Networks and Systems
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Lecture-66
Frequency Response and Stability

In the last lecture, we were considering, the various characteristics and implications of the properties of the system function h of s which is defined as the ratio of Laplace transform of the output to the Laplace transform of the input with 0 initial conditions and the system.

When we say 0 initial conditions and the system, we do not really end of the any loss of generality because any initial conditions this system can be replaced by the equivalent source as we have thought of in the case of the electrical networks. Similarly, in the case of systems as additional inputs we can think of non 0 initial conditions additional inputs.

And therefore, each non 0 initial condition can be thought of as a separate excitation and 1 can use the super position principle therefore, when we say 0 initial conditions this really knows loss of generality, we considered various aspects of the characteristics of h of s.

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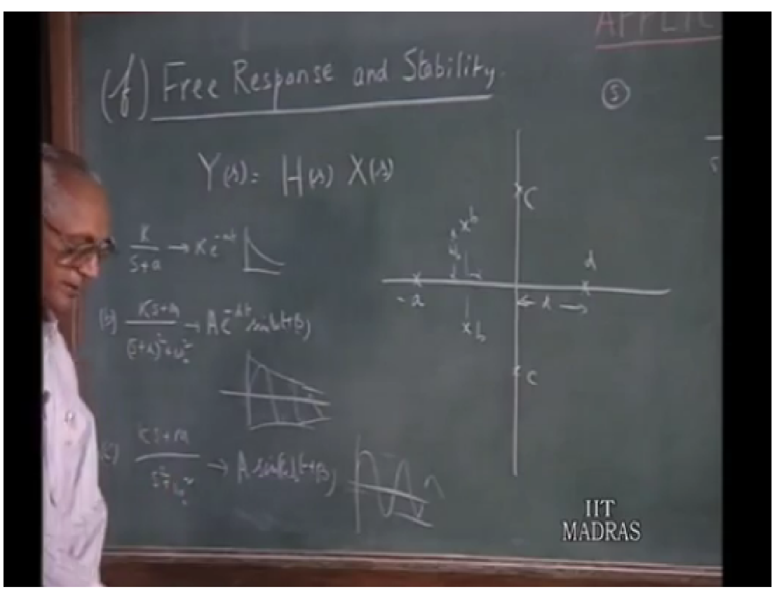

Now let us start, with consider once again the free response and the system as given by the poles of h of s we said in the last lecture. For that as for the output is concerned it consist of 2 parts 1, which corresponds to the poles of x of s and 1 which consist of poles of h of s the poles of x of s give raise to the forced response part of the output.

At the poles of h of s will give the parts corresponding to the free response of the system on the natural response of the system and the poles of h of s and indeed of 1s of f of s which is the denominator of the polynomial in h of s. Now, depending on the location of the poles of h of s we have different types of behavior.

So, in the complex plane, if this is the complex plane, s plane suppose, I have a pole here its location a than that corresponds to in the partial fraction expansion k over s plus this is minus. Let us say, a k over s plus a and this will give rise to a response k e to the power of minus at.

So, that is something which is case with time on the other hand. If I have the pair of complex conjugate poles here say at locations b say this is a location b this will be ks plus jm divided by s plus alpha whole square plus omega 0 square that will be the type of expansion partial fraction expansion that; to get plus 1 into the locations in the b positions this is minus alpha and this is omega not.

And this gives to response, which is sum say a e to the power of minus alpha t sin omega t s beta which means; that we are having a declined oscillations with the amplitude declined exponentially. On the other hand, if you are having 2 pair of poles and the imaginary axis locations c that corresponds to something like: ks plus m once again over s square plus omega 0 square and this gives rise to a response, which is sin omega 0 t plus beta which is sustained oscillations.

Therefore, the poles of h of s take the natural response of the system on the other hand, if the pole in the right of plane suppose, the pole here that corresponds to suppose, this is d s minus d then this corresponds to k e to the power of dt and that is represent an increasing exponential which d indicates in stability.

So 1 can say that, if the system is stable. If you have a bounded input given to x of s which is finite then in natural response must decay with time are at least styles with time. We should not take off cannot go indefinitely to large quantities therefore, first stable system, we require all the poles of h of s to be in the rest of plane and in the as a border line they can be on the imaginary axis because that is the limiting behavior.

However, what happens if the poles are repeated. Repeated poles really need that in the denominator polynomial h of s are factors like s plus alpha whole square are s square plus omega 0 square plus whole square are s plus alpha square plus omega 0 square whole square, if the repetition is other 2. So let us consider, what happens.

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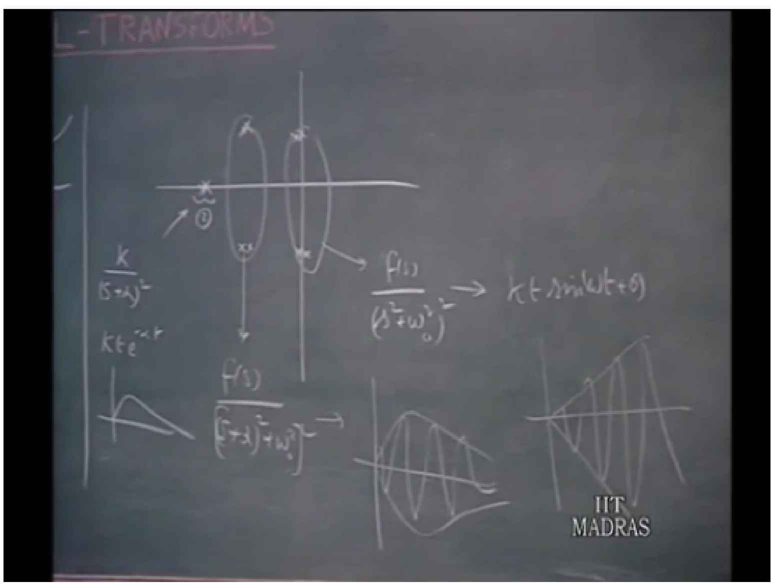

If you in the complex plane you have a pole of order 2 here that means multiple this 2 repeated pole it can be repeated 2 times 3 times on. But let us, illustrate it for the case of 2 that means; if the partial fraction expansion here k over s plus alpha whole square and the time domain response corresponding to this will be of the form kt e to the power of minus alpha t.

So, that means; you have it exponential decays but, starts with 0 amplitude on the other hand, if you are having 2 complex conjugate poles repeated like this that will be of the form some f of s over s plus alpha whole square plus omega 0 square whole square that means; this 2 complex conjugate poles are repeated twice at this gives as to response which will be once again this envelope like this.

So, even though the poles are repeated in the there in the left of plane it gives as a stable response will response ultimately decays, what happens, if the repeated poles in imaginary axis suppose they are the 2. So this corresponds to a term like f of s over square plus omega 0 square whole square.

And this gives as a time response which is some kt sin omega t plus theta that means the time response corresponding to that would be come to like this, So, this certainly should be excluded, if you want a stable system. If you have a reasonably well behaved bounded type of input the output cannot go indefinitely large for a stable system.

So, that means; we cannot have repeated poles on the imaginary axis certainly we cannot have poles in the right of plane whether, it is repeated simple and in the imaginary axis cannot have repeated poles even reputation of by rather duplication of poles that is multiple roles rather 2 themselves or not permitted certainly higher order poles cannot be permitted.

But as for the left half plane is concerned, what we illustrated for other 2 can also be extended for higher order you can have a pole on the negative real axis of order 4, 5, 6 whatever, it might be and similarly, complex conjugate, poles in the interior left of plane can be of any other what so ever, ultimately the response will decay on the imaginary axis.

However, you can have only simple poles that means to conclude this discussion. If, you want to have the free response to decay with time, then we require that the poles of h of s should be in the left of plane the interior left of plane, if they are the imaginary axis they must be simple. They must be unit multiplicity that cannot be repeated and certainly we cannot permit poles of any other right of plane.

So, that is the meaning of the stability the system is stable bounded input must 0 the bounded response and. So, the response cannot go for cannot take off to infinite limits it and go indefinitely with reference to time and the requirement therefore, is that; the poles should be left of plane inform the imaginary axis they should be simple. Let us, know look in some detail the forced response characteristics of system.

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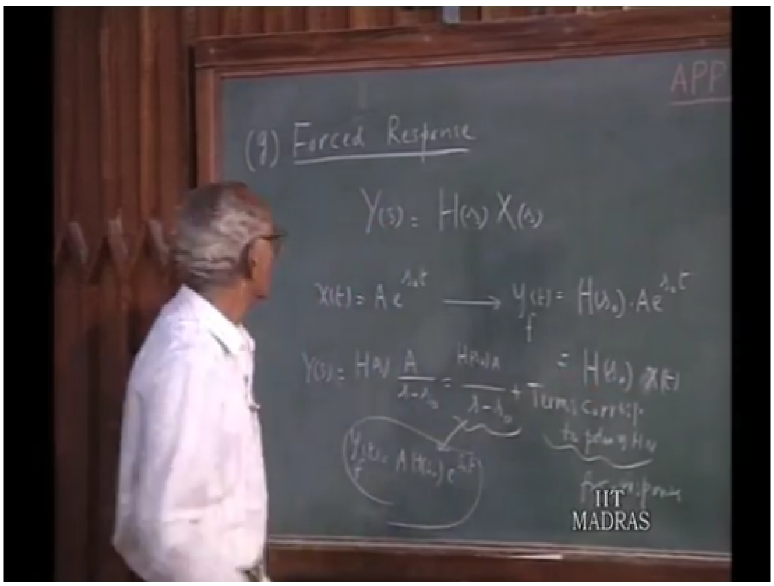

We know that y of s equals h of times x of s. Now, we will take the particular case of the x of s which is of the form xt is a e to the power of say s 0 t this an exponential signal then it gives as a forced response y of t which you have already seen in our pay entry lectures y of t will be h of s 0 times a e to the power of s 0 t which means h of s 0 times x of t.

So whenever, x of t is of an exponential type, exponential signal with the complex frequency x 0 the forced response is obtained nearly multiplied by x of t by h of s evaluated at that complex frequency h s substitute s 0 for s and that is what you get this is something, which we already discussed.

Let us see, how it is arranges we know that if xt is a to the power of x 0 t y of s will be h of s times the Laplace transform of x of t is a divided by s minus s not. So, in the partial fraction expansion of this you have 1 term corresponding to s minus s 0 which is h of s 0 times a plus terms corresponding to poles of h of s.

So, there are other terms correspond to poles of s here you take raise to the free response. The force response is given by this gives you the force response and the force. So, from this y of t the time domain representation of this is a times h of s 0 times e to the power of s 0 t.

So, that is what here having that is what exactly heard is the force response. So, this is how it ties up in the Laplace transform theory, what we already know from your preliminary lectures whenever, an exponential signal is there the output the force response is the input times h of s not.

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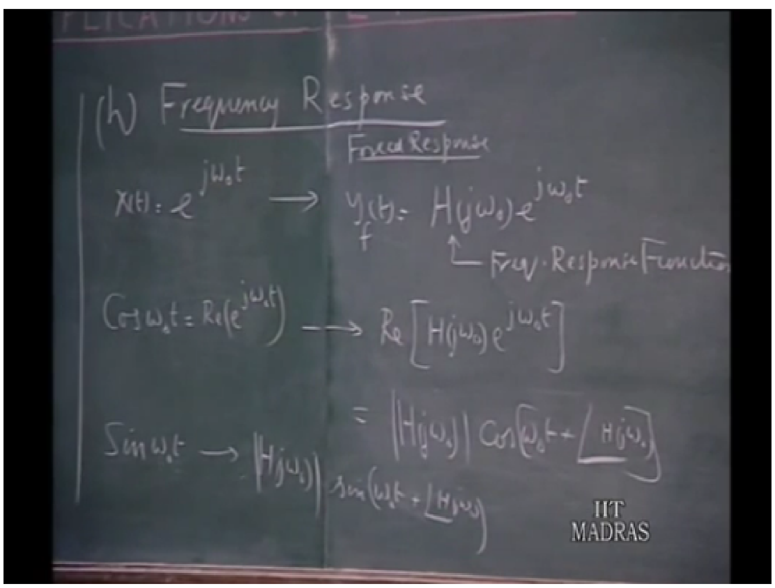

Now arising out of this we can talk about the frequency response characteristic. Suppose, you have e to the power of j omega 0 t as the input quantity then we know the forced response of this the force response of the system yf of t this is x of t the force response is h of j omega 0 e to the power of j omega not.

So, this is the frequency response function but, if you are often interested you are in exponential signals of the circuit e to the power of j omega 0 t but, something like cos omega 0 t let us say. So, cos omega 0 t we can take this as real part of the e to the power of j omega 0 t and therefore, the response to that would be the real part of h of j omega 0 times e to the power of j omega 0 t.

So, this can be written as if h of j omega 0 as the magnitude h of j omega 0 so, h of j omega 0 magnitude times some face. So, the magnitude of this is h of j omega 0 cos omega 0 t plus an angle which is the angle of h of j omega 0 t so that will be what you are having.

So, depending up on if cos omega 0 t is the input quantity the output quantity is given by the magnitude is magnified with the magnitude frequency response function the face is increased by the increase with the angle of the frequency response function cos omega 0 t angle h of omega 0 t.

Similarly, if sin omega 0 t the forced response will be, h of j omega 0 sin omega 0 t thus the angle of h of j omega 0 t. So, this is the frequency response function we have talked about this. When we discuss the Fourier transform and Fourier methods and where h of s substitution j omega will give the frequency response function. So, we can see how this ties up with our Laplace transformation theory. To summarize them, what you are talked about is.

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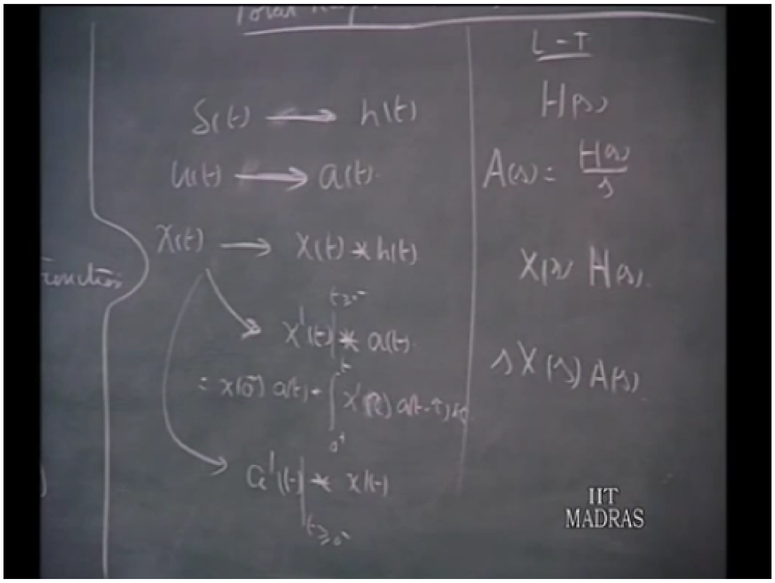

The total response with 0 initial conditions. If the input is delta t it gives raise to h of t and the Laplace transform of that is the h of s, the Laplace transform of the output is h of s, if the input is ut the I am sorry this not arrow this is input this is output we call that, the initial response of step response at and we indicated this as a s that will be h of s over s.

So, we have a general xt then in terms of the impulse response we say xt convolved with ht, which in Laplace transform domain is xs times hs we can also have alternately x dash t convolve with at the convolved with at that is the step response the meaning of this is x 0 plus at plus 0 plus to t of x dash tow at minus tow d tow all this we discussed, in the last class last lecture.

So, in terms of Laplace transforms we can write x of s s times a of s, we also have a alternative representation of this, if you recall you can write this as a dash t convolve with x of t that also have the same Laplace transform and in these 2 expressions a dash t and x dash t, we are taking about the situation take t starting from 0 minus starting from t greater than are 0 minus.

So, that is what we total response of a system can be obtained from the either the step response or the impulse response are in general terms this is how this equations will run.

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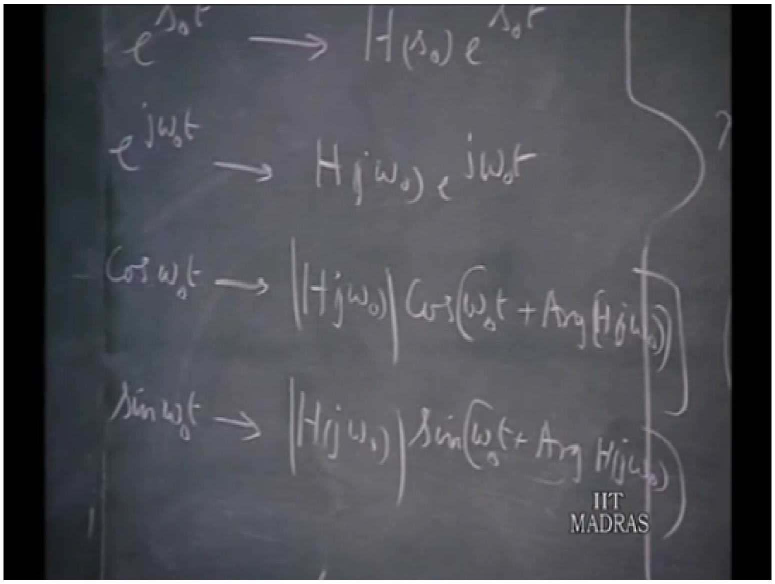

Now, let us talked about set up a similar table for the forced response. So, if the input is the exponential signal e to the power of s 0 t the forced response is h s 0 times e to the power of s 0 t, if it is e to the power of j omega 0 t it is h j omega 0 e to the power of j omega 0 t and if, it is cos omega 0 t it is h j omega 0 magnitude cos omega 0 t plus the argument of h of j omega 0.

So, h of j omega 0 is a complex number and whatever, angle that h s 0 that is the addition to the face of the cosine function, sin omega 0 t this will be like wise h j omega 0 magnitude sin omega 0 t plus the argument of h of j omega 0.

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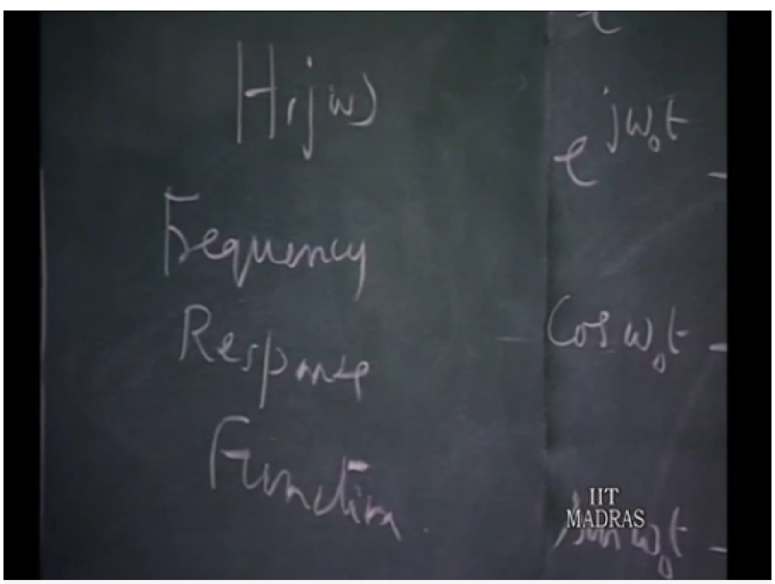

And h of j omega is called the frequency response function, which we had used in steady state sinusoidal analysis of good analysis of h of j omega is something like, your impedance zy and all let h of j omega z is the function of omega h of j omega is the function of omega. So impedance is the ratio voltage to current.

So, if you take the voltage the response quantity and the current as input quantity the system function corresponding to that z in the normally, what we talk about is z is the system function in this context. Let us now, work out in example to illustrate these ideas.