Networks and Systems
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Lecture-59A
Inverse Laplace Transform and Contour Integration

Today at the method of finding out the inverse Laplace transformation, using the partial fraction expansion.

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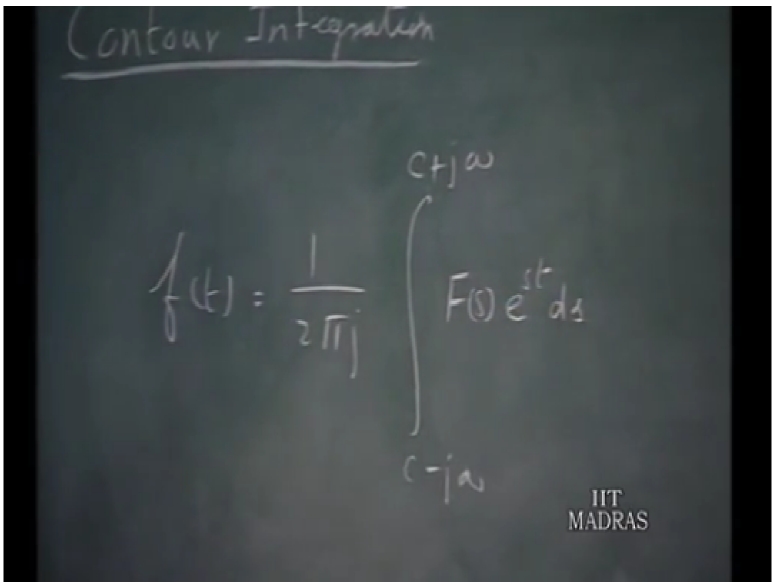

There is another method which is the contour integration, which is more fundamental, and which terms from the very definition of the inverse Laplace transformation. You recall the inverse Laplace transformation f t is 1 over 2 pie j c minus j infinity to c plus j infinity of f of s e to the power of s t d s.

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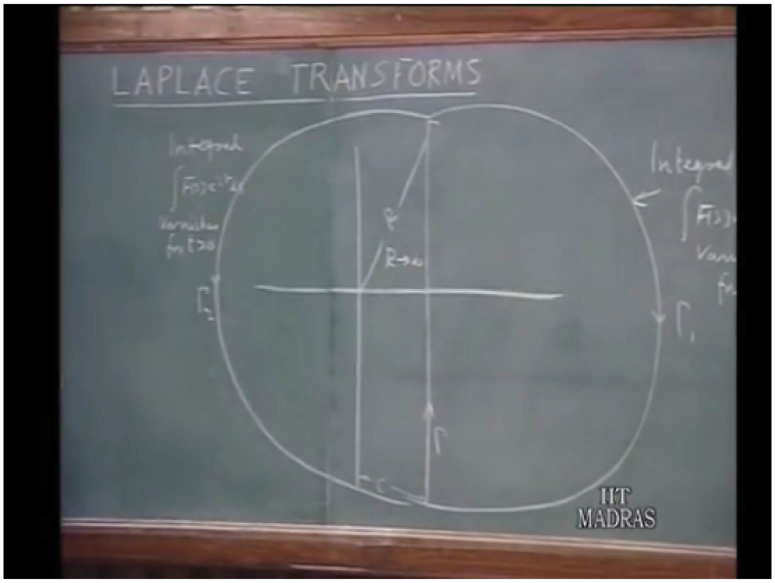

So, in the complex plane, you take the integration contour like this c minus j infinity, all them down to c plus j infinity. Let me say this contour is gamma. So, this integration is a long term, this. Now it is easy to evaluate the integral along a close contour in a complex plane, because from the theory of residues. Once you have integration over a close contour that is equal to 2 pie j time, the residues of the poles inside the contour of the complex variable theory.

So, what one does is, this to close this by means of, too large semicircles where r tends to infinity. Let me call this gamma 1, gamma 2. It turns out that if you have this integral evaluated around this contour for t less than 0 it vanishes. So, integral f of s e to the power of s t vanishes along this contour for t less than j. On the other hand if you take it along this contour gamma 2, this integral f of s e to the power of s t d s vanishes for t greater than 0.

So, since we have, we want to really evaluate this integral along this contour, from this point to this point. But nothing is lost if you have to this contour, this particular large semicircle for t less than 0, and this large semicircle for t greater than 0.

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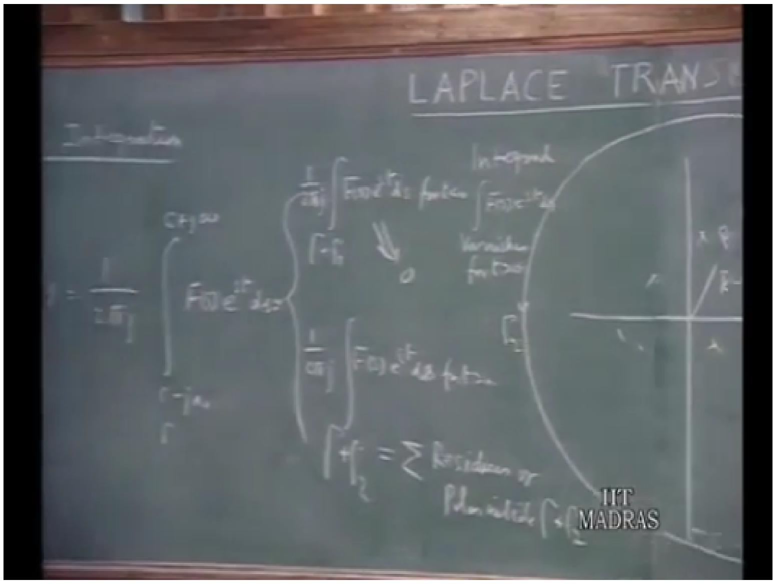

So, in other wards I can say, this is equal to 1 over 2 pie j evaluate the contour cross contour gamma plus gamma 1 of f of s e to the power of s t d s for t less than 0, because of all I made in to that, a quantity which is equal to 0, and this is equal to 1 over 2 pie j integral along this close contour for d s for t greater than 0, and it turns out that. As far as this contour integration is concerned, there are no poles inside as long, because all the poles will be here.

Therefore, since this close contour does not include any poles this transfer to be 0, and as for this is concerned. This will be equal to the sum of the residues of poles inside the close contour 2. So, whatever poles you are having here, you have take the residues of the poles that will be equal to f of t for t greater than 0; this is for t greater than 0, this is for t less than 0. So, in the inverse Laplace transform you will get f of t identically 0 of t less than 0, because of this, and for t greater than 0, it is equal to sum of the residues of the poles inside this.

Our discussion here is lesser to be cursory, because this is not the method that we used for normally evaluate of the inverse Laplace transformation. Just giving this to you the shape of completeness and the partial fraction expansion is. So, much simpler and that takes care of most of our needs.

We do not really have to the contour integration. But, because this is basic one, and this will have to be resorted to, for special types of functions which we may not know the inverse Laplace transform simply, so I included this. So, we will not pursue this topic, just mention the contour integration is alternative method to finding out the Laplace transformation.