, the scalar number remains the same number in both the frames.

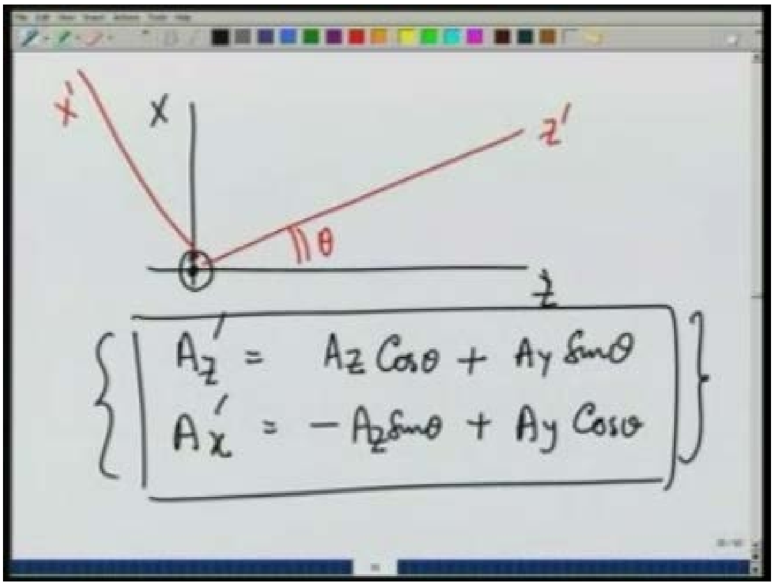
We have looked at in this case, when we rotated the frame about the Z axis because, only X prime and Y prime axis is changed. We can do the same thing for other axis also. So, for example, suppose we rotate about the X axis.

(Refer Slide Time: 49:03)



So, let us say we have X which is coming out, Y and Z and suppose we take the new frame by rotating about the X axis X Y and Z X you see. So, this is going to be Y prime and this is going to be Z prime by an angle theta. Then you can see that what we did earlier by that, Ay prime in the rotated frame and Y rotate about the X axis is going to be Ay cosine of theta plus Az sin of theta and Az prime is going to be equal to minus Az sin of theta or minus Ay sin of theta plus Az cosine of theta.

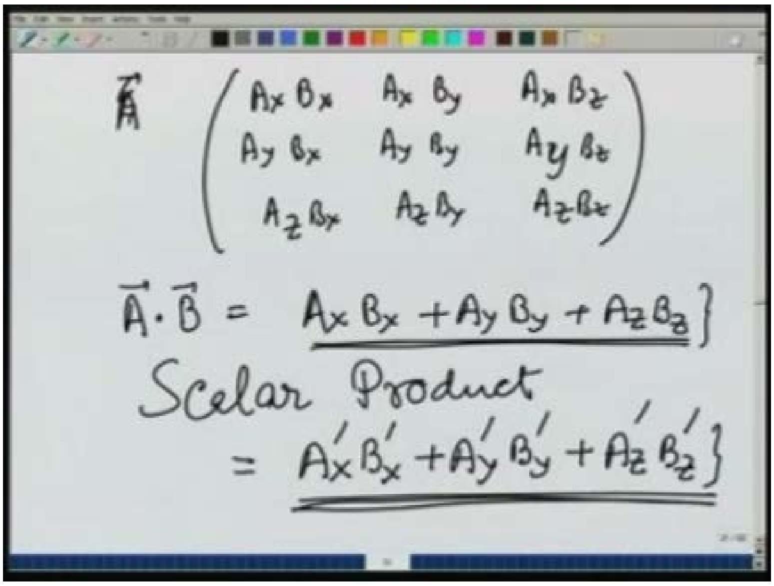
(Refer Slide Time: 50:14)



Similarly, if I rotate about the Y axis let us do that. If I rotate about the Y axis, this is going to be X axis, Z axis, Y axis coming out and make my new Z prime and X prime axis like this, by rotating about the Y axis you will see that Az prime is going to be Az cosine of theta plus Ay sin of theta and Ax prime is going to be minus Az sin of theta plus Ay cosine of theta.

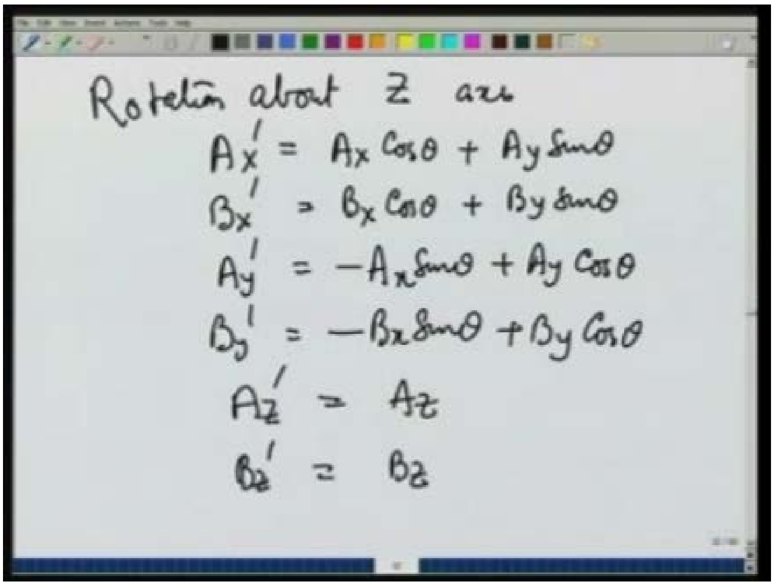
So, this is how the vector components of a vector transform under rotation with respect to X axis, Y axis or Z axis that is, if I rotate about the Z axis I change X prime and Y prime components. if I rotate about the X axis I change Y and Z components and if I rotate about the Y axis I change X and Z component and they change in a very specific manner. On the other hand, the scalar quantity remains unchanged even if it is looked at from a rotated frame and that we are going to use now to define our products.

(Refer Slide Time: 51:36)



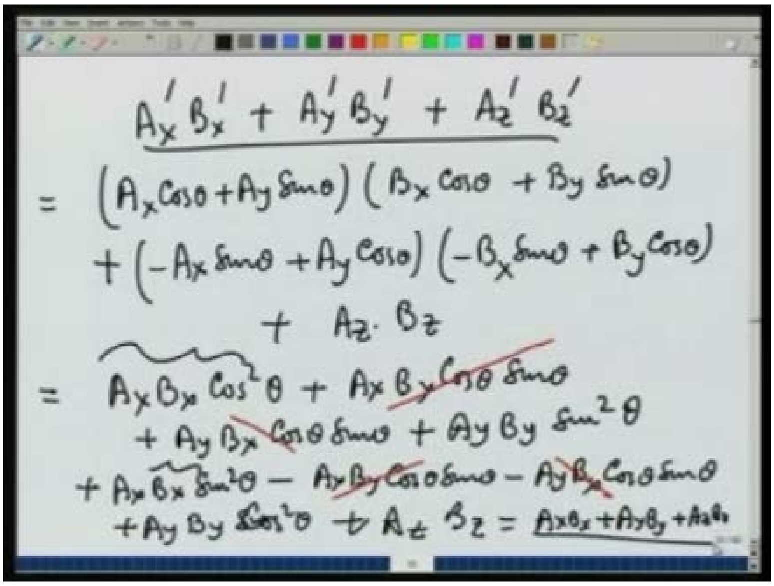
So, first quantity I told you about is of those, let me write all the components Ax Bx Ax By Ax Bz Ay Bx Ay By Ay Bz Az Bx Az By Az Bz. Of all these 9 components we took 3 of them and wrote a scalar product as the sum of Ax Bx plus Ay By plus Az Bz and we call this as scalar product that is, if I take these components and add them up this gives me a number which is scalar how do I prove it? I prove it by going to a new frame and which the scalar product is going to be Ax prime Bx prime plus Ay prime By prime plus Az prime Bz prime and this Ax Bx plus Ay By plus Az Bz must be the same as Ax prime Bx prime plus Ay prime By prime plus Az prime Bz prime if, this is a scalar.

(Refer Slide Time: 53:25)



Let us show that for one specific case and rest I leave for you as an exercise. So, let us look at the rotation about Z axis so that, in that case Ax prime would be equal to Ax cosine theta plus Ay sin theta. Bx prime would be similarly Bx cosine theta plus By sin theta. Ay prime would be minus Ax sin theta plus Ay cosine theta. By prime would be equal to minus Bx sin theta plus By cosine of theta. Az prime would be same as Az because, Z axis is not changing, I am rotating about the Z axis. Bz prime would be same as Bz.

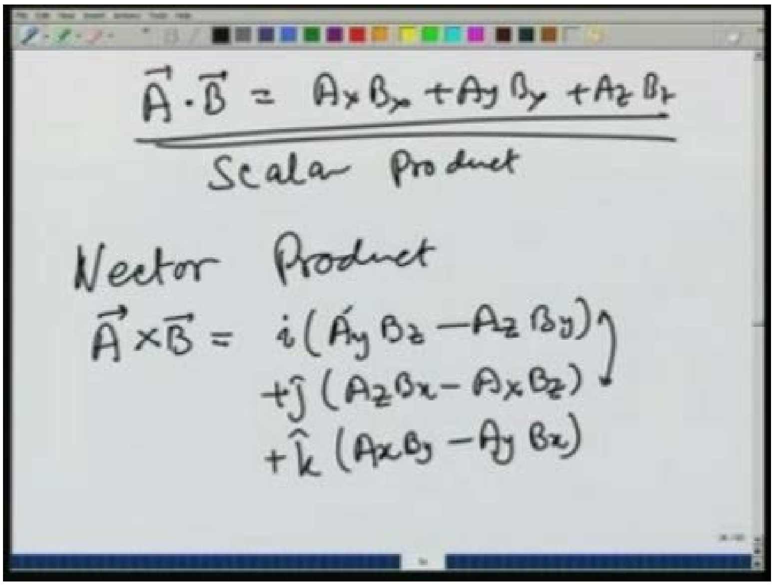
(Refer Slide Time: 54:34)



Now, let us calculate Ax prime Bx prime plus Ay prime By prime plus Az prime Bz prime and this would come out to be equal to Ax cosine theta plus Ay sin theta times Bx cosine theta plus By sin theta plus minus Ax sin theta plus Ay cosine theta times minus Bx sin theta plus By cosine theta plus Az times Bz. This gives you Ax times Bx cosine square theta plus Ay times By sin square theta. The first term you see Ax Bx cosine square theta plus Ax By cosine theta sin theta plus Ay Bx cosine theta sin theta plus Ay By sin square theta plus Ax Bx sin square theta minus Ax By cosine theta sin theta minus Ay Bx cosine theta sin theta plus Ay By cosine square theta plus Az Bz.

Now, you see this term cancels with this, this term cancels with this and Ax Bx cosine square theta plus Ax Bx sin square theta gives me Ax Bx. Similarly, Ay By sin square theta plus Ay By cosine square theta gives me Ay By plus last term is Az Bz. So, what I showed you just now, by transforming the different components that Ax prime Bx prime plus Ay prime By prime plus Az prime Bz prime even under transformation remains the same as Ax Bx plus Ay By plus Az Bz and therefore, this is a scalar quantity and that is precisely why, we call take this particular combination Ax Bx plus Ay By plus Az bz and call this a scalar product.

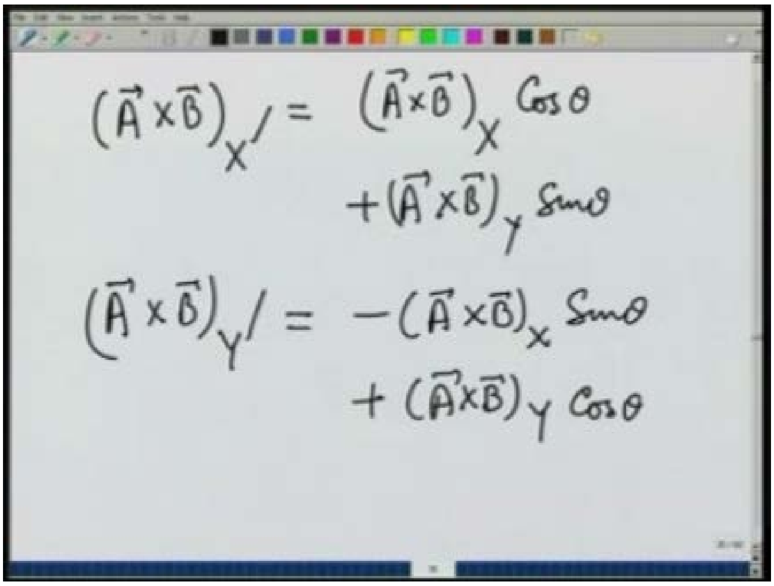
(Refer Slide Time: 57:44)



Similarly, the other quantity that we defined is the vector product in which case, A cross B is given as i, the X component is given as Ay Bz minus Az By plus the Y component is given as Az Bx minus Ax Bz plus the Z component is given as Ax By minus Ay Bx.

What happens under transformation? Under transformation, if I transform each component Ax Ay by the way we did earlier, these components should also mix according to the rule of vector transformation.

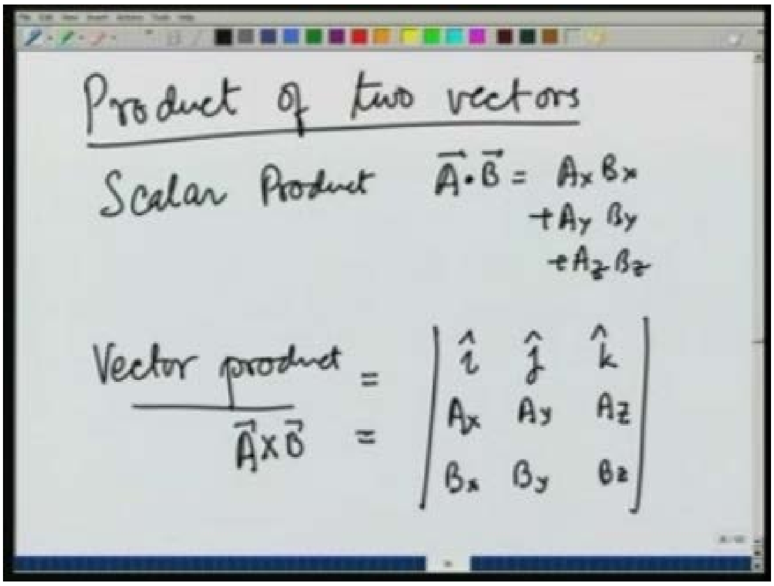
(Refer Slide Time: 59:09)



and I leave it as an exercise to show that, A cross B, if I take this component in X prime direction it is equal to A cross B X component cosine of theta plus A cross B this Y component sin of theta. Similarly, A cross B Y prime component if I am rotating; if I am changing only X prime and Y prime axis and by rotation about the Z axis, should be equal to minus A cross B the x component of sin theta plus A cross B y component cosine of theta. I leave this as an exercise for you.

So, it is this combination, Ax By minus By Ax and things like those that defines the vector product for us. So, product of 2 vectors

(Refer Slide Time: 01:00:18)



1 is a scalar product which is A, written as A dot B which is nothing but, Ax Bx plus Ay By plus Az Bz and the other is the vector product which gives me a vector quantity, which in a short term, I can as a determinant i j k Ax Ay Az Bx By Bz a determiner of this and this is A cross B. I leave it for you to check that, this gives you same expression as we used earlier.