Networks and Systems
Prof. V.G.K. Murti
Department of Electronics Engineering
Indian Institute of Technology – Madras

Lecture-3
Linear Systems

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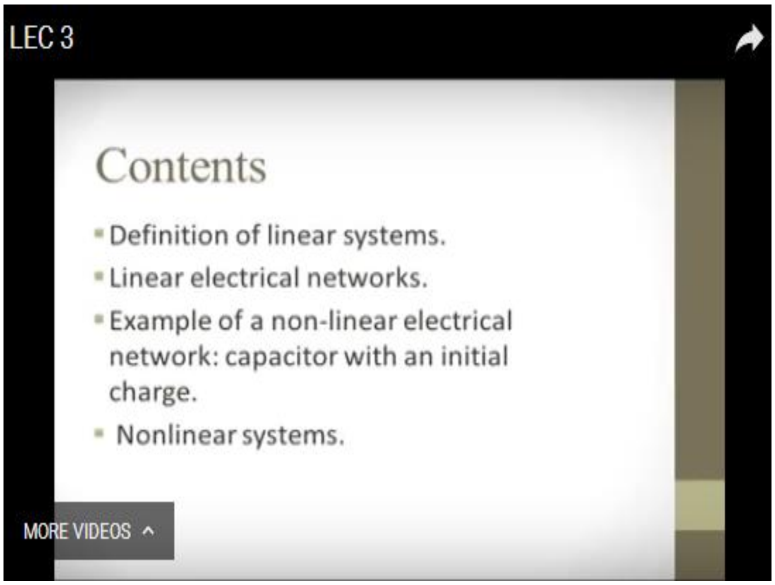

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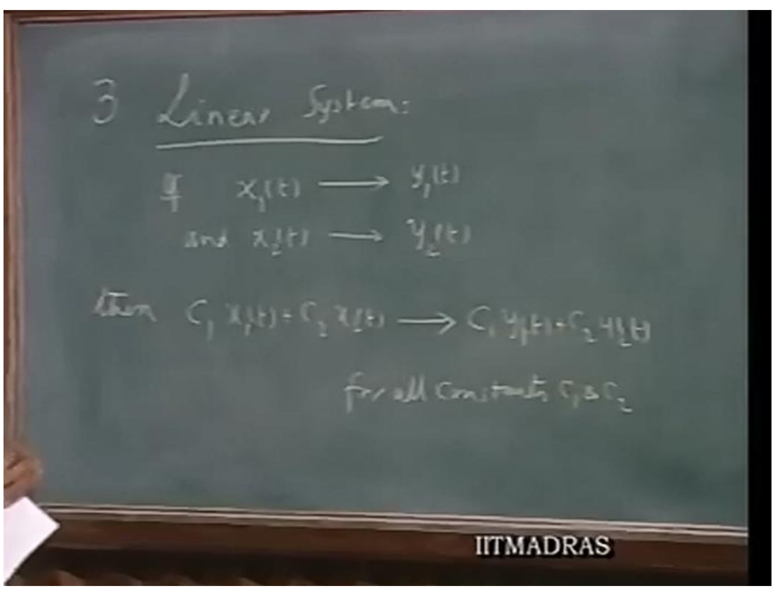

A third important classification is the concept of linear system and non linear system. In a linear system if an input x1 of t I am talking about a continuous time system. Now this is our discussion will be mostly in terms of continues time system at later point in time of time in this course we will talk about the discrete time system in a specially but wherever examples are given we talk in terms of continuous time system now for the most part.

If x1 of t gives rise to an output y1 of t and x2 of t gives rise an output y2 of t then for a linear system no matter what x1 and x2 are a constant time x1 of t plus another constant times x2 of t will give rise to an output which is c1 y1 of t plus c2 y2 of t for all constants c1 and c2. This is what is refer to the principle of super position some times people refer to this break this up into two parts homogeneity and additivity.

What they say is if x1 of t gives rise to y1 of t then c1 x1 of t gives rise to c1 y1 of t that is called the principle of homogeneity. Then the second part is if x1 of t gives rise to y1 of t and x2 of t gives rise to y2 of t then x1 plus x2 will give rise to y1 plus y2 that is called additivity. But now we will combine these together and we will say c1 x1 of t plus c2 x2 of t will give rise to c1 y 1 of t plus c2 y2 of t for all constants c1 and c2.

And this is the class of systems which are called linear systems one which obey this principle of super position. And most of the electrical networks that we have sawed earlier belong to this class. But you must keep in mind that this particular super position principle will be valid for example in the case of a electrical circuit with zero initial stored energy. If a capacitor has got some stored energy in that this principle of super position will be not valid.

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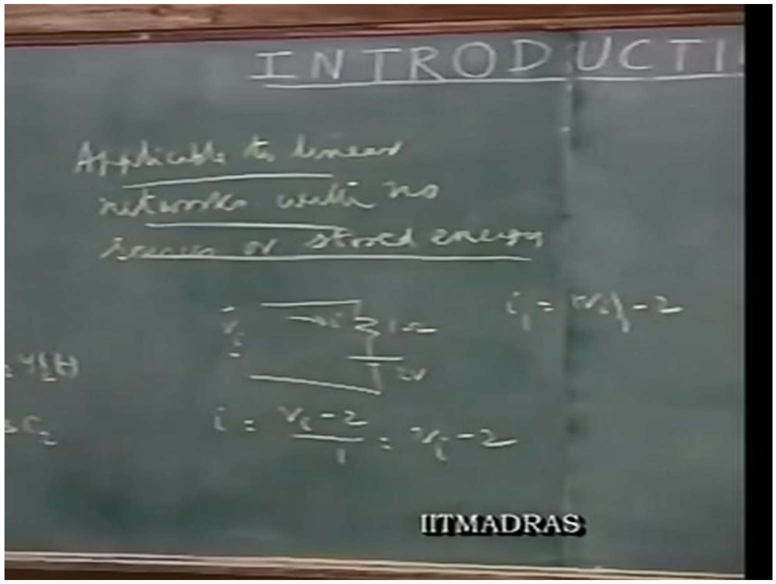

Let us an taken example for example I will say this applicable to linear networks as a special in of course with no independent sources or stored energy important. For example this is a linear network resistance suppose I have a source here 2 volts 1 ohm then if I have an output an input here vi and I will look to take the current to be by response quantity i equals vi minus 2 divided by 1 or vi minus 2 and if I have 2 different voltages v1 and v2.

And find the currents i1 and i2 and if I say if I have v1 plus v2 then I will not get i1 plus i2 as you can see for example i1 here is vi1 minus 2 for one particular excitation. For second excitation i2 equals it is vi2 minus 2.

But suppose I have a single excitation which is equal to vi1 plus v2 then the current would be vi1 plus vi2 minus 2 that will be the current when I have a single excitation vi1 plus vi2 but i3 is not equal to i1 plus i2, why because of the source here i3 is not equal to i1 plus i2. So when you apply this linearity principle as superposition principle you must make sure that you are applying it to a network which does not have sources or stored energy for that matter.

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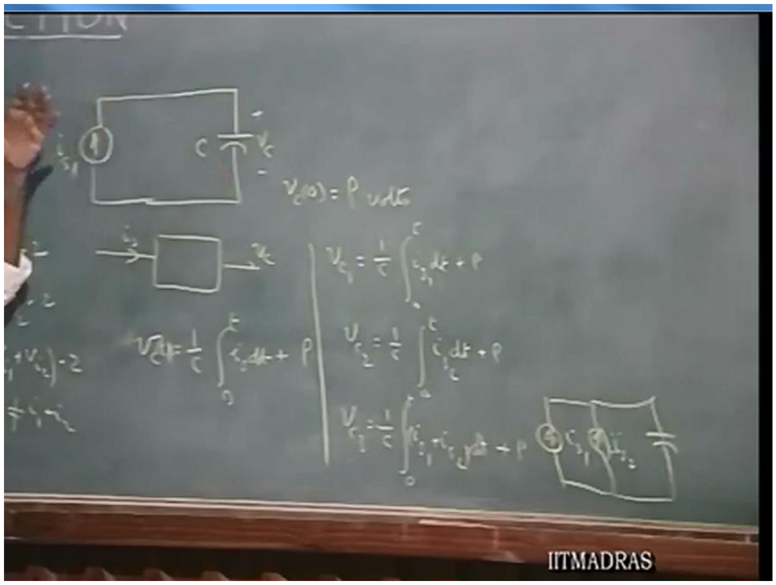

Let us take a second example to illustrate the second point which I have in mind, suppose I have a capacitor which has got an initial stored vc this is c and I have a current source as excitation. So I think of these as the system with i s as the input quantity and vc as a output quantity. Okay, and let me assume that vc0 equals some Rho rho volts that is initial capacitor voltage is rho volts.

Then you have the relation is equals vc equals 1 over c 0 to t is dt that is the initial charge that is convert to that capacitor in the integral from 0 to t, this is vc of t plus the initial voltage that is across the capacitor Rho that is this initial voltage plus the increment in the voltage due to the additional charge that has been conducted to the capacitor in the integral 0 to t. This is the equation between the input quantity is and the output vct.

Now let us take two situations where I have is is this 1 is 1 then it you will get vc1 equals 1 over c integral is 1 dt plus rho 0 to t of course. And let us take a second excitation which is 0 to t is 2 dt plus rho that is vc2. So is1 and is2 respectively will give rise to this voltages vc1 and vc2. Suppose I had a single source which is1 plus is2 both of them are active in parallel simultaneously is1 and is2 together charging the capacitor then the third time the voltage vc3 will be 1 over c integral is1 plus 1s2 dt plus rho again because the initial is the capacitor charge rho volt.

Now you can see vc1 plus vc2 which are the response to the is1 and is2 will not add up because vc3 is not equal to vc1 plus vc2 what's spoil is this initial charge in the capacitor because when you add this 2 up you get 2 rho but it only one rho which means that the argument is that this principle of superposition is valid only for systems which are purely linear in the sense that there should not have any sources inside the independent sources are initial charges in the capacitors or initial currents in the inductors which are always consider to be equivalent resources.

So this is an important principle and to keep in mind when you talk about linearity of systems. A system which is not linear is of course called non linear. The difference between the linear system and a non linear system is a linear system is governed by linear differential equations and a non linear system by non linear differential equation. So what is the meaning of a non linear differential equation, linear differential equation is one in which the coefficients of the various derivative terms coefficients are not functions of the dependent variable.

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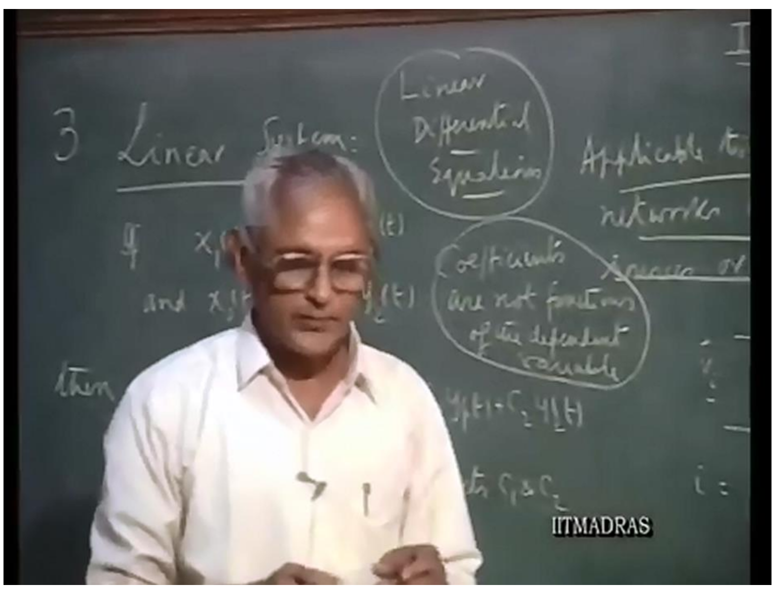

I will take some examples later on so if I have a term like b squared by dt squared its coefficient should be either constant it could be function of time but it cannot be a function of another derivative of y or y itself. So linear differential equations are the one which govern the operation of the linear system and in the system if the i equation fails to be linear then it becomes non linear and that the super position principle will no longer valid.

We will take a some examples of these a equations which governs the performance of the difference kinds of systems in the next lecture. But let me stop at this time and briefly summarize what has been discussed so for. Starting with the scope of this course which is mainly the dynamic performance of linear time invariant systems and networks. We had a look at the number at the difference and text books that you may use for following this course.

Then I mentioned the background material I assume that you have had before taking up this course then we looked up looked at the meaning of a system broadly speaking we said system is a collection of objects. A system components united to some form of interdependence among the various consequent components. We said a network is a special kind of system in which all the variables ill be of a particular kind a one kind in a electrical network and mechanical network given as examples and then we also talked about the modeling of a system.

Modeling of a system is done in the form of ideal components put together the constituted the system interconnected fashion and we do this idel idealization keeping in on one hand keep in mind on one hand that the complex the system should be not too large to ficiate convenient handling at the model at the same time it is should be succinctly realistic so that there is all obtained for the model are generally realistic in practice.

Otherwise the model have to be find once again also mention that the modeling system we you use the two terms inter changeably almost as if the model itself is a real system which is of course not true. Then we looked at the representation of the system and we talked about the multiple input, multiple output system and a single input, single output system and then we went on to discuss the various kinds of classification of systems because as I mention we are talking about a linear time invariant system therefore we must know what they mean.

So for that with that in mind we looked up at the we were trying to find out the classification of the important classification of systems. We talked about the difference between static systems and dynamic systems. We talked about the difference between continuous time systems and discrete time systems and lastly we have talked about the difference between linear system and non linear systems.

The discussion on the linear and non linear systems is not complete because I would like to give some examples of the governing differential equation for both the type of systems and we will pick up at this point in the next lecture and continue our discussion from that point onwards.