Networks and Systems
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Lecture-55
Complex Convolution and Periodic Functions

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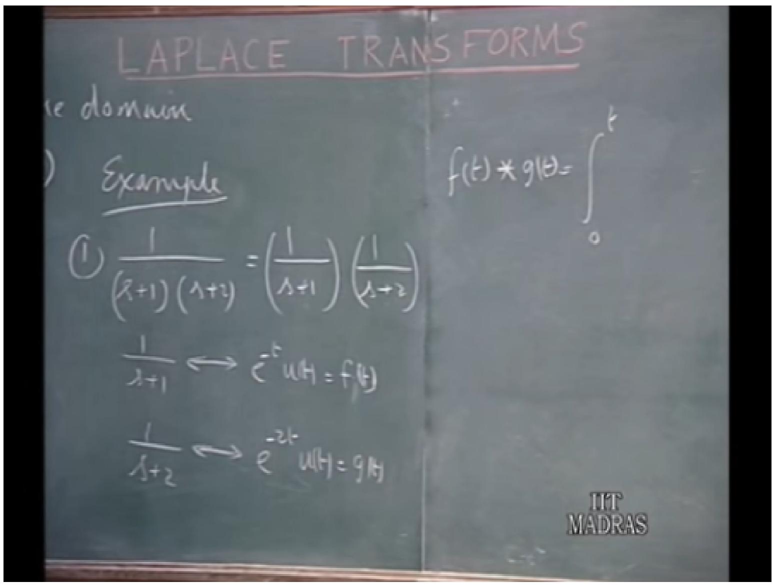

Let us consider an example suppose I have 1 over s plus 1 times s plus 2. Now, we can spilt this 1 over s plus 1 multiplied by 1 over s plus 2. So, this f of s is the product of 2 functions and suppose, I call as 1 over s plus 1 the inverse Laplace transform of that is; e to the power of minus t ut. Let me call this as f 1 ft are simply f of t the inverse Laplace transform of this 1 over s plus 2 is e to the power of minus 2 t ut. Let me call this g of t.

So, the product of this 2 Laplace transforms must corresponds to the convolution of these 2 time functions f of t and g of t because convolution product in time domain is equivalent to multiplication the transform domain. So, the inverse Laplace transform of that: is ft star gt ft star time gt ft star gt. So, that will be 0 to t there will be no impulse special may have start away 0 and t 0 minus.

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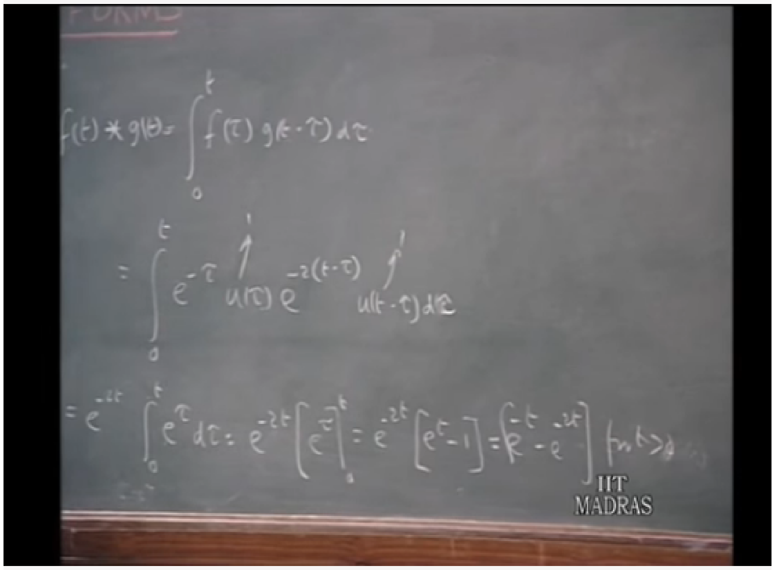

And we can simply say 0 to t of f of tau gt minus tau d tau. So, 0 to t f tau e to the power of minus tau u tau and gt minus tau g is e to the power of minus 2 t ut therefore, e to the power of minus 2 t minus tau ut minus tau. Now, in the range of integration when tau goes from 0 to t this is equal to 1 u tau is equal to 1.

Because tau takes only positive value obtained tau positive values and ut minus tau also the argument the t minus tau is going to be positive. Because tau is going to be less than t therefore, this is also equal to 1 you need not regard them we need not pay any attention to them. Now, the integration is only with reference to tau therefore, e to the power of minus 2 t is the constant as for the integration is concern.

So, we can full this outside in the integral sign. So, you have 0 to t of e of minus tau multiplied by e of plus 2 tau. Therefore, it is e tau e of minus tau multiplied by e to the power of plus 2 tau. So, e tau t tau; that means, this e to the power of minus 2 t the integral of this is e to the power of tau itself by 1 with the limits 0 and t.

So, that will be e to the power of minus 2 t e to the power of t minus 1 and that will be e to the power of minus t minus e to the power of minus 2 t. And this is valid for t greater than 0 in all over arguments. If t is greater than 0 only we assume, to take ut minus u tau to be equal to 1. Therefore, t greater than 0 it is e to the power of minus t minus e to the power of minus 2 t.

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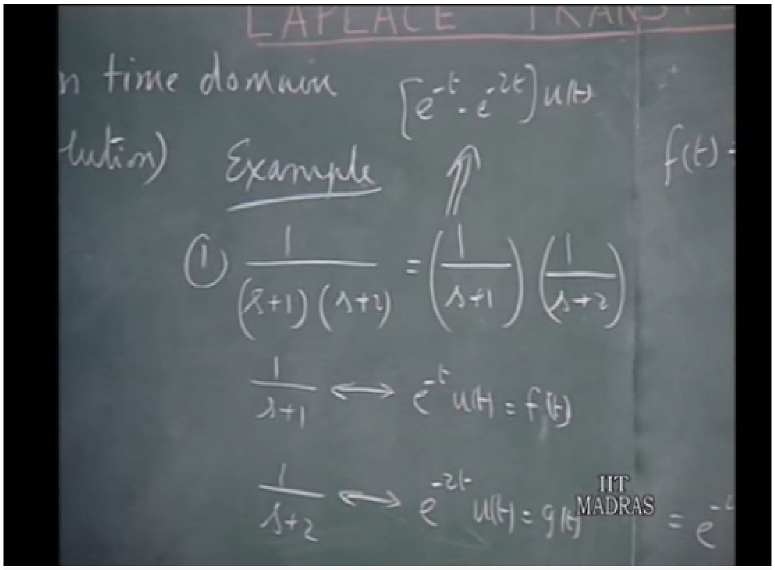

And therefore, this particular Laplace transform has got by the simplest Laplace transform e to the power of minus t minus e to the power of minus 2 t ut. Now, we wanted to illustrate this as the application of the convolution theorem you can get at this result even simply by considering this by making partial fashion expansion which will talk about in get a detail in nature.

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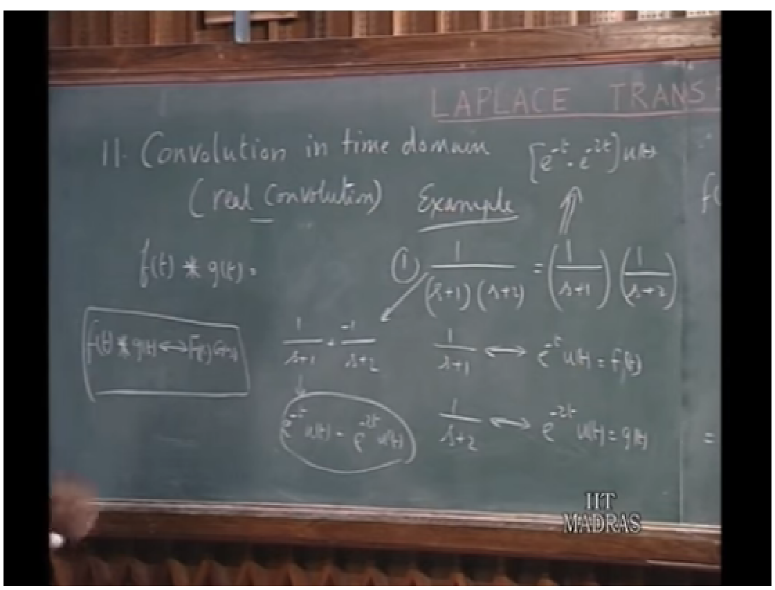

But this can always be 1 over s plus 1 thus 1 over s plus 2 and you can write this as 1 minus 1. So, you consider this you have common denominator you have s plus 2 minus of s plus 1 that will be 1 this is equal to this and we know that, the Laplace inverse Laplace transform of this e to the power of minus t ut.

The Inverse Laplace Transform of 1 over s plus 2 is e to the power of minus 2 t ut. So, this exactly the result we obtain here we have more in fashion the simpler fashion. But I would wanted to illustrate the application of the convolution theorem, by considering this as a product of 2 transforms and in the time domain it will terms of to be convolution of time functions this is verification of our result.

So, we have seen, the convolution in time domain corresponds to multiplication of the respective Laplace transform in the frequency domain.

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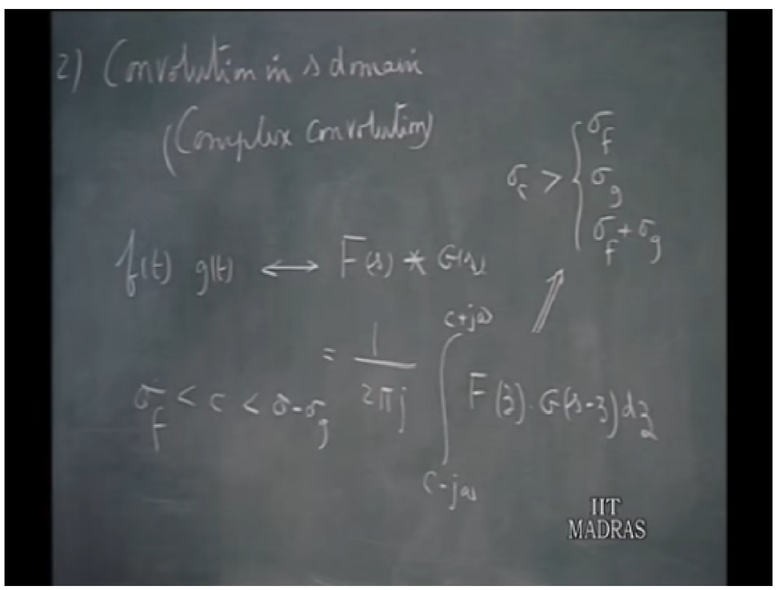

As a dual role, as a dual rule we have convolution in complex domain convolution in s domain this is called complex convolution. I will just mention this rule and leave it at that we will not pursue this because we there are the difficult to apply and we do not have really necessity to use this as a greater deal.

Therefore, I will simply say that, it to multiply time functions ft and gt multiplication in time domain corresponds to convolution the frequency domain f of s convolve with gs, what is meant by this convolution the frequency domain is an integral, which by this 1 over 2 pie j c minus j infinity to c plus j infinity of f of z times g of s minus z dz. So, you do the integration of product fz times gs s minus z, where z is running variable.

And therefore, the entire integral will be a function of s and that would be the Laplace transform of the product of f of t. And we have to take see the absence of convergence from this sigma c must be greater than abs of convergence function f of t Laplace transform of f of t Laplace transform of g and Laplace transform of delta f sigma f plus sigma g. The abs of convergence the product f of t gt Laplace transform must be larger than all this.

So, whichever is the maximum that, should be smaller than a sigma c. And this contour integration c should be larger than sigma f and then should be sigma minus sigma g; that means, you are actually what it turns; how it is you have to take this contour integration in a narrow band separated by 2 regions.

But to as I said we will not pursue this all it we can say is that multiplication time domain corresponds to convolution of the frequency domain those of who are interested can look up the reference books on this and you will get further details on this. But we will not pursue this.

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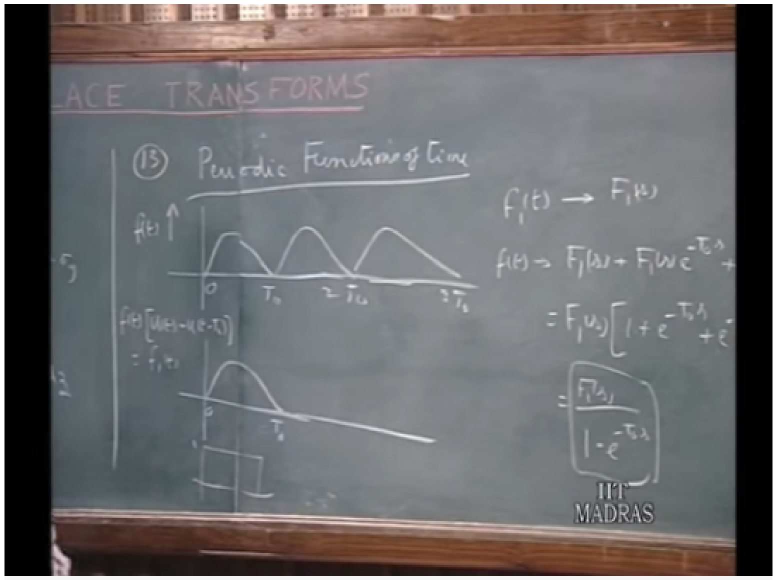

We will now take up an interesting application of the Laplace transforms which we will also thing of this is the property of the Laplace transforms of periodic functions of time Laplace transforms are periodic functions of time. Suppose, I have a periodic function of a time, which is given by like this 0 to t 0 2 t 0 3 t 0 this is f of t.

Now, consider what period of this blank this, I will call this ft multiplied by ut multiplied by minus u of t minus t 0; that means, you are multiplying this a gate function u of t and u of t minus t not that is just like a pulse like this unit amplitude you can multiply this if this you can this.

So, this will call that f1 of t. So, if you take a single pulse the variation of the f of t over 1 particular period only if, you know the Laplace transform of this you can find the Laplace transform of this easily it goes like this suppose, f1 of t as the Laplace transform f1 of s. That is the single pulse it have the Laplace transform of f1 of s then what is the Laplace transform of f of t, f of t is this pulse plus, second pulse plus, third pulse each pulse is this place by this by an amount equal to t 0.

The Laplace transform of this is f1 of s the Laplace transform of the second pulse duration over the second period is f1 of s times e to the power of minus t 0 of s the Laplace transform of the third pulse is f1 of s times e to the power of 2 t 0 s and so, on so.

So, the entire periodic pulse function will have 1 plus e to the power of minus t 0 s thus e to the power of 2 t 0 s extra and this can be put in the form f1 of s over 1 minus e to the power minus t 0 s. So, if you have the periodic function of time you want to find out the Laplace transform all units to do this consider the Laplace transform of over 1 period of the variation over the 1 period.

If it is Laplace transform of it is f 1 of s the Laplace transform of the entire periodic function is f1 of s over 1 minus, over the e to the power of minus t 0 s, where t 0 is the period of the particular wave form. To sum up we have in this lecture, consider further interesting properties of the Laplace transform in particular. We have considered, the Laplace transform of ft up on t given the Laplace transform of the f of t.

How we can find the Laplace transform of f of t over t than we consider, the initial and final value theorems, which enable as to find out the initial value and final value of function of time without from there Laplace transforms without having to find out f of t explicitly.

Then we also talked about the, convolution properties, convolution in time domain corresponds to multiplication frequency domain of the 2 particular transforms and the convolution in frequency domain correspond to multiplication in time domain the later 1 we said is a limited interest was then. So, we pursue this in some detail we also observed that, if you have the periodic function it s enough to find the Laplace transform over 1 period.

Then you can use that information to find out, the Laplace transform of the entire periodic function by multiplying that, the Laplace transform of 1 period by 1 1 minus e to the power of minus t 0 of s, where t 0 is the period of the periodic function of time. In the next lecture, we will work out several examples illustrating, the application of the various properties, that we have discussed so, far.