Networks and Systems
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Lecture-48
Recap, Poles/Zeros and Laplace Transform Notation

We had introduced ourselves to the concept of Laplace transformation of the function of time. In the last class and we had a look at the Laplace transforms of some important functions of time like the unity impulse function the unit step function and the 2 sinusoidal functions cos omega not t and sin omega not t. Let us, first recapitulate some of the important points that was discussed in the last class.

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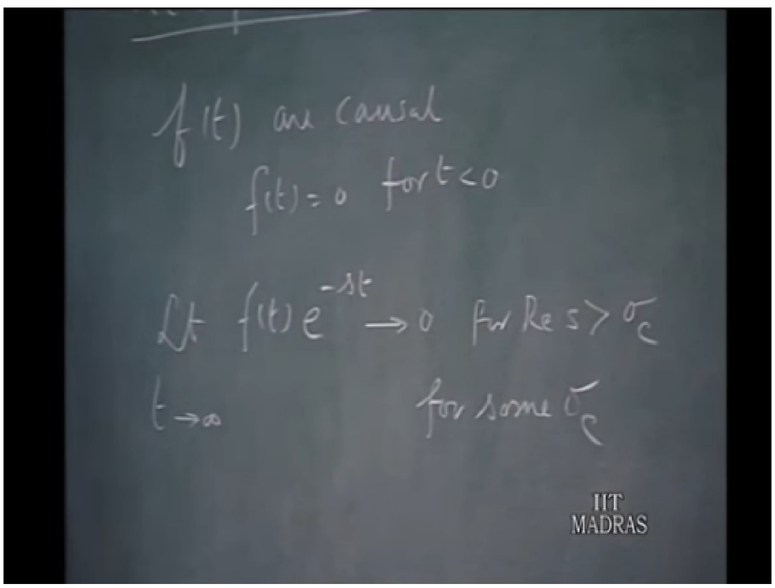

We are concerned with functions of time which are causal. So, the f of t which are concerned with are causal that is f of t is 0 for t less than 0. And further we said their exponential order; that means, limit as t goes infinite of f of t e to the power of minus st close to 0 for real part of s greater than some number sigma c for some sigma c which is the function of f of t. So, if the function of time is the exponential order that is the t goes to infinite the e to the power of minus st is able to pull thus out the negligible proportion.

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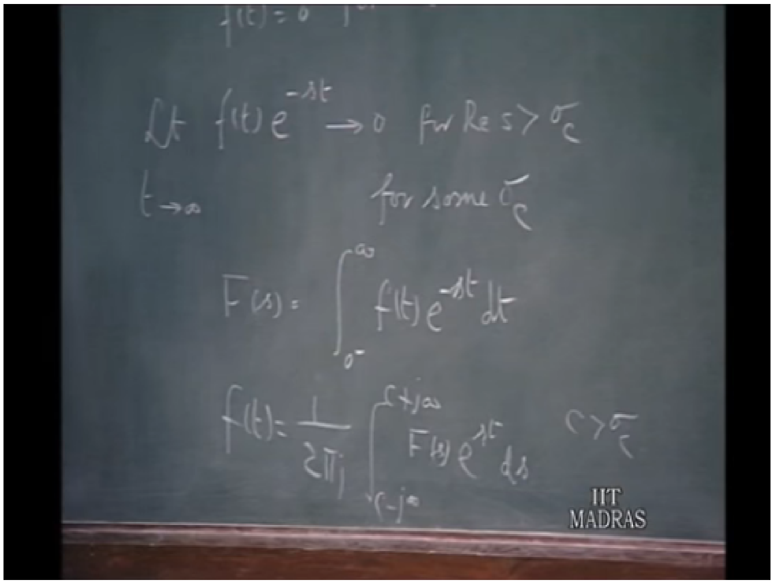

Then for such time functions we said we have a Laplace transformation f of s. Which is given by the integral sub 0 minus to infinite of f of t e to the power of minus st dt. That is; the forward Laplace transformation formula and the inverse Laplace transform is given by 1 over ft is 1 over 2 pie j c minus j infinite to c plus j infinite of f of s e to the power of st ds this is the inverse Laplace transformation formula.

This cantor of integration which extends from c minus j infinite to c plus j infinite is the vertical line in the region of the convergence of the Laplace transformation; that means, c must be larger than sigma c. And this cantor of integration as i mentioned is referred to as the ground which cantor. Now, as the result of this the Laplace transformation that we get is usually a rational function for the type of functions of time that we consider.

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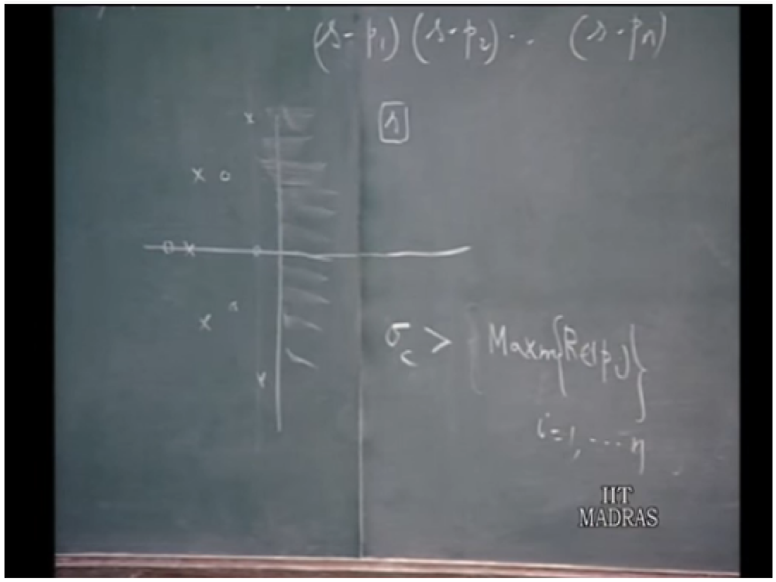

So, for the given f of t that we having we have an f of s which is the rational function. And this rational function by means, the ratio of 2 polynomial and this could be the 2 polynomial could be put in the form m is the numerator is factorized as s minus z1 s minus z2 down the line z minus zm. And the denominator likewise the factorized as s minus p1 s minus p 2 s minus pn, then the Laplace transformation epodes is defined by the values of the z value z1, z2, z3, zm and the values p1 up to pn apart from the constant multiplied factor m.

So, this f of s can be represent in the complex plane by the location of the zeros of f of s and poles of f of s. So, zeros are normally marked by means of small circles. So, this may be 0 locations poles may be are marked by crosses. So, there may be a pole here pair a poles here extra. So, the pole in 0 locations are marked in the s plane.

So, the pole of 0 locations of f of s completely defined f of s expect for the multiplied factor m. And we also lot that as long as coefficients in the 2 polynomials are real then early complex pole is accompanists by its conjugate any complex 0 is accompanists by its conjugate and the absence of convergence is.

Sigma c the absence of convergence is larger than the maximum value of the real part of pi. So, whatever the pi you are having is the maximum value or the real part of pi i from 1 to n. So, that will be define the absence of convergence; that means, the region of convergence starts from the right most pole.

So, if this is the right most pole anything beyond that if there is in the conversions. So, the maximum value of the real part of the poles defines the reason of convergence. And we have seen this in the case of simple functions like the unit stuff function which as the sigma c as 0 of the e to the power of minus 2t where sigma is minus 2 and so an so for, as that is what we have seen earlier a few comments about.

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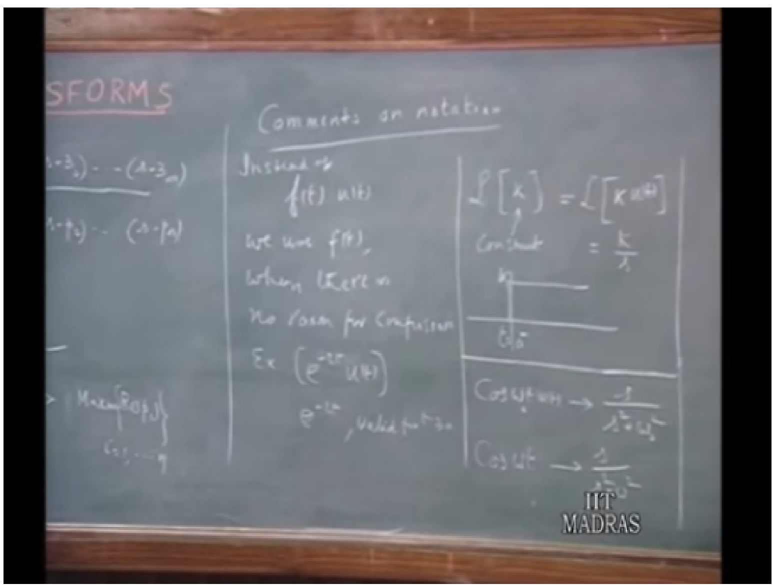

The notation as i mentioned the Laplace transforms exists for unilateral Laplace transform that we are talking about exist only for causal time functions. So, if f of t by definition has the value for negative values of time as well, to make sure that we are talking about the truncated part of that function of t we normally write f of t to ut. To make sure that f of t the composite function is 0 for negative values of time.

Often instead of f of t ut we use simply f of t then there is no cause for consumption when there is no room for consumption. In other words example, suppose e to the power of minus 2t this is f of t that you are talk that you cant find Laplace transform such a function. We have to find the Laplace transform and e to the power of minus 2 t times ut.

So, when you are got 1 over x plus 2 the inverse Laplace transform of that e to the power of minus 2t ut. Instead of that we write e to the power of minus 2t itself, with the understanding that we are talking about e to the power of minus 2 t valid for t greater than or equal to 0. So, as long as we understanding that whatever time function we are having is valid only for t greater than or equal to 0 and it is 0 for negative values for time.

Instead for e to the power of minus 2 t ut we ofer by e to the power of minus 2 t to save or write. Another point is suppose we have a some constant. Laplace transform or some constant k again by definition by constant k exists for all values of time, but in the context of Laplace time form. We assume the distance function is the value k starting from t equal 0 minus. Because we can teak only functions which are 0 for negative values of time in the Laplace transformation consideration.

Therefore, even if you teak the going to Laplace transformation constant. We regard that constant that will have the function will have the value k starting from t equal to 0 minus and 0 for values of t less than this. So, when we are talking about the Laplace transform the constant. We treat this as laplace transform of k ut at say this is k apart s.

So, we do not sometimes exhibits specifically this u of t which understood just to say, a right. Another feature of common notation is you recall that i said cos omega not t ut has Laplace transformation s over s square plus omega not square. We have taken a particular angular frequency omega not and truncate it a t equals for negative values of time and arrive get this Laplace transformation.

Now, s is equal to sigma plus j omega. The imaginary part of s is also given the simple omega it is a running variable and to distinguish that running variable omega with this omega. So, we have given as a special simple as a omega not. But in our manipulation we very rarely decompose s sigma plus j omega looks at the imaginary part of s.

So, where there is no confusion we even right simply cos omega t having a Laplace transform. As s square plus omega square as this is return in this manner in this notation we understand that this omega is not to be confused with the imaginery part of the s. This is the omega a particular value of omega associate a trigonometric function this should not be confused with the imaginery component of s.

And secondly, as we mentioned earlier actually we are talking about cos omega t ut. But this additional function u of t is often dropt. Assuming that, we are talking about a function which has this functional notation only t equal to 0 plus onwards 0 minus onwards. So, often instead of righting like this we simply said cos omega t is the laplace transform s over s square plus omega square.

These are minor deviations in the notation we often find to simplify the righting of the various expressions this type of simplification is distorted 2. But basically, we must note that whatever function we are talking about is assume to be 0 for values of t less than 0 minus.