Networks and Systems
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Lecture-61
Laplace Transform for Resistor and System Analysis

We have considered the terminal relations of an inductant capacitor in terms of the Laplace transform variables, what about resister this is of course quit simple.

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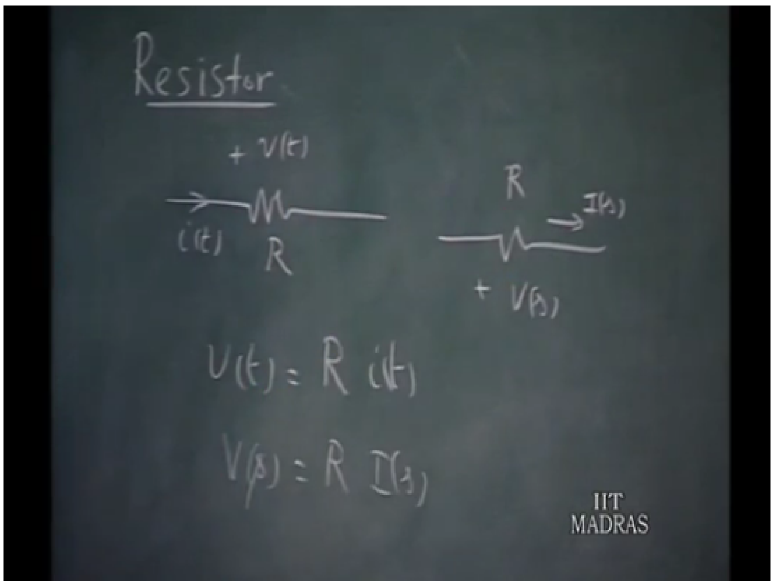

If I have a resister R, carrying a current i t and the voltage across this is V t. Then, straightaway, we know that, V of t equals R times i of t at all points of time. Therefore, in terms of Laplace transforms V of s equals R time I of s. So, in the transform diagram as well you have an R, this is V of s and the current is I of s.

So, there is nod difference between the transform diagram and the normal time domain representation of a resister. The ratio of the time domain voltage to the current is R. The ratio of the Laplace transform voltage the current is also to R.

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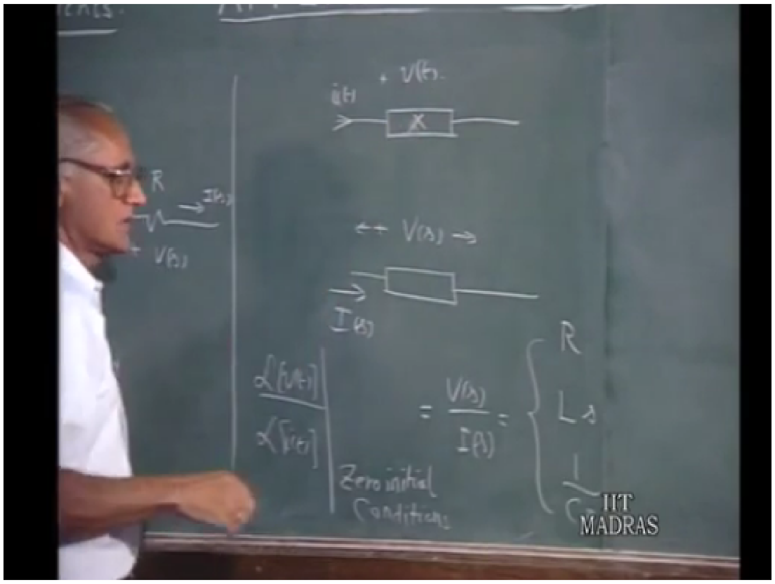

So, in general, if you have a general element like this and this is I of t and this is V of t. In the transform domain, we have the Laplace transform current indicated as the current variable. And the Laplace transforms voltage as the voltage variable. And the ratio of the Laplace transform of the voltage to the Laplace transform of the current, let us say with 0 initial conditions.

That means, we are assuming in the case of the capacitor and current, capacitor and inductor. There is no initial charge on the capacitor. There is no initial current in the inductor, 0 initial conditions. So, Laplace transform of the voltage to Laplace transform of the current 0 initial conditions, if this are V s over I of s respectively. This ratio will be equal to R, for the case of the resister.

If this box contain the resister R; that will be V s over I s is R. If this box represents the pure inductor, this will be L s and the box represents the capacitor it will be 1 over c s.

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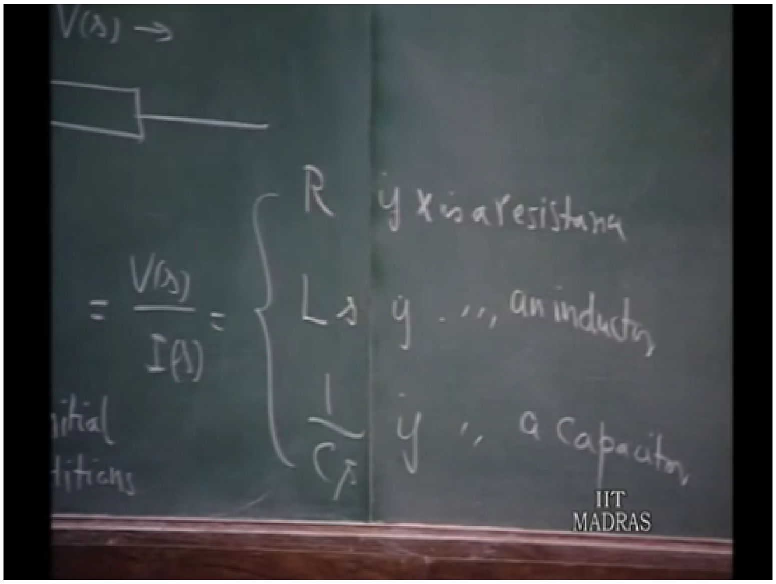

So, if this box, if this is x, R, if x is the resistance, if x is a inductor and 1 over c s, if x is a capacitor. So, we can think of this as a generalized impedance of this element, either R L s 1 over c s. And this of course, can be extended a combination of element. Suppose, the inductance assistance or in series, we can say the generalized impedance R L s and so on and so forth.

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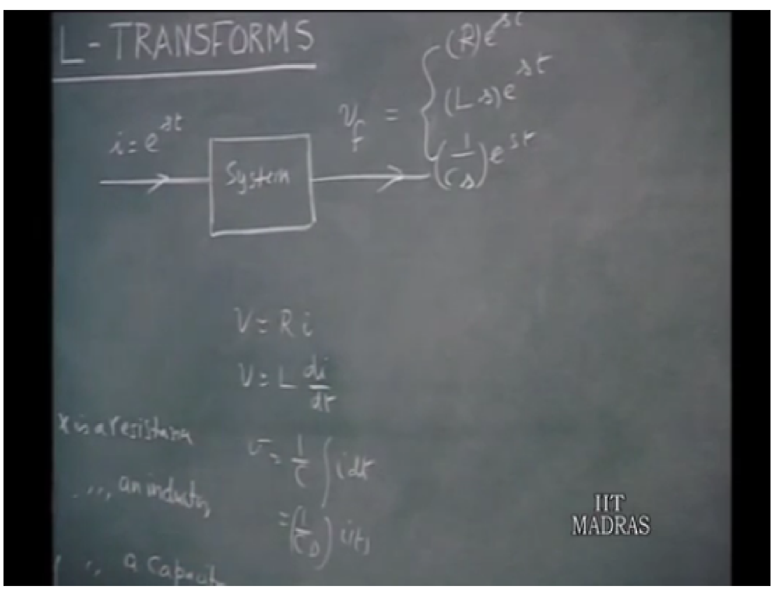

We can also think of this, from the system point of view, suppose I have represents this element of a system and we have a current, which is given by e to the power of s t. And you want to know the output the force response of the system, for this input condition.

This will be turn out to be, if the system happens to be a resistance current passing through a resistance.

If this i, e to the power of s t is the excitation to the system and the force response is equal to after all V equals R i is the equation for the capacitor. For the resistance, V equals L d i d t is the equation for an inductor, V equals 1 over c integral i d t the capacitor. Therefore, using this three fundamental relations, if the forcing function is the e to the power of s t at the voltage will R times e to the power of s t.

That is the force response of the resistance. If the system happens the pure inductor, the relation between the voltage and the current is L times d i by d t. Therefore, this is L s times e to the power of s t. And if the system is the pure capacitor the relation between the force response voltage to the current, suppose treat this as a kind of differential equation and I is the input e to the power of s t. This is 1 over c d operating on i of t.

Therefore, if i of t, is the exponential input the force response for the voltage is 1 over c s times e to the power of s t. So, these generalized impedances are R, L s and 1 over c s, can also be thought of as the ratio of voltage to the excitation. When, when the current excitation is form e to the power of s t.

So, this is an alternate way of looking at this. So, now that we have got this, we would like to set up the transform diagram of a whole circuit or a whole network, in terms of the generalized impedance. To aid as in this process, let us recapitulate, what you did in the case of Ac circuits under sinusoidal excitation function.

We have the general circuit that given to you. So, instead of dealing with i of t and v of t sinusoidal voltage and currents. We represent in the circuit diagram, the impedances of the various elements j omega L, 1 over j omega c or as the case may be. And instead of the raw currents and voltages time domain, we represent them by means of phases.

So, the whole entire circuit diagram can be written in terms of the variables being the phases of the respective quantities voltages and currents. And the element values, instead being R, L and c; we write in terms of R j omega L, 1 over j omega c the steady state impedance values.

We do much the same thing in the transform diagram. What we do is, we write up represents the circuit in the same form manner as we have in the time domain. Except that, we put down the generalized impedances of the various elements. And certainly, the variable that we are taking about are instead of the time domain expressions will talk in terms of the Laplaced transformed variables.

The Laplace transform the various current, the Laplace transforms of the various voltages. A third point, which is new here is, wherever the inductance carrying initial currents and whenever the capacitors carry initial charges or initial voltages equivalently. Then, we represent those initial conditions by equivalent sources in the transform diagram.

So, we can have a voltage source; that represent in the current inductor, initial current in inductor or a current source. Similarly, we can voltage source representing the capacitor voltage or a current source equivalently. So, the only difference is, that we have initial currents represented by additional sources, excitation sources.

Otherwise, the diagram approach is very similar to what we do in the case of Ac circuit analysis. We deal in terms of transformed variables in some phases is you deal with generalized impedances, instead of sinusoidal steady state impedances. That is, wherever j omega occurs, we use s instead of j omega. That is the only difference.

So, this will be illustrated by means of an example, which will now take up, we illustrate of the technique of the Laplace transform to the transient analysis are problem in the transform diagram.

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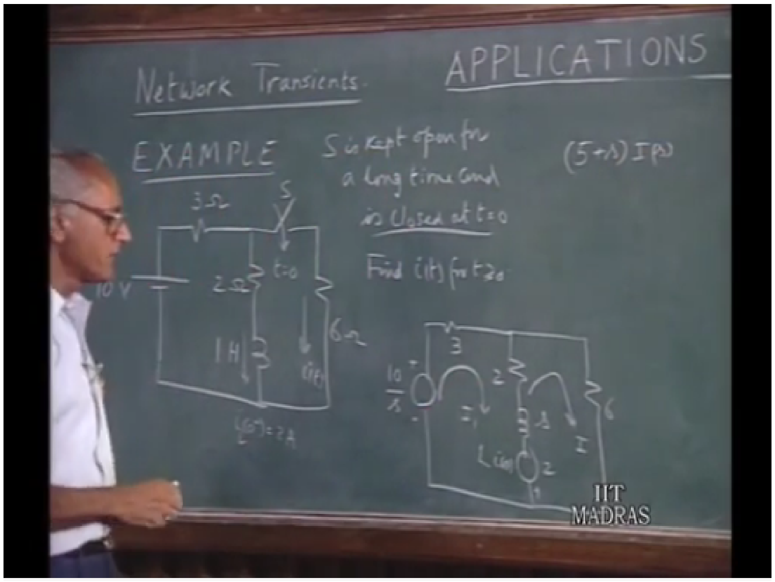

Let us take an example, we have 10 volts source, operating in a circuit comprising a resistance, 3 Ohms, 2 Ohms, 1 Hendry is the switch. This is 6 Ohms and the switch is closed, t equals 0. After having it been kept, open for a long time, S is kept open for a long time and is closed at t equals 0. So, we are asked to find out, i of t, find i of t for t greater than are equal to 0.

So, we what you need to do is, set up the transform diagram for this. So, you have for the 10 volts in straight away take the Laplace transform of this. So, that becomes 10 over s. Because, after all this is source, the transform of a constant 10 volts will be 10 upon s. The resistance 3 Ohms is represented by 3 Ohms again is 3 Ohms.

You do not have to write Ohms, it generalize impedance of the resistance is 3. Now, this is 2, resistance here is 2. As for the inductor is concern it generalize impedance is s, because is 1 Hendry inductance therefore L s is s. But, you must also indicate have here, L i 0 a voltage represent the initial current in the inductor.

And what is the initial current in the inductor, when the switch is kept open; 10 volts raise the current as 3 plus 2, 5 Ohms. Therefore, the current here I of 0 minus is 10 divided by 3 plus 2, 5, 2 Amperes. Therefore, L i 0 is 2, the strength of the voltage signal is L times I

0. I 0, which is 2, where circuit 0 minus values, in this case 0 minus 0 plus or 1 in the same.

But, we are substitute in 0 minus values, because come naturally it was and then you are having 6. So, we solve this equation in the terms of the loop currents. After all, once we have the transform diagram the equations are return down in exactly the same manner as you do for dc circuit. So, you write down the loop equations for the two loops. In terms of the variables I 1 of s and I of s, I of s is what we are interested.

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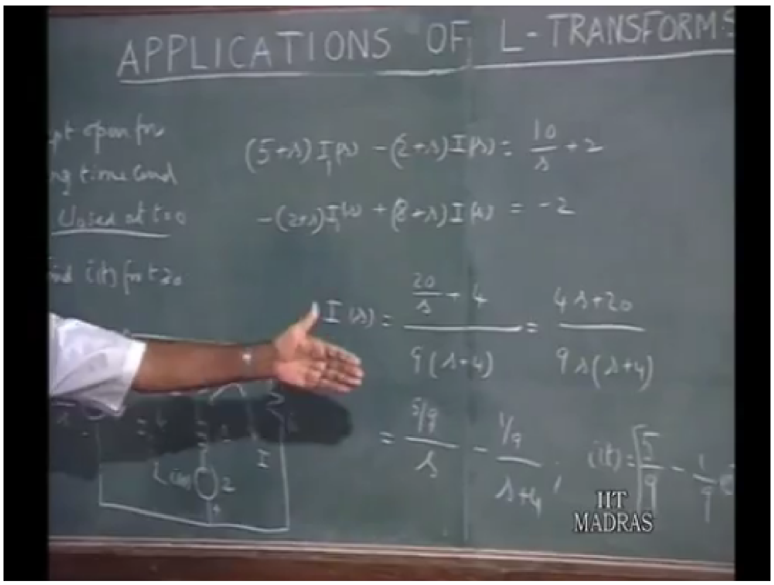

So, the first loop, you have got total impedance is 3 plus 2 plus s, 5 plus s times I of s. The mutual impedance between loops 1 and 2 plus s and the 2 currents are oppositely directed in this. So, minus 2 plus s times, this is I 1 of s, this is I of s. Equals the total driving voltage in the first loop is in the direction of I 1 is 10 plus s plus 2, 10 over s plus 2.

And as for the second equation is concerned the total self loop impedance is 6 plus 2 plus s 8 plus s times I of s. And the mutually impedance is loops 1 and 2 is the s plus 2 as before minus 2 plus s times I 1 of s. The total emf in the second loop in the direction of I is minus 2, because this voltage drives to current in the opposite direction, therefore this minus 2.

So, you have two algebraic equations involve in I 1 and I, you can solve for I of s, through elimination of I 1 to determinate. So, whatever method you having and you arrive at the expression for I of s as 20 upon s, s 4 divided by 9 times s plus 4. And this can be written further as 4 s plus 20 divided by 9 s times s plus 4.

So, at his stage, you make the partial fraction expansion for I of s in order to find i t. So, you carry out the partial fraction expansion, it becomes 4 up on 9 divided by s, 5 upon 9, the other 5 up on 9 over s plus minus 1 by 9 s plus 4. So, in other words, I of t would be once we have I of s, i of t would be 5 by 9 minus 1 by 9, e to the power of minus 4 t.

That is the expression for the current i t. So, let us see, what we have done. We started with this circuit diagram; replace this by a transform diagram. And from the transform diagram, you set up this algebraic equations so the pertain to the transform diagram. And solve for the current I of s required variable and find out the inverse Laplace transform.

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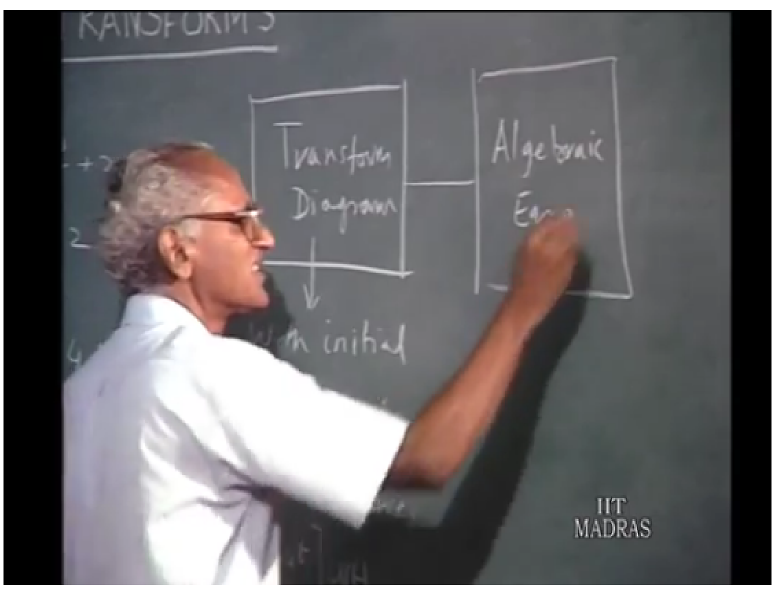

So, the steps that involved would be to set up the transform diagram in the transform diagram. You also with initial current sources with initial condition sources, with initial condition replaced by sources. Appropriately, the voltage sources or current sources from the transform diagram it set up algebraic equations.

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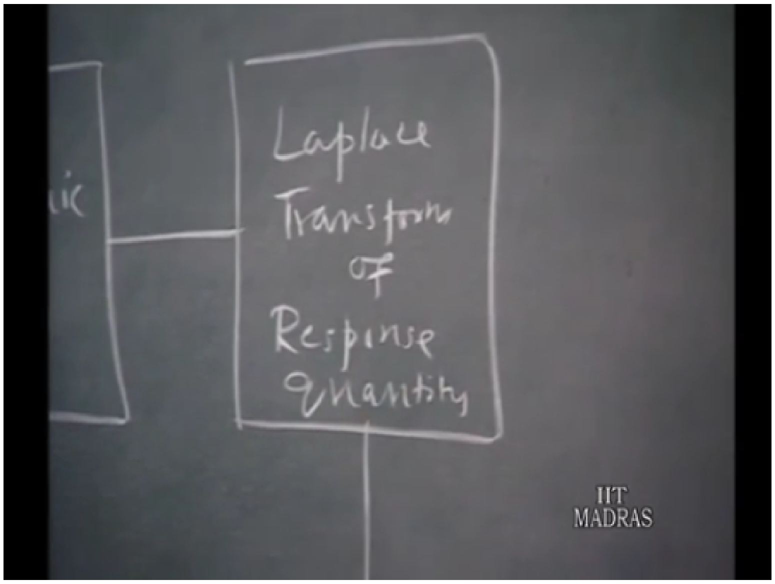

Then, get the Laplace transform of the response quantity, whichever you are after.

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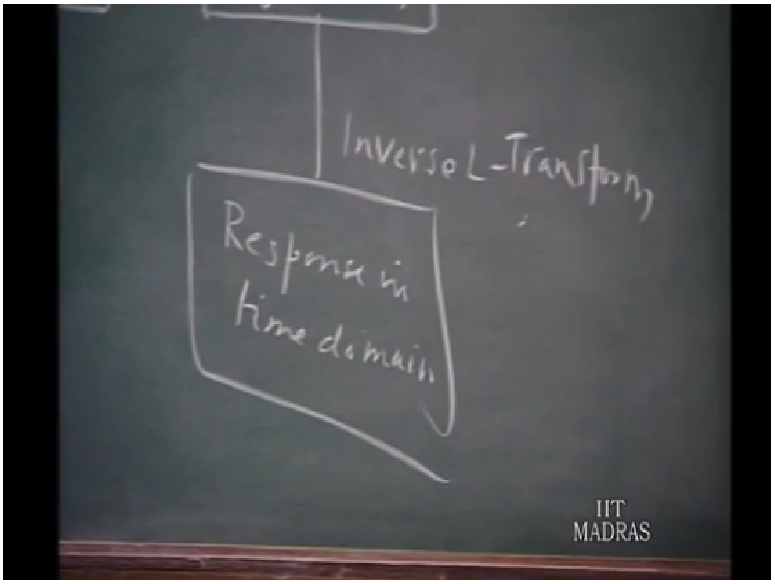

And then, find the inverse Laplace transform response in time domain. For this, you do the inverse Laplace transform. That there is the way, which we handle the network transient problems, during the transform diagram. So, we avoid writing down the differential equations pertaining to this.

By once, we have the transform diagram the solution runs on very similar parallel line to Dc circuit analysis. The equations are written in the same fashion, expect of this Dc quantity, we write in terms of the. Functions of s, what currents and voltages are Laplace transform variables and instead of pure resistances, we write in the terms of functions of generalize impedances.

This also very, very similar to, what we do steady state circuit analysis. Again, instead of phases we use Laplace transform here. And instead of impedances, j omega, you write generalized impedance in times of s. As a matter of fact the writing down the equations is simpler than in the case of Ac circuit analysis, because, we avoid complex number. Even though, this is complex expressions we have only using s.

S is the complex variable, but the coefficients all the f and the higher powers are real coefficients. Therefore, we do not see the complex numbers explicitly on the, you writing down these equations. So, this the j terms, which as turn all over the place in the case of Ac circuit analysis are avoid or repeat slot seeing here.

And therefore, it proceeds in a very smooth fashion. The major difficulty of course is, the finding out the inverse Laplace transform of this. If I of s have a complicated rational function, then the partial fraction expansion and finding out the inverse Laplace transform is that much more complicated.

But, however the procedure is straight forward. So, today in this lecture, we took for the consideration the application of Laplace transform in network transient's problems. We saw the two different approaches; one is to start from the differential equation. This is the very basic approach and general approach and we should always keep this as the back over mind, because fundamentals are always are very important.

But, then we would like to simplify the initial labor, that is involve, because we have may to do this type of problem again and again, not it simplify the work. Instead of writing down the Laplace transformed oceans of the differential equations, which turn out to be algebraic equations. We try to find out short cut by the writing down the algebraic equations. Then, we develop the theme of the transform diagram, which is alternative representation the circuit diagram.

And from the transform diagram, we straight away write the algebraic equations performance of the network. And from that, we get an expression in Laplace transform the response quantities and you find out the inverse Laplace transform to find out the response quantity in time domain. That constitute the solution of the transient problem require; more about this in next lecture.