Networks and Systems
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Lecture-59A
Partial Fractions: General Case

Let us now consider a general case of situation where multiple order poles occur.

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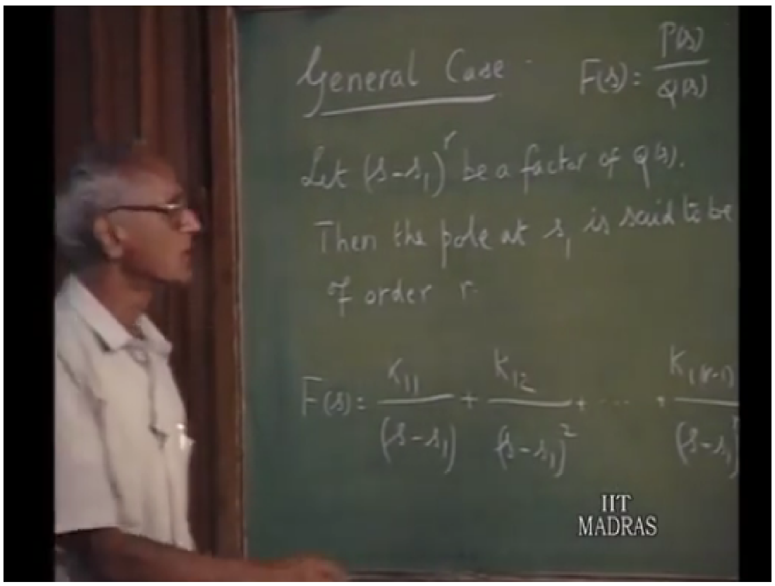

So, given f of s which is p s or q s. let s minus s 1 power of r be a factor of q of s. in this event we say that the pole at a sequence s 1, is of order r, because the particular factor is repeated r times. Then the partial fraction expansion f of s will have the form, corresponding to the pole is equals s 1. We can write this as k 1 one over s minus s 1 plus k 1 2 over s minus s 1 whole square plus.

So, the denominator will have expanding powers of s minus s 1, and finally, you have k 1 r minus 1 by s minus s 1 raise to the power of r minus 1 plus k 1 r over s minus s 1 to the power of r plus terms which involve other poles of f of s plus terms which involve other poles that is poles other than let us equal to s 1.

Let me call this group of terms r of s, because we really. Our focus now is on how to calculate k 1 and k 1 r. We are not interested in other terms. So, now our discussion will now sent around, methods of finding out k 1 one k 1 2 or up to k 1 r.

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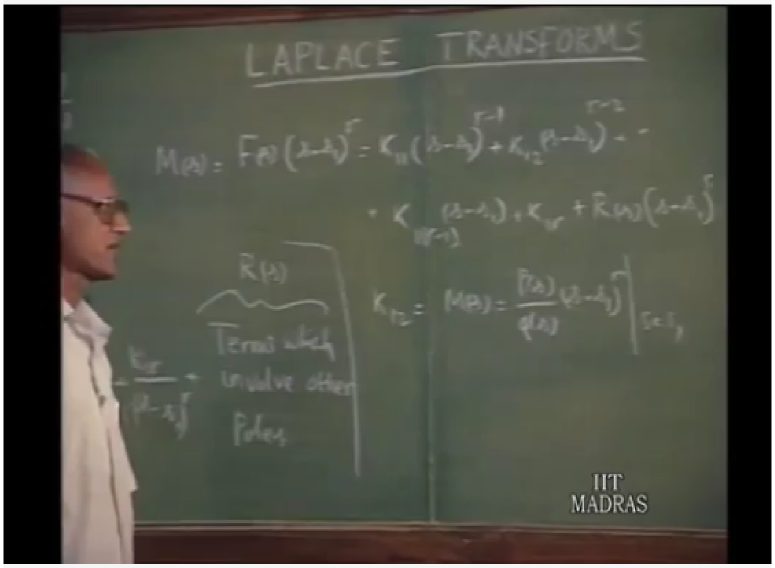

Now, taking the q from the example which we discussed just now. Let me say; m of s is obtained by multiplying f of s over s minus s 1 raised to the power of r. So, this is the partial fraction expansion f of s. We multiply this by s minus s 1 to the power of r. Then this entire group of terms will be multiplied by s minus s 1 raise to the power of r.

Therefore, that will turn out to be k 1 1 times s minus s 1 raised to the power of r minus 1 plus k 1 2 raised to the power of s minus s 1 raised to the power of r minus 2 down the line plus compare to this last 2 terms k 1 r minus 1 times s minus s 1 plus k 1 r plus the entire group of term r of s multiplied by s minus s 1 raised to the power y that is how this m of s look like.

So, every 1 of this term is multiplied by s minus s 1 raise to the power of r and this is what the result is. Now it is easy to see now that k 1 r is simply found out by multiply by substituting s equals s 1 in this expression, because every term manipulate expects this, when we substitute s equals s 1.

So, you substitute s equals s 1 in m of s k 1 r falls out. So, k 1 r equals m of s, which is really p s over q s multiplied by s minus s 1 raised to the power of r with the substitution s equal to 1; that is quite straight forward that is k 1 r. Now, to find out k 1 r minus 1, we adopt the same trick as use in this example. What you recall.

We have taken the derivative of m of s with respect to s. If you do that these terms drops out, and the derivative of this is simply k 1 r minus 1. And since these are all accompanied by higher powers of s minus s 1 when you take the derivative, they still continue s minus s 1 continuous to a factor here; therefore, they all drop out.

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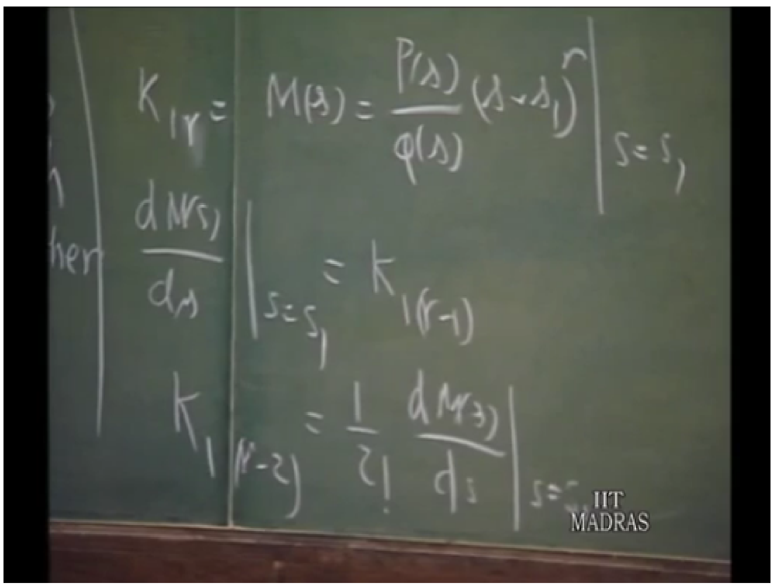

So, when you have got d m s over d s, if you do that, and substitute s is equal to s 1. Then the only term which means is k 1 r minus 1, because k 1 r disappears. All the other terms have a factor s minus s 1; therefore, we substitute s is equal to s 1 that remains; that is k 1 r minus 1. Now, suppose you take the second derivative, then the previous term will come.

The previous term here you recall, it will be k 1 r minus 2 s minus s 1 whole square. So, when you take this first derivative, you get 2 times s minus s 1 when you take the second derivative, it will be 2 times k 1 r minus 2. Therefore, you have k 1 r minus 2 will be 1 upon 2 d m s over d s substitute s equal to s 1.

That is a matter of fact instead of 2 I will write just 2 factorial, because that is how the progresses, at the previous term when to take k 1 r minus 3 you will get, by the final analysis 3 factorial that is how it goes.

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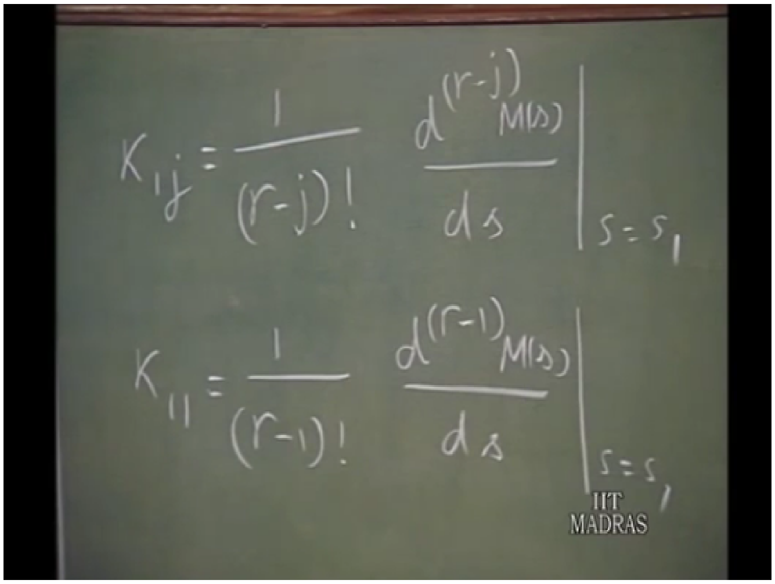

So, continue in this discussion it is easy to verify, we can show the k 1 j is obtained as 1 over r minus j factorial. The r minus j derivative of m of s over with respect to s. The substitution s equals s 1. And in fact you get k 1 1; that is the first term here k 1 one and this is called the residual of a multiple of the pole let us k 1 1, which is the hard to find out. It is obtained by taking r minus 1 derivative of the r minus 1 m of s over d s.

So, in principle it is same as what we have done in the first example we worked out. Only thing is, when you have got a large number of a multiple order pole, you have to take as substitute derivative of this term m of s f of s s minus s 1 raised to power of r; that is you lock of the term the denominator which is s minus s 1 raised to the power of r, and the rest of the rational function, you take successive derivatives and substitute s equals s 1.

So, you get all the terms the partial fraction expansion, starting from k 1 r write up to k 1 1 .k 1 1 is the numerator of the linear factor, k 1 r is the numerator of the factor s minus s 1 to the power of r. So, the principle is straight forward, you do not have to really remember this formulas, provided you know the principle that is involved. You can always figure out how is done.

You have to take the rational fraction as such, given rational fraction, multiply by s minus s 1 raised to the power of r and you get a resultant rational fraction, function which is called m of s to take successive derivative and substitute s is equal to s 1. So, the principle of finding out the partial fraction expansion, when you have got multiple order poles, is again a straight forward extension what you have done in the simple case pole.

And when you have a combination of simple poles and multiple order poles you have to use a combination of both the techniques. Simple poles is quit straight forward, multiple order poles, we have to take successive derivative of the factor m of s. A combination of this 2 has to be adopted.

In the case of complex conjugate poles, if you find the residue with respect to 1 complex conjugate pole, it is enough. This second its conjugate residue corresponding to its conjugate, is the conjugate of the residues itself, this is one way of handling this. Alternately as when the example that you worked out are suggested, you can combine the terms corresponding to 2 conjugation poles, and find out have a quadratic denominator, and find out a linear factor in the numerator.

And associate this with terms of the type e power minus alpha cos omega t and e power minus alpha t sin omega t that can also be done. Otherwise you can leave them as linear terms, and then find out the residues corresponding to complex conjugate poles, find out the inverse Laplace transform of this and combine this resulting time functions, which again yields finally the same result.