Networks and Systems
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Lecture-59D
Exercises

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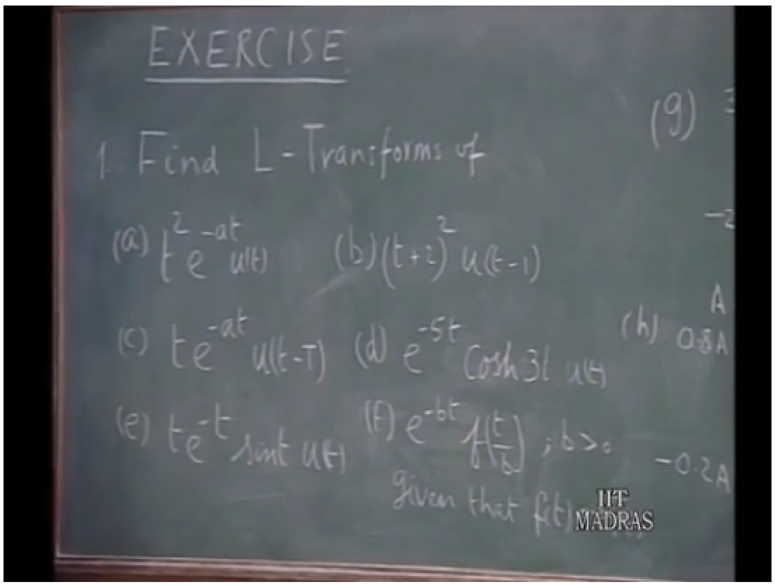

The first question is, find the Laplace transforms of the set up time functions that are given here; a, t square e to the power of minus a t u t. b, t plus 2 whole square times u of t minus 1. So, this time function really starts t equals 1 onwards. The third function; time multiplied by e to the power of minus at t e to the power of minus a t multiplied by u of t minus t; a delayed step function.

So, once again this functions starts from t equals T onwards, its truncated from values of t less than capital t. d, e to the power of minus 5 t times hyperbolic cosine cosh 3 t u t. We are not talked about Laplace transforms hyperbolic functions, but you can always express them, as a sum of exponential functions, and then combine this e to the power of minus 5 t and you can find the Laplace transform of that. e, t e to the power of minus t sin t u t.

Here again you have the combination of the factors. We have worked out examples where e to the power of minus t sin t, we found Laplace transform, but that multiplied by t. So, whenever you multiplied by t equivalent to taking the derivative in the transform domain. So, you have to use those properties, to find the Laplace transform of this. f, given that f of t has the Laplace transform of f of s.

What would be the Laplace transform of another function derived from f of t in this manner; e to the power of minus b t times f of t minus b. So, its scale by a factor b, in addition you multiplied by multiplied it by exponential term e to the power of minus b t u t. Take b to be a positive number, and then find this out. Find the Laplace transform of this, in terms of b and f of s.

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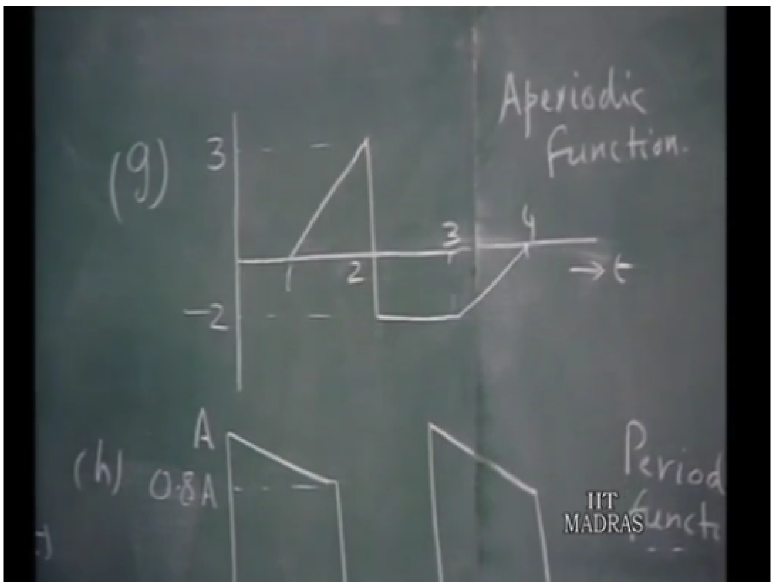

Then we are given the wave forms of two time functions; one is a periodic function. This is all the function there is, for outside this range from t equals 1 second to 4 second it is 0. So, within this interval t equals 1 to 4 it behaves its manner, it linearly raises from 0 to 3 units, and then claims it down claims down to minus 2 steeply, and then remains at minus 2 from 2 seconds to 3 seconds, and then once again comes back to 0 at 4 and stays there for p greater than 4. So, it is a periodic function, described in this manner.

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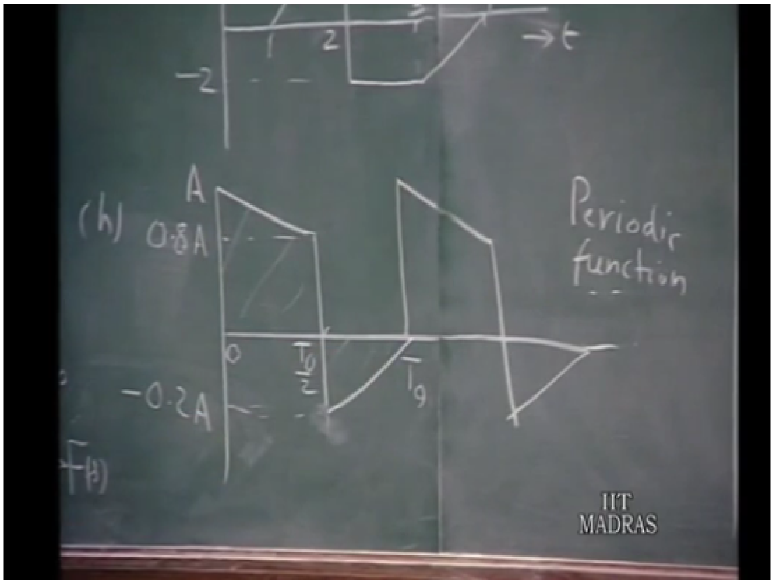

H is a periodic function; the variation for 1 period 0 to t not is described as here. It starts from a and comes down to point 8 a in half a period linearly, and then it jumps to minus point 2 a at this point, and again increases linearly and reaches 0 at equals t 0. So, this is the basic variation in one period, and using the, find the Laplace transform of this variation in one period, and then use the property of Laplace transforms of periodic functions to find out, the Laplace transform of the entire time function.

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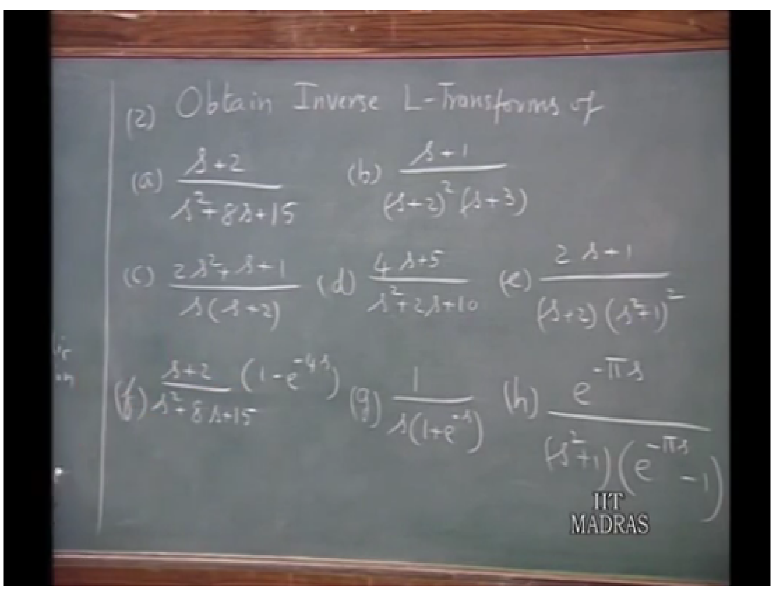

The second example listed here a set of Laplace transforms, find the time functions to use the correspond; that means, find the inverse Laplace transforms of A; s plus 2 over s square plus 8 s plus 15. B, s plus 1 over s plus 2 whole square times s plus 3; that means, the repeat at pole here. So, you have to use the rule for finding out the residues when poles are involved. C, 2 s square plus s plus 1 over s times s plus 2.

You observe here that the numerators have the same as the denominator; therefore, apart from the factor which correspond to k 1 over s plus k 2 over s plus 2. You also have a constant term 2 which is the ratio of the 2 leading coefficients. So, 2 plus sum k 1 over s plus k 2 over s plus 2 is the partial fraction expansion of this. So, corresponding for the 2 the inverse Laplace transform will be 2 delta t.

So, whenever the numerator degree fabers in the same as the denominator degree, you have constant term in partial fraction expansion and the inverse Laplace transform you have a delta function. D, your 4 s plus 5 over s square plus 2 s plus 10, these are lead to complex conjugate poles. So, instead of finding out the residues of each of the complex conjugate poles, we can do it; that is one way of doing this.

Other way would be to treat this as s plus a whole square plus omega square, and then try to find out the inverse Laplace transform, in terms of cosine and sin functions with an exponential decay amplitude.

Then you have 2 s plus 1 over s plus 2 times s square plus 1 whole square. Here you repeat poles and the imaginary axis slightly more complicated, but you can handle this in same way as you have done earlier. Then F, s plus 2 over s square plus 8 s plus fifteen times 1 minus e to the power of minus 4 s. In tackling this problem you already know the inverse Laplace transform of s plus 2 over s square plus 8 s plus 15.

So, all you have to do is use this information here, because e to the power of minus 4 s means whatever f of t you have obtained here, is the same f of t delayed by 4 seconds is what corresponds to this. Therefore, use this information that you already obtained here, to find out the f of t corresponding to this. 1 over s 1 plus e to the power of minus s.

So, if you are 1 minus e to the power of minus t s we recognize this to be, the Laplace transform of periodic function of time. So, all you have to do is multiplied both the numerator and denominator by 1 minus e to the power of minus s. So, you have 1 minus s 1 times 1 minus e to the power of minus s by s times 1 minus e to the power of minus 2 s.

Therefore, you can recognize this to be the Laplace transform of some periodic function with the Laplace transform for the basic period being 1 minus e to the power of minus s over s, you can find that out. Similarly here you have e to the power of minus pie s by s square plus 1 e to the power of minus pie s minus 1.

So, you multiply both sides numerator and denominator by minus 1. You have 1 minus e to the power of minus pie s. So, you can remove that, then find out the Laplace transform of the remaining function, and once you introduce it becomes the Laplace transform of periodic function of time. So, using all this properties that we derived, you find the inverse Laplace transforms of all this 8 functions of s.
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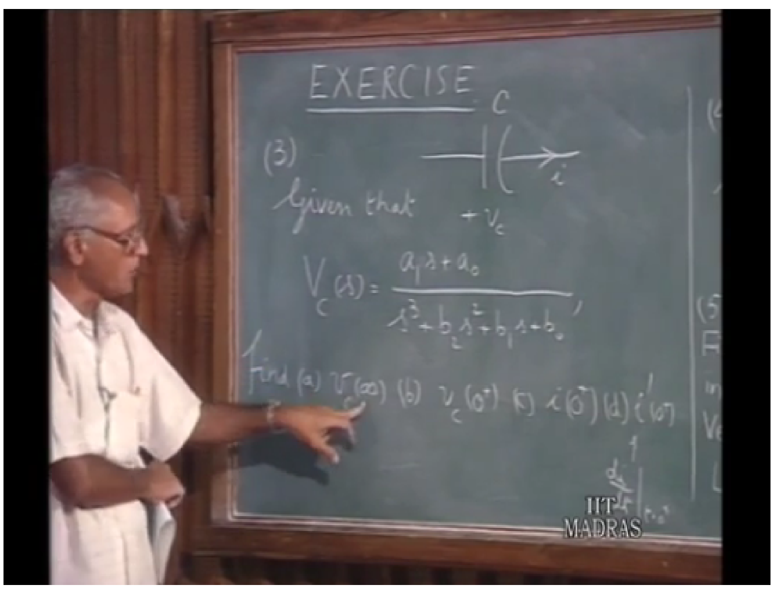
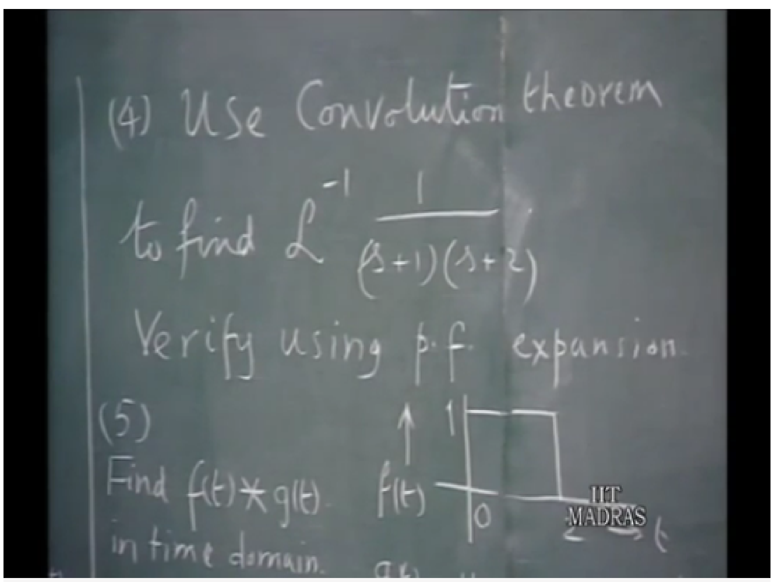


Question number 3: You have a capacitor circuit, current I is current passing the capacitor, and v c of t is the voltage across to the capacitor. Suppose the solution of the network we found out, the Laplace transform of the voltage across the capacitor v c of s.

Let it be in symbolic form a 1 s plus a 0 over s cube plus b 2 s square plus b 1 s plus b 0. That is the Laplace transform of the voltage of the capacitor.

Now, using this information, in terms a and b constant. Find the final value of the capacitor voltage v c infinity. b the initial value of the capacitor voltage, which is v c 0 plus immediately at after t equal 0. Also find out the initial value of the current i 0 plus. And lastly find the initial value of the derivative of the current i prime 0 plus. This means this is d i by d t evaluated t equals 0 plus.

So, in application of the initial value theorem, final value theorem, and the derivative rule; all this are needed for solution problem. In other words you do not have to explicitly find v c of t using this initial value theorem, final value theorem in the derivative rule. You should be able to find out all this quantity in terms of the a constants and b constants. (Refer Slide Time: 09:01)



Question number 4: Use convolution theorem to find the, inverse Laplace transform of 1 over s plus 1 times s plus 2. Verify using the partial fraction expansion. So, this Laplace transformation can be thought of, have the product of 1 over s plus 1 time 1 over s plus 2. You can find out the time functions corresponding to this two factors, and since it is the product that is involve the transform domain, in the time domain, is the convolution.

So, it is the convolution of the time function corresponding to 1 over s plus 1 and 1 over s plus 2. Find out the corresponding time function. Now, you can also find out the inverse Laplace transform, using partial fraction expansion verify, using partial fraction expansion. And compare the 2 answers, then both of them must naturally agree. So, this is an exercise in not only partial fraction expansion, but also the using the properties of convolution.

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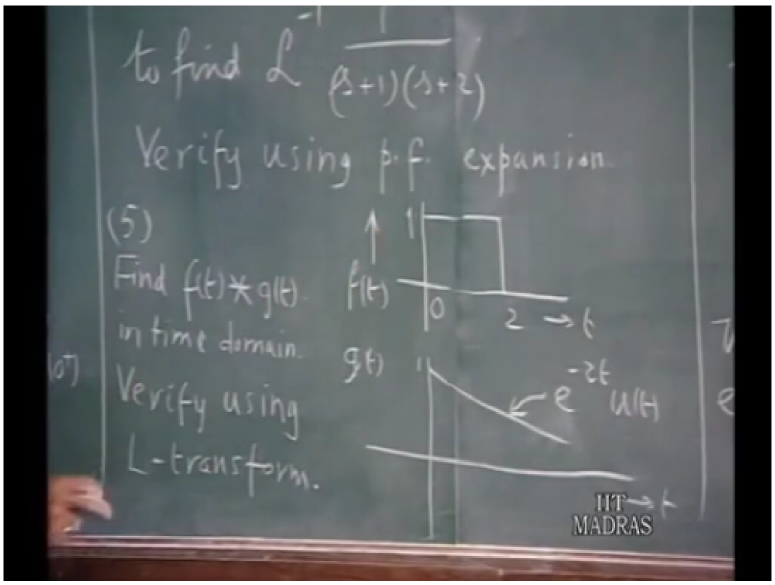

Question number 5: You are given 2 time functions f of t is a pulse which loss from 0 to 2 seconds, having an amplitude 1. And g of t is an exponentially decay time function e to the power of minus 3 t u t. Find ft star g t; that is the convolution of f t and g t. Both are causal time functions, both are emendable to Laplace transformation.

Find this in time domain, purely working out in time domain from the basic definition of the convolution integral, you find out the ft star g t in time domain. As an alternative verify using Laplace transform. In the second step is what you have to do find the Laplace transform of f of t, find the Laplace transform g of t.

Since we interest in the convolution of this two; that whatever is the result of this convolution must have for its Laplace transform f of s time g of s. So, you find the

Laplace transform of this, find the Laplace transform of this, both are simple to find out. Find the inverses Laplace transform, you get f t star g t.

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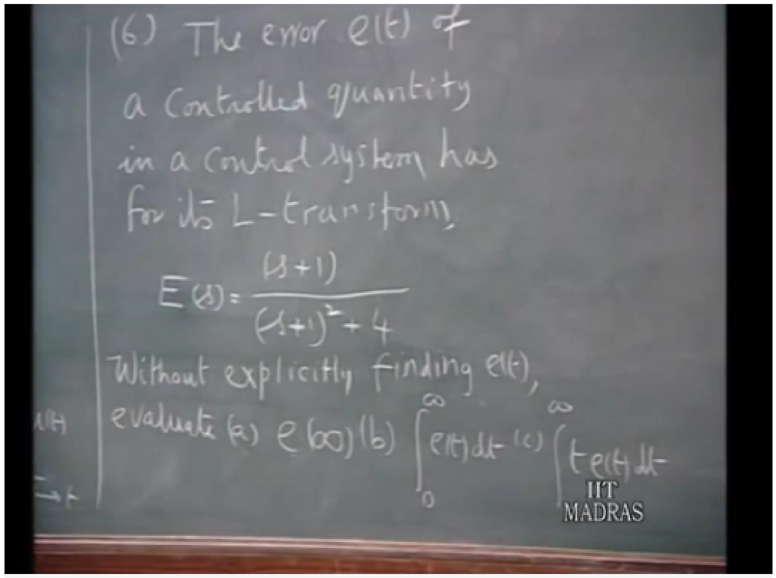

Lastly, you have the error e t of a controlled quantity in a controlled system, has for its Laplace transform. This expression e of s equals s plus 1 over s plus 1 whole square plus 4. The error et of a controlled quantity in a controlled system has for its Laplace transform this expression e of s equals s plus 1 over s plus 1 whole square plus 4.

Without explicitly finding e of t, you do not have to find inverse Laplace transformation of this to find e of t, without explicitly doing that evaluate e infinity; that the final value of this error. B, integral 0 to infinity e t d t. The integral of that error over the period 0 to infinity. C, 0 to infinity t e t d t; that is time multiplied t e t d t.

This you should be able to do it without actually finding out e of t, using initial value theorem, the rule for integration so on and so forth. You can also extent this to, I can ask you find out integral 0 to infinity of e square t d t; that is the integral of the square error. But that requires a convolution in the complex domain, since we have not discussed that we will not include that type of question.

So, this exercise covers the various properties of Laplace transformation that we discussed in the earlier 6 lectures. In the next lecture, we will discuss the application of Laplace transformation technique to evolution of transients in networks and systems.