Networks and Systems
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Lecture-54
Properties: Convolution in Time Domain

We shall now, consider another important property, which is useful in system studies particularly when you want to find out the response to an arbitrary input, when you know the response to an impulse input through the medium of convolution integrals. So, you would like to know, how the convolution in time domain how it transforms itself in the s domain.

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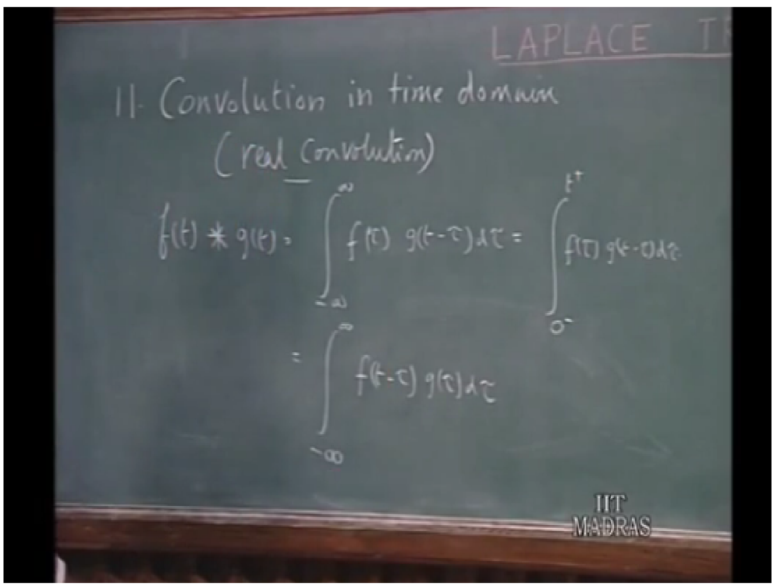
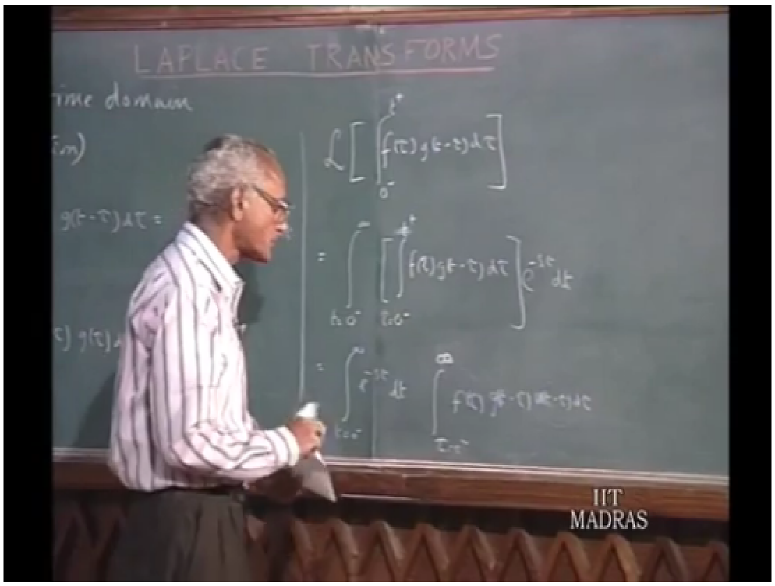


Convolution in time domain, this is sometimes referred to as real convolution. This is said to be real convolution in the sense that carrying out this convolution in time domain which is the real variable not in the transform domain which is the complex value. So, this is real convolution you recall that ft convolve it g of t is the definition is f of tau g of t minus tau d tau you can also write this as ft minus tau g tau d tau this are both are equal and you take, the variation of tau the minus infinity to plus infinity in the general case.

And for causal functions, where ft and gt are causal time functions, you take the limits because, f tau as the value only from tau equals 0 onwards possibly f of tau as f of t as an impulse the origin will take 0 minus. Then, when tau exceeds t this becomes negative therefore, there is no point in carrying out integration. Beyond t equals tau they are tau equals t therefore, we take this as t then f tau gt minus tau and take care of possibility of an impulse, if g of t the origin we take this as t.

So, this is what we are having by means of the meaning of f of t convolve with g of t. (Refer Slide Time: 03:00)



Now, let us try to find out the Laplace transform of this. So, Laplace transform of f tau t minus tau d tau 0 minus to t plus that is: what we are seeking. So, this will be 0 minus to infinity of this quantity 0 minus to infinity a t plus f tau gt minus tau d tau and this integration is carried out on tau and whatever the results is the function of t you multiplied by e to the power of minus st dt this integration is with reference to t.

So, this is the definition of Laplace transform of the convolution of the 2 time functions ft and gt which are assumed to be causal. Now, I would like to do the integration must with reference to t and then go to tau later therefore, to do that I must, I have also a function t plus s. So, to avoid that what can do is I will have e to the power of minus st dt because I would like to do the integration with reference to I will do the integration with reference to tau itself start with toe equal 0 plus to t plus.

Now f of tau gt minus tau suppose, I introduce ut minus tau d tau t equals 0 minus to infinity. Now, ut minus tau is going to be 0. When tau exceeds t, when tau exceeds t ut minus tau is 0 therefore, this integrant will become 0 when tau exceeds t therefore, I have instant having t plus I can as well infinity. Now, that I introduce the symbol ut minus tau here I may as well take the limit of integration of tau from 0 to infinity this is 0 minus infinity.

So, to ensure that; the same integral valid even here same the values of the 2 integrals are the same I will introduced purposely ut minus tau and take the limit as up to infinity because the value of ut minus tau is going to be 0 for tau greater than t the reason why I did this because, we will like to relate them to the Laplace transform integrals therefore, the limits must be 0 to infinity. So, in order to have that kind of property, I have to purposely introduce this alright f of t minus t d tau.

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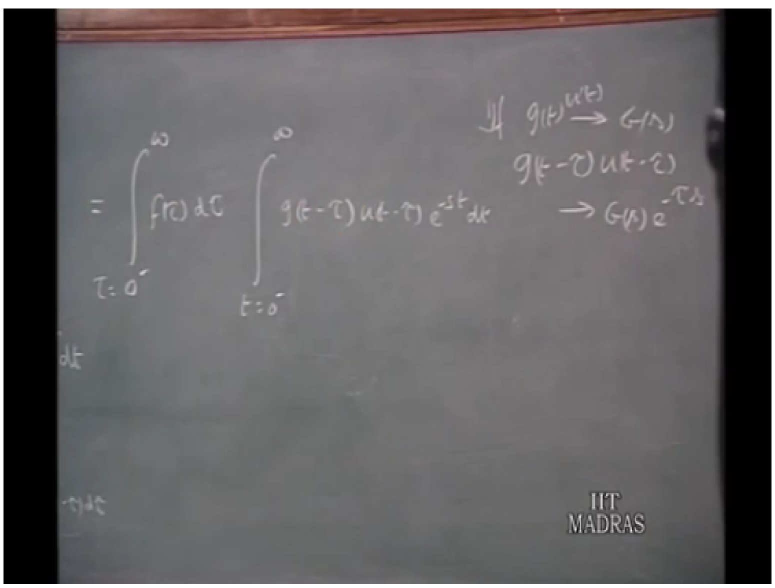

Now, this will be equal to 0 minus to infinity. Now I would like to interchange the limits of integration this is originally the first integration is refers to tau next integration with reference to t. Now, suppose I reverse the roles. So, I do the integration with f of tau later and then do the integration with reference to t first. So, whatever we are having as constants whenever, we integrating with reference to t must be got outside which is f tau for 1 example, d tau can be brought outside.

And all functions which are involved with t are to be taken into account in the first integration that will be gt minus tau ut minus tau e to the power of minus st dt. Now, this is the Laplace transform of the delayed function of gt delayed by tau seconds. If the Laplace transform of g of t is g of s Laplace transform of the delayed time function t minus tau is e to the power of minus s tau time g of s if, g of t has the Laplace transform g of s g of t ut as the Laplace transform g of s g of t minus tau ut minus tau as Laplace transform g of s time e to the power of minus tau s that is; something which we already know.

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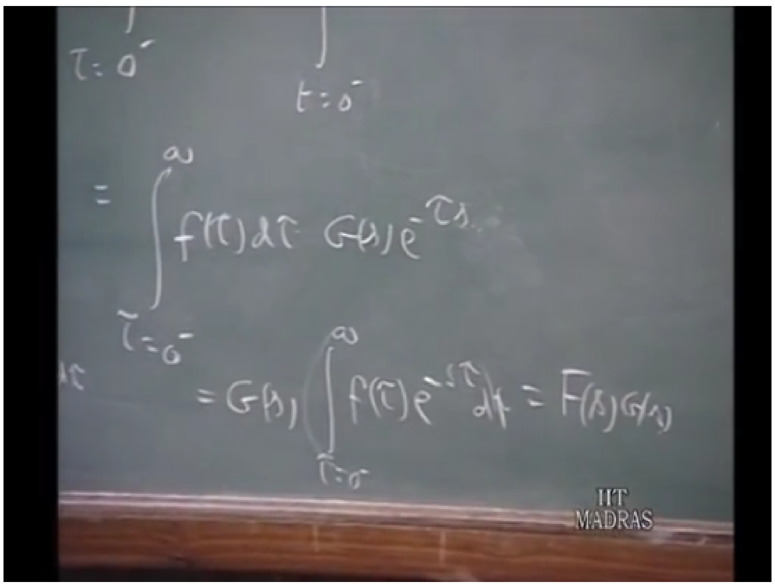

Therefore, we can write this as tau equal 0 minus to infinity of f tau d tau g of s times e to the power of minus s tau d tau sorry d tau come from here. Now, in this g of s is the constant. Therefore, g of s times tau equal 0 minus to infinity of f tau e to the power of minus s tau d tau, I have writing d tau twice here is not necessary. And this is indeed f of s f of t e to the power of minus t dt integrated from t equal 0 minus to infinity is f of s instead of t we have the dummy variable tau therefore, this is f of s multiplied by g of s.

So, the neat result that we are having here is finally, that ft star gt star the convolution of 2 time functions ft star gt star as the Laplace transform the product of the 2 corresponding Laplace transform.

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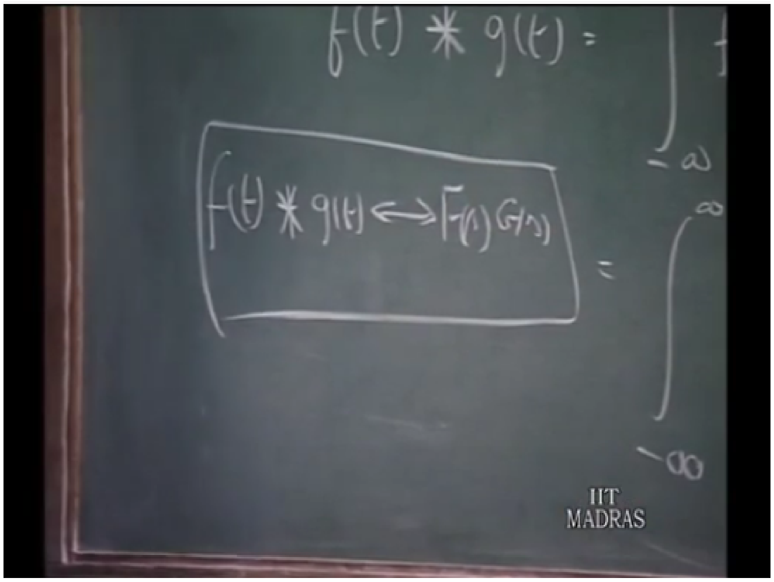

So, this is the important facility convolution, the time domain corresponds to multiplication of the respective transforms in the frequency domain are: the complex frequency domain s domain this is quite neat. And this is a rule which will be useful for our as a side in a system studies, where when we know the impulse response h of t to find out the response for arbitrary function of time. All we have to do is the Laplace transform of the output will turn out to be the product of the Laplace transform. So, the impulse response and the input function.