Biomedical Ultrasound: Fundamentals of Imaging and Micromachined Transducers

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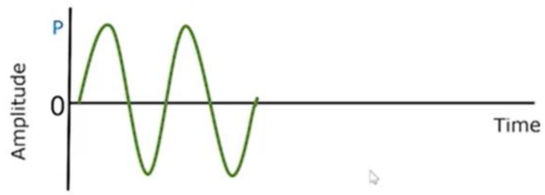
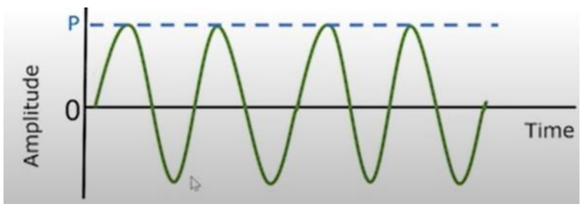
Lecture: 05

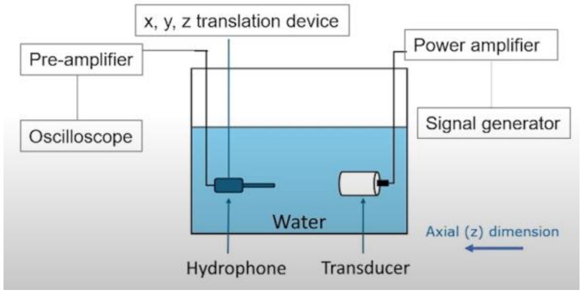
Intensity, Reflection, Transmission

Hello, welcome to Biomedical Ultrasound Fundamentals of Imaging and Micromachined Transducer. I'm Professor Karla Mercado-Shekhar, and in today's lecture, we'll discuss concepts on intensity, reflection, and transmission. Let me introduce some ultrasound field descriptors. So typically, the ultrasound wave field is described in terms of pressure amplitude, typically in terms of Megapascals. So, as you can see in this schematic, this is what a sinusoidal waveform would look like, and it's typically characterized by this pressure amplitude. As we know, pressure does not give the entire picture of the acoustic energy because the acoustic energy also depends on the duration of the pulse.





If we compare these two pulses, the one in the bottom being have more cycles compared to the one in the top, they have the same pressure amplitudes, but the one in the bottom will have more acoustic energy. And the way we would measure the pressure amplitude would be based on this practical setup.

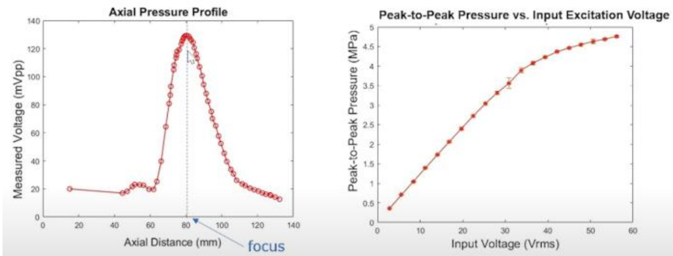


So, what we had here is a setup that consists of a hydrophone, which is used to measure acoustic pressure distributions, as well as the shape of the ultrasound beam. Now this hydrophone, you can think of it as a microphone for ultrasonic frequencies, and this hydrophone can be immersed.

It's a piezoelectric element that can be immersed in water. So, a typical ultrasound experiment would look something like this. So, in this schematic we have both the hydrophone and the transducer of interest inside immersed into the water tank. Now, this transducer is excited by a signal generator, which then you can use to generate the ultrasound pulses that you will send to the transducer. And the power amplifier here is needed to be able to amplify the signal to the transducer.

Now the transducer will send an ultrasound beam in the direction from right to left known as the axial or the Z dimension right here. And typically, what the hydrophone is connected to is the preamplifier as it will receive and amplify the ultrasound signal received from the transducer, and the signal will then be digitized with the oscilloscope. So, this hydrophone is also connected to a XYZ translational device here, and that will allow the hydrophone to scan the beam in three dimensions inside the water tank. The way we would measure the pressures based on the signal received from the hydrophone is based on calibration data. So, the voltage measured using this calibration data can be converted to pressure based on the hydrophone sensitivity.

Typically, the calibration data is in terms of Millivolts per Megapascals. And this allows us to convert this measured voltage into pressure. So, what you can see, an example of the plot of the axial beam profile is to the left here, where we have on the x-axis the axial distance from the transducer and as well as the measured voltage here. So, you can see that on the axis, as you move towards the focus here, the amplitude of the signal increases. And this is where the transducer has a focal length of 80 millimeters.



As you go further away, as the hydrophone goes further away from the transducer, the beam profile decreases in amplitude, as you can see here. After receiving, also after receiving this waveform, what we also do is compute the peak-to-peak pressure. So, what the peak-to-peak pressure is, for example, if this is what the ultrasound signal would look like, which you would collect from the hydrophone, you would get the amplitude from the highest maximum peak here to the lowest. So, this is our peak-to-peak pressure amplitude. And we would usually plot that as a function of the input voltage to the transducer here, which we would also measure.

And as you can see, as we increase the voltage that is being sent to our transducer, the peak-to-peak pressure also increases. But as the voltage increases much further, notice that the peak-to-peak pressure also tends to saturate. Now, these are the excitation limitations of the transducer. Now, when selecting a hydrophone to use, we have to make sure that the hydrophone has a small aperture, small enough relative to the wavelength, the ultrasound wavelength, to avoid spatial averaging, as well as disturbing the ultrasound field. So, what you would receive from the hydrophone is a signal at a particular point location in the beam.

And if the hydrophone is much bigger than the ultrasound wavelength, then the signal that is being received will be spatial averaged along the aperture of the hydrophone. Also, if the hydrophone is too big, then it can disturb the ultrasound field, and then the wave profile changes. An ideal hydrophone would be a point-sized detector. It should have an infinite bandwidth. That way, it can be used for ultrasound in a wide range of frequencies.

And it also should have isotropic sensitivity. What this means is that the directivity pattern of the transducer is equivalent in all directions. So, if you send an ultrasound beam in one direction and the hydrophone is receiving it at a particular direction, the sensitivity of that hydrophone should be same as when a transducer is sending a beam at all different directions. So, this is what an ideal hydrophone is. An ideal hydrophone does not exist in practical, in real life, but there are recommendations for which the type of hydrophone you would need based on its size.

So, these recommendations are provided by the International Electrotechnical Commission or the IEC. This commission produces standards for various electronic instruments all over the world, and what they have recommended for the maximum effective hydrophone's radius, which is defined by in this equation by b_{max} is according to this equation, which is a function of the wavelength of the sound inside the water, also the axial distance between the transducer and the hydrophone.

So, if you have a transducer face here, you have a hydrophone here, the distance between those two would be defined by this "l" parameter, as well as the radius of the transducer as defined by this "a" parameter here.

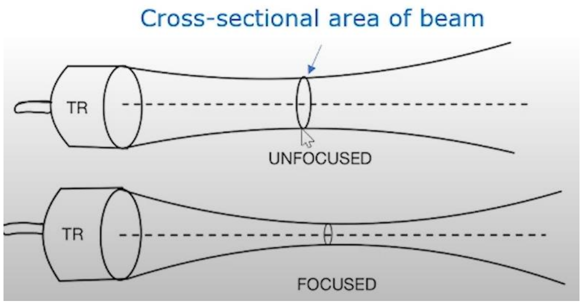
$$b_{max} = \frac{\lambda}{4}\left[\left(\frac{l}{2a}\right)^2 + 0.25\right]^{1/2}$$

So, if you had calculated, if you know these other parameters here, you can calculate this maximum effective hydrophone radius, and you would select the hydrophone whose radius does not exceed the size that you have calculated using this equation. And as we know, as higher transducer frequencies are being used, then you would need a smaller hydrophone size, a smaller aperture to be able to detect the transducer beam signal with good resolution.

So, after you have measured the pressure distributions of your ultrasound beam, you can also calculate intensity. And intensity is related to power. Power, by definition, is the rate at which the transfer of energy is done from the transducer into the medium. And we also note that the distribution of ultrasonic energy is important when we're mapping the beam. So, in this case, intensity parameter becomes quite important and intensity equals to the power divided by the cross-sectional area of the beam.

$$Intensity, I = \frac{Power(mW)}{Area\ (cm^2)}$$

Typically, the power is in units of milliwatts and the area that we're looking at in terms of centimeters squared. So, intensities are usually reported in terms of milliwatts per centimeter squared. or in some cases watts per centimeter squared. So, when we talk about different ultrasound beams, here I show a schematic of what an ultrasound beam would look like for an unfocused transducer right here. The area of the beam that we consider in this equation of intensity is the cross-sectional area.



So, what I highlighted here is the beam at its focus. So, A would be the area, would be the cross-sectional area of the beam at its focus. When we use focus transducers, we can imagine that the energy of the ultrasound beam would be highly focused at a particular region. So, these focused transducers would have a smaller beam size here, so smaller beam cross-sectional areas. And so if you just look back at this equation, for the same amount of power that you were driving the transducer, the acoustic power that is being generated in a focus beam at a smaller area would be having higher intensity.

So focused ultrasound beams typically have higher intensities and these types of beams are typically used for bioeffects applications such as in high intensity focus ultrasound which falls into the therapeutic range of ultrasound. The way we would calculate the intensity of utilizing the pressure waveform that we received from the signal, we look at this equation right here, where the pressure at a particular location is denoted by "p". We also recall from a previous lesson that "rho" corresponds to the density of the medium, "c" corresponds to the longitudinal sound speed, and "t" being the period which corresponds to the one wavelength of the sound, one cycle of the sound wave. So, the intensity can be calculated as a integral of the pressure squared at that particular location looking at the period.

$$I = \frac{1}{\rho c}\frac{\int_0^T p^2 dt}{T}$$

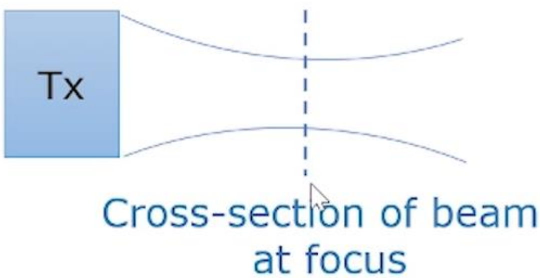
So, we also assume that wherever we are measuring the pressure wave, we also assume that the wave is a plane wave.

In focused fields, we assume that at the region of the focus there is a distribution of plane waves within that focus. Now, if we assume that the wave is such that the sinusoid with a fixed amplitude, such as this shown, then this integral reduces to an equation of this form, where the intensity is the pressure squared divided by two rho c.

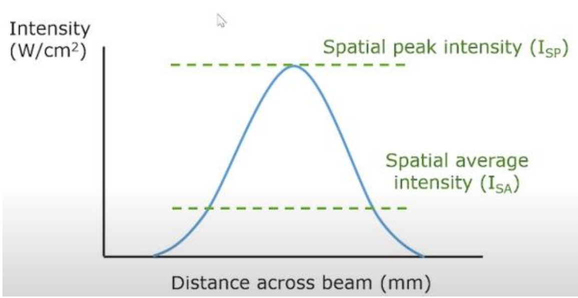
$$I = \frac{p^2}{2\rho c}$$

So, this is a particular special case. And many pressure waveforms may not have this fixed amplitude. So typically, what we do is calculate this integral.

Now intensity is highly important in terms of when we consider the safety and bioeffects induced by ultrasound. And in the future lecture, we will discuss how this intensity parameter is important for these aspects especially in ultrasound imaging. Now, there are several ways you can specify the acoustic intensity. We know that intensity varies over space and time, and there are several intensity metrics that you can look into when characterizing the intensity in space.



So, if we have a look at the schematic on the top right here, we have a transducer, a focus beam right here, and let's take a cross section of the beam at this focus and we plot its intensity profile as a function of time.

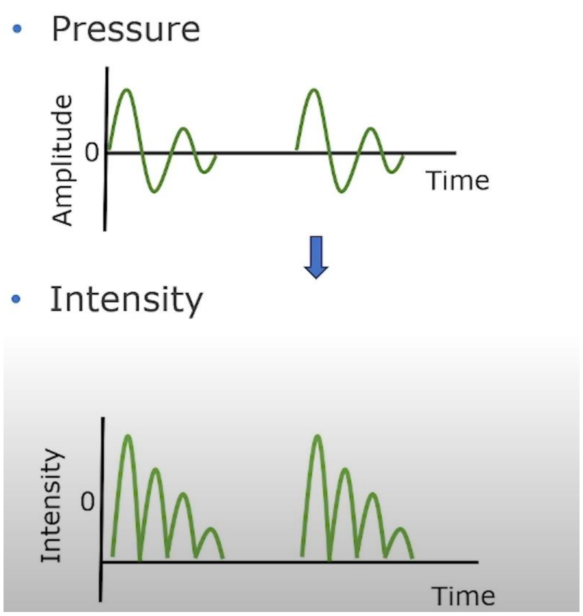


So, in the y-axis we have our intensity and in the x-axis we have the distance across the beam, at the center being the on-axis distance. So, there are two metrics you can calculate. You can calculate the spatial peak intensity right here, which is the intensity at the peak of the beam here. You can also average the spatial average intensity, which averages the spatial peak according to the cross-sectional area of the beam here. One can relate these two types of intensities using the beam uniformity factor, where the beam uniformity factor here is the spatial peak intensity divided by the spatial average intensity.

$$BUF = \frac{I_{SP}}{I_{SA}}$$

And this metric is typically used to assess how uniform the beam is at that location.

Since intensity also varies with time, we also calculate several temporal intensity metrics. So here is an example of what we would get of a pressure waveform can detect from a hydrophone. And in this example, the amplitude of the pressure is not the same throughout the duration of the pulse here. You can apply the integral to get an intensity.



This is just a schematic example. And a couple of intensity metrics that you can calculate are first being the temporal peak intensity, or I_{TP}, which is the peak intensity at that instant when the pulse peaks. Now, this is the maximum value of the intensity within that pulse. And then we can also calculate I_PA, which is much lower than the temporal peak intensity. I_{PA} corresponds to the pulse average intensity.

And this is the average intensity during the on time of the pulse or during the pulse duration. We can also calculate the temporal average intensity, which is I_{TA}. Now, this is the intensity that is averaged during both the on time and the off time of the pulse, and this is over the entire pulse repetition period. So, this value is typically much lower. Temporal average is much lower than the pulse average, and pulse average is much lower than the temporal peak.

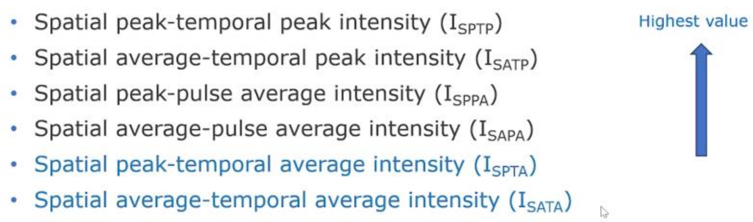
$$I_{TA} = I_{PA}\ x\ Duty\ Factor$$

$$Duty\ Factor = \frac{Pulse\ duration}{Pusle\ repetition\ period}$$

You can also relate the temporal average and the pulse average via the duty factor. In the previous lecture, we had discussed what the duty factor is, and that is called the pulse duration or the on time divided by the pulse repetition period. which is also a summation of the on and off time.

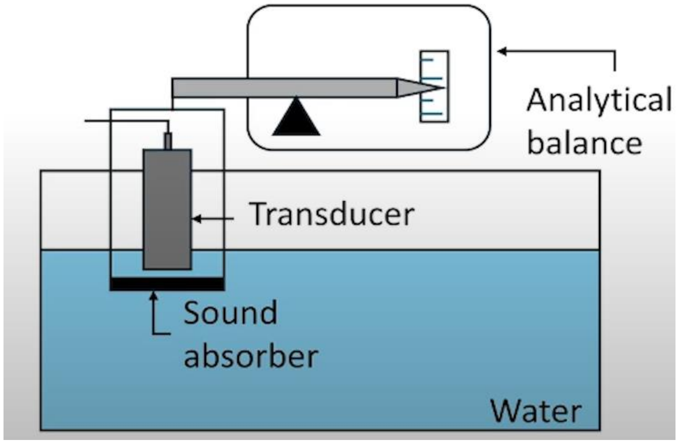
So, here's a summary of all the intensity metrics that we had discussed. We can combine both these spatial and the temporal intensities together, and you can get intensities such as the spatial peak, temporal peak intensity.

You can have spatial average, temporal peak intensity, or I_{SATP}. can have a spatial peak pulse average intensity, I_{SPPA}, the spatial average pulse average intensity, or I_{SAPA}. There are two parameters that are widely used in ultrasound. instance, the spatial peak temporal average intensity, or the I_{SPTA}, and the spatial average temporal average intensity, I_{SATA}. So, as we average the intensity spatially and along the poles, you can see that the value of the intensities decreases.



So, our spatial peak temporal peak has the highest value. In terms of using I_{SPTA}, I_{SPTA} is actually widely used when we're talking about radiation forces, especially when we talk about elastography. And we'll have a subsequent lecture on shear wave elastography that utilizes radiation forces to be able to induce a disturbance or a wave in the tissue which can then be used to calculate the elastic properties. So, this I_{SPTA} metric is important for systems that enable these type of shear wave elastography mode. Where I_{SATA} is widely used is when you want to measure the power of the transducer.

So, this is useful in calculation of the acoustic power. And one way we can have a relationship between measuring the power and the average intensity is by using a radiation force balance. So, what this is, it's a force balance is used to measure a type of radiation force that exerts by an acoustic field. So, this occurs when the ultrasound field is impinging on a target that then induces a transfer of momentum and that causes a force to be created.



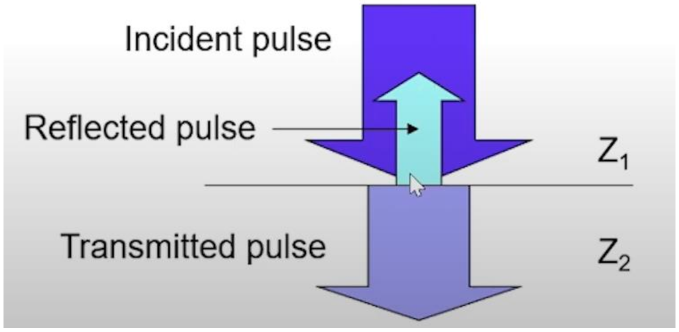
So, this is what radiation force is. We can use this radiation force balance here. So, this is a schematic of what experimental setup using a radiation force balance looks like. We have our transducer here that is then sending an ultrasound field towards a sound absorber right here. So, this will be the target. And this sound absorber is connected to an analytical balance that you can use to measure the force.

And from that, you can measure the power and considering also the cross-sectional area of the beam, you can then measure the intensity, the spatial average temporal intensity.

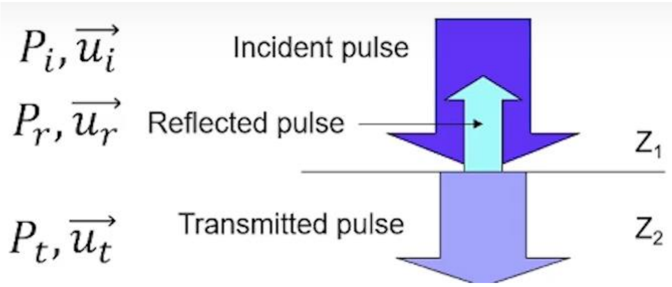
Now let's talk about reflection, reflection at interfaces. So, we know from a previous lecture that acoustic impedance is important and how acoustic impedance is important in producing reflections and scattering. If there are any acoustic impedance mismatch between media that are adjacent together, then it will produce a reflection or a scattering. So, we had recalled from a previous lecture that the acoustic impedance is a function of the density of the medium "rho", the longitudinal sound speed "c", also as a function of the pressure divided by the particle velocity.

$$Z = \rho c = \frac{P}{u}$$

So, there are several ways we can look at reflection. The first one, we'll talk about reflection when the incident pressure is perpendicular from the interface. So here is an example where we have an interface between two media. The top media has an acoustic impedance of Z_1.



The bottom media has an acoustic impedance of Z_2. The incident pulse is firing from top to down. Once it reaches that interface, the pulse can either be reflected or transmitted depending on the difference in the acoustic impedance. And one way to quantify the reflection, how much ultrasound pulse is being reflected is by the reflection coefficient parameter. So, we define here the amplitude reflection coefficient by "R" here. And in this same schematic here, our medium, our pulse is characterized by the incident pulse with the pressure P_i, the particle velocity u_i, The reflected pulse is characterized by the pressure, P_r, the particle velocity, u_r, and the transmitted pulse is characterized by the pressure, T, and the particle velocity, u_t.



Now the particle velocities are vectors, so there's a directional dependence on them. And when we compute the reflection coefficient, we are actually computing the fraction of the incident pulse that is actually being sent towards the pressure of the reflected pulse.

$$R = \frac{P_r}{P_i}$$

Now let's consider some boundary conditions. There are two boundary conditions that we would consider when computing the reflection coefficient with regards to the acoustic impedances of the two media. So first we look at this equation wherein we assume continuity of the particle velocity.

$$\overrightarrow{u_i} + \overrightarrow{u_r} = \overrightarrow{u_t}$$

$$u_i - u_r = u_t$$

$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_t}{Z_2}$$

So, the vector u_i plus the vector of the particle velocity and that of the reflected force equals the particle velocity that's transmitted. So here we assume continuity in particle velocity because if not, if it's not continuous, then the media will separate. Now taking also into account the summation of this boundary condition where the pressure of the incident wave and the pressure of the reflected wave equals the pressure of the transmitted wave here.

$$P_i + P_r = P_t$$

Now this boundary condition is important because if the pressure of the incident and the reflected waves do not equal the pressure of the transmitted wave, then that interface will move, that boundary will move. So, these two boundary conditions are important for deriving the reflection coefficient which we will do now.

So, the particle velocity equations here will then be converted in its scalar form here and this minus or this negative is because of the reflected pulse traveling into the backward direction relative to the incident pulse here. So, if we then substitute the equation of the impedance that I had shown you earlier into these variables of particle velocity, we will get P_i divided by the impedance of the first medium, since the pressure of the incidence pulse reflected onto the top Z_1 medium and minus the pressure of the reflected pulse here divided by the impedance also in the first medium. And that would equal the pressure of the transmitted pulse divided by the impedance of the second medium since the transmitted pulse is traveling in this second medium here. So then, if we use this boundary condition related to the pressures and plug it into the transmitted pressure here, we get this equation here.

$$\frac{P_i}{Z_1} - \frac{P_r}{Z_1} = \frac{P_i + P_r}{Z_2}$$

We just plug it in. Subsequently, if we just rearrange the equations in terms of isolating P_i on one side and P_r on the other side, the equations reduce to this.

$$P_i \left(\frac{1}{Z_1} - \frac{1}{Z_2}\right) = P_r \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

And if you rearrange such that the Pi goes into the right side of the equation to get this, and the right side of the equation incorporating the impedances would move to the left, and we would get this expression, where the reflection coefficient will now equal the impedance of Z_2 minus the impedance of Z_1 divided by the summation of these two impedances. So, this is the final reflection coefficient that one would calculate.

$$R = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

We can also compute an intensity reflection coefficient of this form where our I now is our intensity reflection coefficient. Here we're trying to assess the fraction of the intensity incident pulse that is being reflected back into the medium towards the transducer.

$$R_I = \frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

And this is the form of that intensity reflection coefficient. You can use similar equations to be able to derive this expression, but at this point, we don't have time to do it now.

We'll move on to computing the transmission coefficients. Now, looking forward to how we'll be able to assess how much signal is being transmitted through the interface and how we would quantify that. we quantified using the amplitude transmission coefficient here, which is denoted by this parameter T, where that equals the fraction of the incident wave pressure that is going and transmitted into the second media.

$$T = \frac{P_t}{P_i} = 1 - R = \frac{2Z_2}{Z_2 + Z_1}$$

And that equals one minus the reflection coefficient here. In terms of impedances, this is the expression of the transmission coefficient two by two times the impedance of the second medium divided by the summation of the impedance of both mediums.

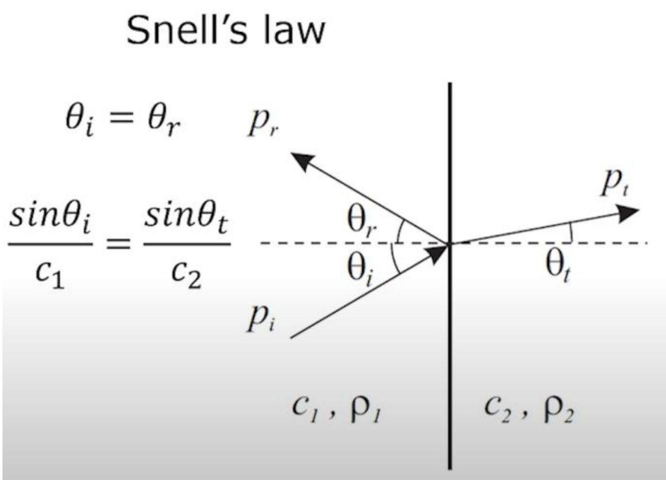
We can also calculate the intensity transmission coefficient. So, it's just a fraction of the intensity that's being transmitted through into the second medium. And it has the following expression here.

$$T_I = \frac{I_t}{I_i} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}$$

Now all these terms can be derived as well using the previous relationships that we have gone through to calculate the reflection coefficient.

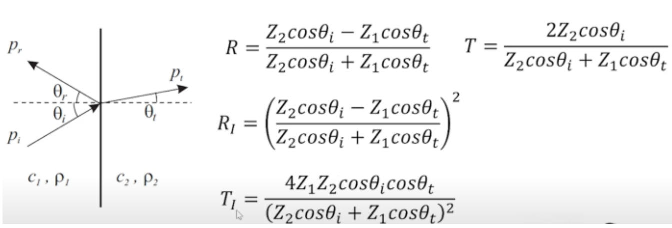
So, we've just discussed the reflection at an interface that is perpendicular to the ultrasound beam. Now let's look into about the oblique incidence. So, what this means is that the incident wave characterized by pressure I here is now traveling at an angle, θ_i, towards the interface. And that can produce some interesting effects.

As you know, you might have already learned about Snell's law. So, Snell's law also applies to sound propagation. So here, what we have as the incident wave goes through the medium into the interface, because of the acoustic impedance mismatch of the two, then there would be a reflected pulse characterized by P_r that would be reflected at an angle similar to the angle θ_i. So, the angle that this pressure field would go through would be θ_r. And because of the difference in acoustic impedances, the transmitted pulse will then be sent towards at an angle that θ_t.

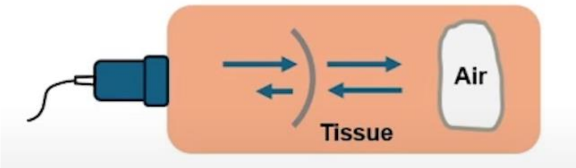


So we're familiar with this Snell's law equation here. And what happens through the transmitted pulse here is that it is being reflected. What does refraction mean? It means that the ultrasound wave is not going through a straight-line path anymore. It is being refracted at a certain angle because of the differences in the tissue properties, in the media properties. And this is important for imaging because if an ultrasound wave is being refracted then it can misregister a target inside the image. We can also quantify the reflection and transmission coefficients in this type of scenario.

So here I show the equations. I'm not going to derive the entire expressions here. But what you can see is that the reflection and transmission coefficients are defined by the angles of incidence as well as the angles of transmission. And similarly, here for the reflection and the transmission coefficients in terms of intensity. So, note that in these equations, there's no dependence on frequency. So, these are just dependent on the angles and the pressure amplitudes for which the wave is being traveled.



When we look at these interfaces in tissue, the reflections are weak except when the interfaces involve air or tissue or bone and tissue interfaces because of their high acoustic impedance mismatch. So, let's take an example of an ultrasound beam that is traveling and is perpendicular to an interface here. So, incident normally at an interface. And we ask the question, what percentage of the intensity will be reflected if the surface is a muscle fat interface? Such as in this case, let's imagine that this is our transducer, this is fat tissue and muscle tissue here. So, what would be the intensity of the reflected sound at this interface? And also, let's consider a fat and air interface right here.



This is fat and this is air. So, we know that the acoustic impedance of muscle, fat, and air are 1.65, 1.33, and air being a less dense tissue or less dense medium compared to tissues has really, really low acoustic impedance. So, what we would calculate is this using our equation of intensity reflection coefficient.

$$R_{I,muscle/fat} = \left(\frac{1.65 - 1.33}{1.65 + 1.33}\right)^2 = 0.012$$

$$R_{I,fat/air} = \left(\frac{1.33 - 0.004}{1.33 + 0.004}\right)^2 = 0.988$$

We just plug in our values to that expression. And we see here that the reflection intensity coefficient between the muscle fat interface right here would be about 0.012, meaning that this is the fraction of the wave intensity that is being reflected back at that interface. But if you consider this fat-air interface here, we're plugging the values of the acoustic impedances of fat and air, we see that it's a really, really high value, 0.988. And this is high because if a reflection intensity equals one, as it approaches one, it means much of the ultrasound wave is being reflected back towards the transducer.

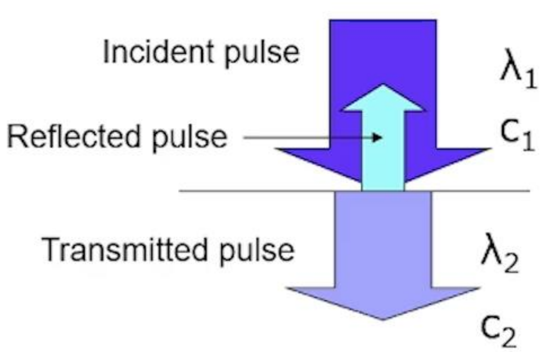
And when that happens, very, very little of the ultrasound wave will be transmitted through the air. So, this is an issue for imaging especially if you have an ultrasound image and there's an air interface then not a lot of the ultrasound signal would go through regions that are beyond the air interface. So, this can be a challenge. And here I have a table of different interfaces and their reflection intensity coefficients right here, where in that you can see that blood-brain 0.3, blood-kidney 0.7. When you talk about water and brain, there's also a substantial amount of reflection intensity coefficients. muscle fat. Just note that these are in terms of percentages. When you look at muscle and bone here, also a high intensity reflection coefficient where 100% equals total reflection.

Interface	R_I (%)
Blood-Brain	0.3
Blood-Kidney	0.7
Water-Brain	3.2
Blood-Fat	7.9
Muscle-Fat	10
Muscle-Bone	64.6
Brain-Skull	66.1

So, bone also has a different acoustic impedance. You would have remembered that the speed of sound in bone is very high. It can go up to 4,000 meters per second. And in soft tissue, it's 1540 meters per second. So, because of this high sound speed and bone is also really dense, it has a really, really high acoustic impedance.

And that can cause a high intensity reflection coefficient as well. So, it is also challenging when you're doing ultrasound imaging of bone to look at regions that are deeper in the other side of the bone.

Let's also look into how the wave speeds can change depending on the frequency of the waves can also change depending on the speeds of sound of the medium. So if a wave was traveling from one media to another. Typically, the frequency remains unchanged. Let's ignore any frequency dependent attenuation for now. But if the media has different speeds of sound, then the wavelengths will change according to this expression right here.



$$\frac{\lambda_2}{\lambda_1} = \frac{c_2}{c_1}$$

So, if you recall this equation, the relationship between frequency, the longitudinal sound speed, and the wavelength here, if we were to rearrange it such that it includes the wavelengths of the sound at medium one and another wavelength at medium two, then these speeds of sounds would also be different. Just the fundamental properties of the tissues can change even the frequency of the wave speed of the sound.

So, what did we learn today? We discussed various ultrasound exposure metrics. We also looked into ways to measure the ultrasound fields, such as hydrophone measurements, mapping the spatial profile of the beam, as well as calculating the pressure amplitude, as well as the intensity. We also looked into various metrics of intensity, the spatial peak temporal average intensity, as well the spatial average temporal average intensity.

So, these two-intensity metrics are frequently used in ultrasound in terms of imaging and assessing the bioeffects and safety. We also talked about the phenomena of reflection, refraction and transmission at different interfaces between different media of varying acoustic impedances. And we've looked into different coefficients that can quantify how much of the wave is being reflected and transmitted through these interfaces. So, what we have discussed today has set up a basic framework for understanding the scattering phenomenon that we will be using for imaging in ultrasound.