Networks and Systems
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Lecture-51
More Properties of Laplace Transform: Shift in Frequency Domain

Looked at certain important properties of the Laplace transforms in the last lecture. The particular properties which are of interest are those when f of t gets multiplied by t what happens in the transform domain. If f of t is differentiated f of t is integrated there effects the transformed domain are 3 important properties which we discussed in the last lecture.

So, if you continue this discussion of the properties of Laplace transforms which will be useful when, we handle problems involving the application of Laplace transform between the solution of electrical networks and systems under dynamic conditions.
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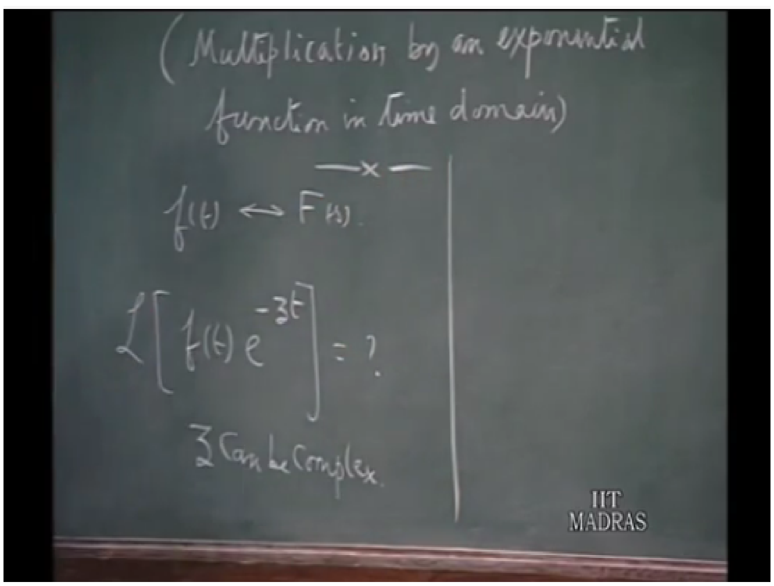

So, let us consider now the next property which concerns what is called shift in frequency shift in frequency domain that is s domain which is equivalent to multiplication by an exponent multiplication by an exponent exponential function in time domain. What we are going to show is that if you multiply an f of t the exponential function like, e to the power of minus alpha t it corresponds to a shift in the frequency domain that is explain are s domain.

Let us, consider an f of t is the Laplace transform f of s. the question which we like to ask is what is the Laplace transform of f of t multiplied by e to the power of minus e zt assuming all this of course as assuming that f of t is causal time function. What is that equal to? Both for generality z can be constant. So, this is the what we are going to look at?

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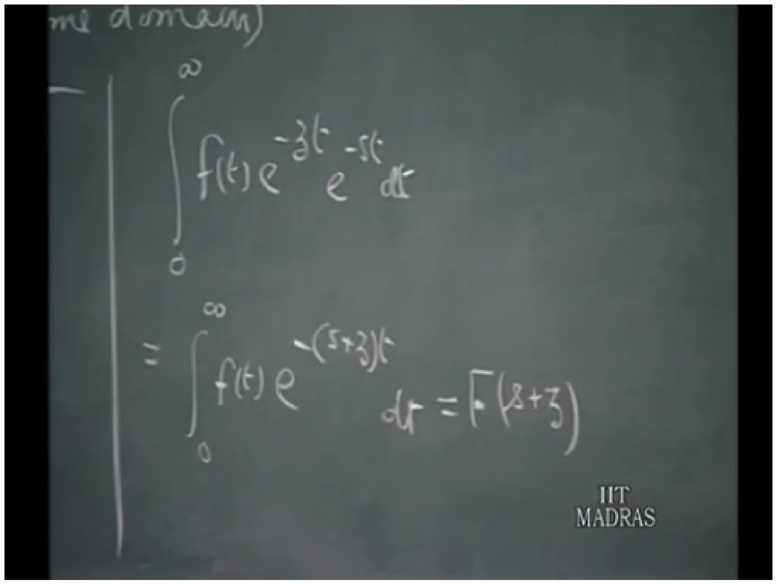

So, to using the formula for Laplace transform of the time function this particular Laplace transform would be from 0 to infinity f of t e to the power of minus zt e to the power of minus st dt. I am not putting specifically 0 minus because, depending up on our interest whether you take from the 0 minus or 0 plus depending up on what should take f of t to be the type of f of t you are interested.

So, we leave it in that this of course, can be written as 0 to infinity of f of t e to the power of minus of s plus z t dt. Now, you will see that e to the power of minus st dt ft e to the power of minus st dt integral will be f of s all we have now is instead of s we have s plus z. So, if you get the f of t e to the power of minus st dt this should have been calculate f of s this Laplace transform of f of t now instead of the variable s we have s plus z.

So, this will be f of s plus z. So, we have if f of t has the Laplace transform f of s if you multiply f of t e to the power of minus zt, the Laplace transform is simply f of s plus z. That means, this translation are shift in the frequency domain. So, this is the very important property which will before quit useful in finding out the Laplace transform of certain time function as well as, the inverse Laplace transforms of certain functions are s.

And of course, in the new function if you are having the f of convergence whatever it was for f of t we will get modified. Because, of this the factor e to the power of minus zt depending up on the real value of z. Then, the abscissa of convergence can be modified for the new function that is not important for us.

We note of course, that is the possible the abscissa of convergence are the new function different from the abscissa of convergence for the original f of t.

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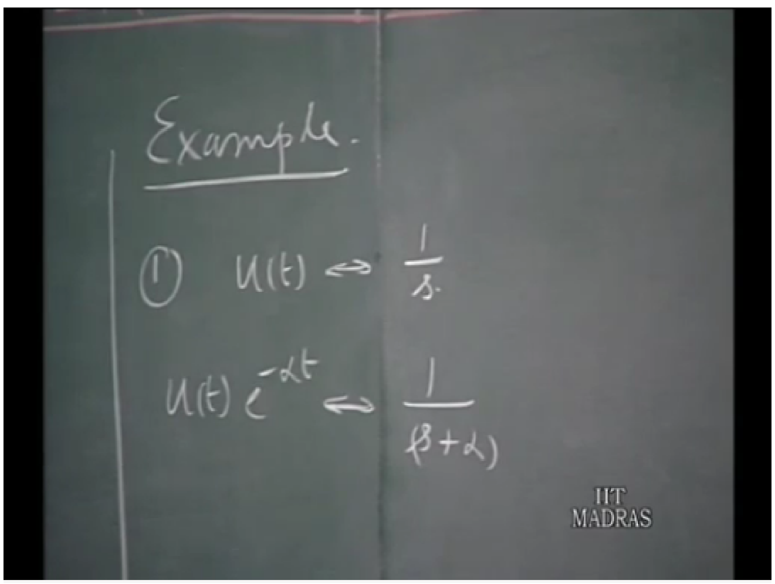

Let me, take an example suppose we have u of t we know that the Laplace transform is 1 over s now let me multiply u of t e to the power of minus alpha t. So, according to this rule instead of we have replaced s as alpha this all we have to do therefore, this is 1 over s alpha. So, in other wards e to the power of minus alpha t ut which we have derived earlier from fundamentals could have been derived using the applying this particular rule of shift in frequency domain corresponding to multiplication by e to the power of minus alp ha t.

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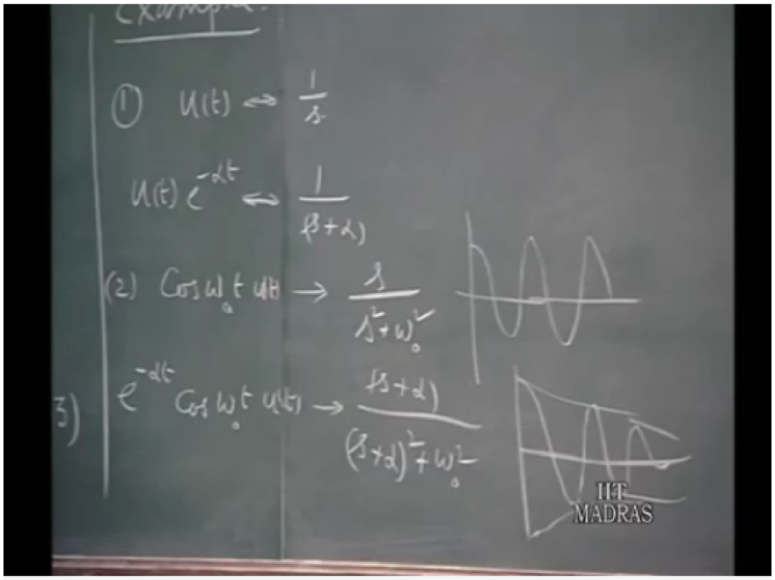

Now, let me take other example suppose I have task omega not t ut a cosine function we know its Laplace transform is s over s square plus omega not square that would be a trigonometric function which starts from t equal 0 on the other hands, suppose I take e to the power of minus alpha t cos omega not t ut this particular time function is again oscillating.

But with decreasing amplitude depending up on this value of alpha. So, its plot would be if this is the envelope so, you have something like this. Where the amplitude is decay as e to the power of minus alpha t. So, in exponential damped sinusoidal function of time. Now, all this differs from the original function is by multiplication of e to the power of minus alpha t therefore, according to this rule always we have to do is replace s by s plus alpha.

So, you have s plus alpha divided by s plus alpha square plus omega not square. So, this gives you a new function of time Laplace transform par e to the power of minus alpha cos omega not t ut will be s plus alpha over s plus alpha square plus omega not square you do not have to remember this.

Because, once you know the formula the Laplace transform of cos omega not t whenever it gets multiplied by an exponential factor of that we should able to deadly reduce this. But more over less put this down in our table of Laplace transform and f of t pairs I will go to the other end of the blackboard even if our camera man will frond at this but let me, do that I will take that risk.

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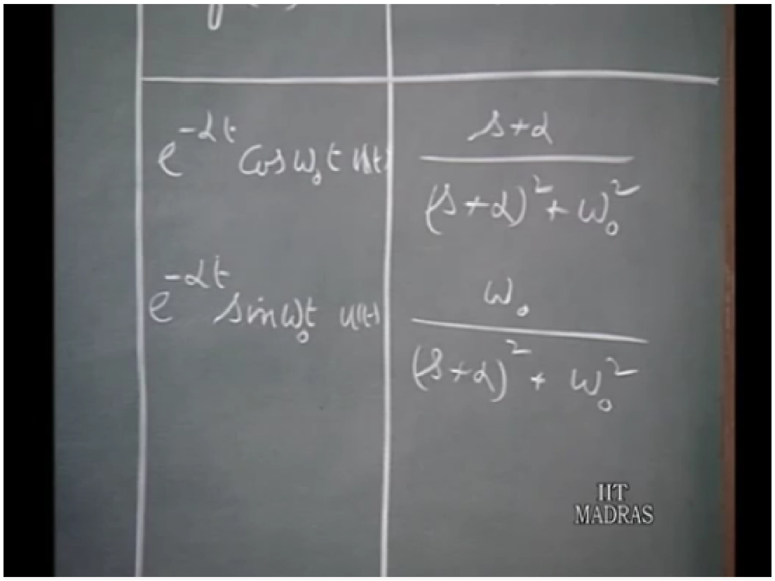

So, we have e to the power of minus alpha t cos omega not t ut will have Laplace transform s plus alpha over s plus alpha square plus omega not square. In the same manner suppose, I have e to the power of minus alpha t sin omega not t ut we know that, Laplace transform of sin omega not t ut is omega not over s square plus omega not square.

So, in that particular function we have to substitute s plus alpha for s. So, omega not will remain as before the denominator will be s plus alpha square omega that would be the Laplace transform e to the power of minus alpha t sin omega not t ut. Visualizing this in this fashion will be very important when, you are taking some times inverse Laplace transform of functions where a quadratic exists in the denominator.

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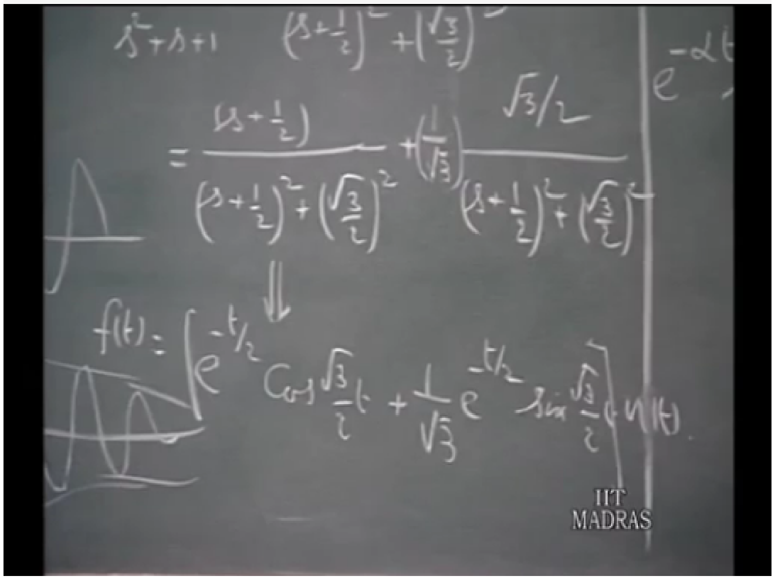

For example: Suppose I ask the question what is the inverse Laplace transform of say s plus 1 over s square plus s plus 1 suppose, I ask the question. Now, note that the poles of this the rational function are the zeros of the denominator polynomial are complex in that case, s square plus s plus 1 can be put in this form s plus alpha whole square plus omega not square try to put in that form then, you can combine you can decompose this function into sum of 2 such functions.

Then, you can readily integrities them to be the inverse Laplace transform of sinusoidal functions where decreasing the amplitude. So, let us go about that see this: so, s plus 1 over s square plus s plus 1 first of all I should write this in denominator as s plus alpha whole square plus omega not square. Therefore, I write this s plus half whole square so, that account for the coefficient of s here s square plus s plus one-fourth.

But still you have 1 here therefore, to cover that I will put this root 3 by 2 whole square. So, you observe that 3 quarters plus 1 quarter is equal to 1 and therefore, we have this the complex conjugate the poles the denominator of the function are clearly displayed as minus half plus or minus root 3 by 2. So, the numerator has been s plus half you would identified this to be e to the power of minus half t cos root 3 by 2.

Therefore, let us do that so you have s plus half divided by s plus half whole square plus root 3 by 2 whole square this is 1 term and what is left behind is another half here if the numerator has been constant root 3 by 2 then, you would put this as e to the power of minus alpha t sin omega not t by 2. So, if this had been root 3 by 2 divided by s plus half whole square plus root 3 by whole square. Then, we would have set this to be the Laplace transform of e to the power of minus half t times sin root 3 up on 2 t. But had only half here therefore, to cover that put 1 over root 3.

Then, you have s plus half this term we know is the Laplace transform of e to the power of minus t by 2 cos root 3 by 2 t and this term will be 1 over root 3 e to the power of minus t up on 2 sin root 3 by 2 t. So, of course, the entire thing is multiplied by u of t that is the f of t which corresponds to this Laplace transformation.

So, this rule are what happens when, you multiply the time function by an exponent will come in handy not only in find the Laplace transform as a certain time functions. But when you are finding the inverse Laplace transform of quadratic factors where the denominator zeros are complex conjugates can be easily be handled in this fashion.