Networks and Systems
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Lecture-47
Laplace transforms of important functions

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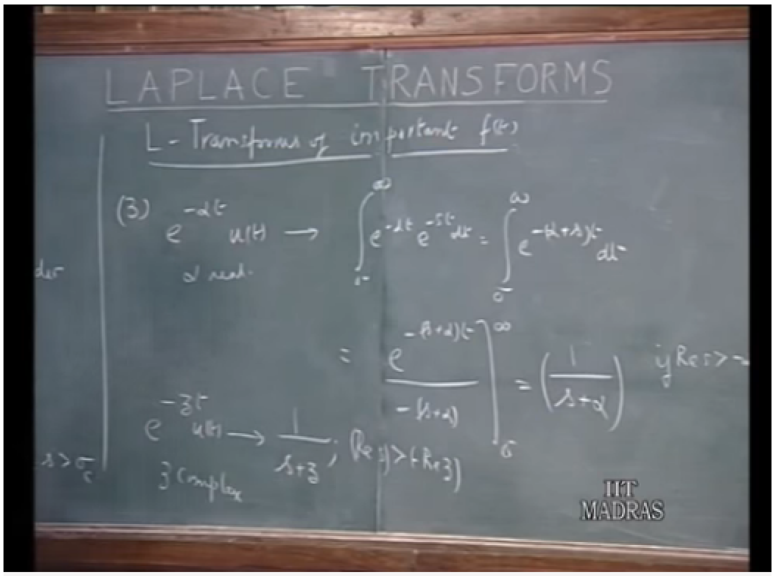

Okay, now we will take a third function: e to the power of minus alpha t u t. Then, take the Laplace Transformation now, e to the power of minus alpha t, u t is of course is 1 in the range of integration e to the power of minus st dt which is equal to 0 minus to infinity e to the power of alpha plus s times t dt, which is equal to e to the power of minus s plus alpha times t divided by minus of s plus alpha evaluated between the limits 0 minus to infinity.

Now, once again we like to make the integral converge at t equals to infinity by making sure that, the real part of s plus alpha is positive number. Real part of minus s plus alpha must be greater than 0. So, when you make that if real part of s is greater than minus alpha, the real part of s greater than minus alpha.

At the upper limit real part of s plus alpha is greater than 0. Therefore, at t goes to infinity this integral becomes 0 the limit of this at t equals to infinity will become 0. At the lower limit 0 this will be of course, will be equal to 1. And therefore, this will be minus sign already here. Therefore, this will be 1 over s plus omega. So, we have s a function e to the power of minus alpha t u t; the Laplace Transform is 1 over s plus alpha and this integral will converge provided that, the real part of s is greater than minus alpha here we are taking alpha to be real in this case. If, alpha is real this is what we have.

Now, as far as this integration is concerned it does not matter, even if you have taken e to the power of minus zt z is complex then, we go through the same arguments, same analysis it will be 1 over s plus z provided, the real part of s is greater than minus of the real part of z or the real part of s plus z is greater than 0. So, even this u t of course. So, alpha did not be real, it could be even a complex number even if z is z complex, the same relation should be valid.

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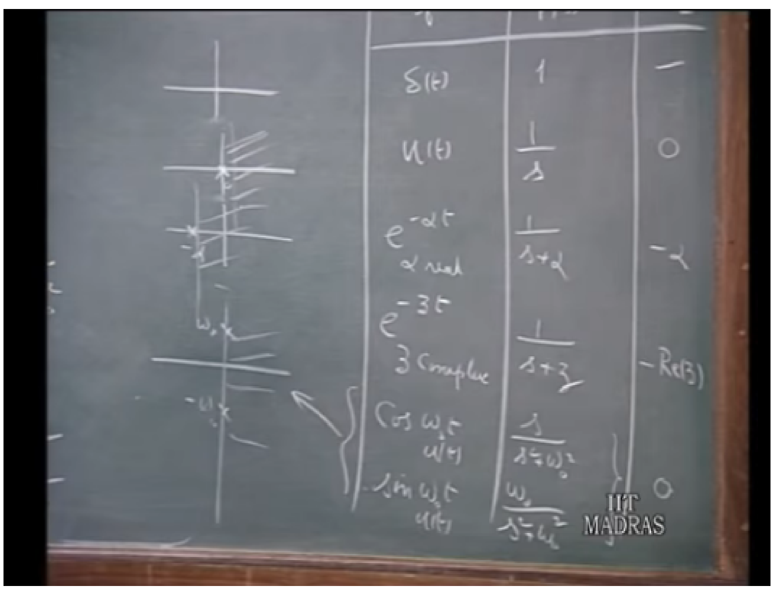

So, I will write here: e to the power of minus alpha t alpha real 1 over s plus alpha, this is minus alpha minus zt z complex 1 over s plus z minus real part. So, once we have the relation, we can find out the Laplace Transformation of trigonometry function

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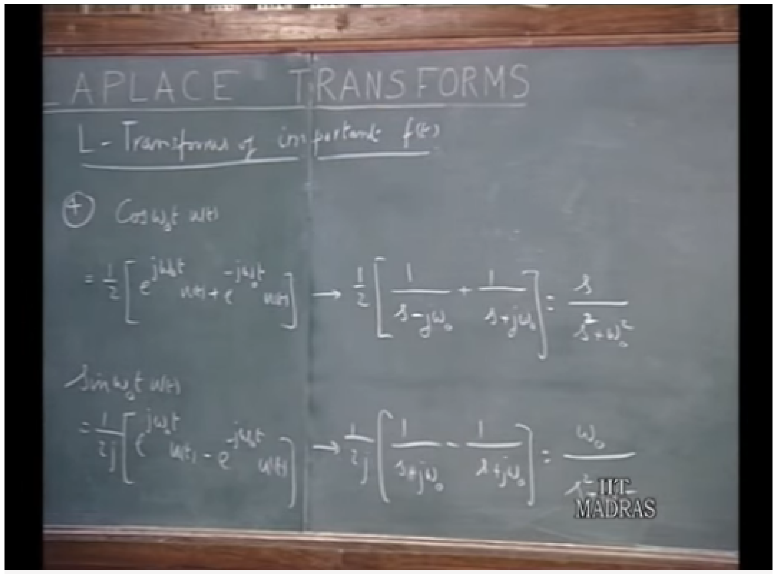

Suppose, I take cos omega 0 t u t. This can be written as one-half of e to the power of j omega not t u t plus e to the power of minus j omega 0 t u t. And since, we have found out that e to the power of zt minus zt will have 1 over s plus z as its Laplace Transformation then, we have instead of that we have j omega not.

Therefore, the Laplace Transformation of that would be one-half of 1 over s minus j omega not. Because, e to the power of minus zt has the Laplace Transformation 1 over s plus z. Instead of minus z you have j omega not. Therefore, you have s minus j omega not and the Laplace Transformation of this would be 1 over s plus j omega.

So, if you complete this, this will become s over, when you rationalize; denominator will become s squared plus omega not squared and the numerator will be 2s and divided by 2 and this become s by s square plus omega not squared. In a similar fashion if you find out the Laplace Transformation of sin omega not t u t as 1 over 2 j e to the power of j omega not t u t minus e to the power of minus j omega not t u t that is after all, the sin function can be described in this fashion.

This can be written as 1 over 2j 1 over s minus j omega not minus 1 over s plus j omega not, exactly the same fashion. And in the numerator you get 2j omega not by rationalizing the denominator and the 2j will cancel with this. So, you get omega over s squared plus omega squared, so that is what we are having.

And in this derivation the abscissa convergence should be the real part of z; this is the running with z. The real part of this is 0 means: the abscissa convergence for both these is 0. So, we have the final result cos omega not t. I can write this as omega t because, as long as omega t or if you like you can still continue.

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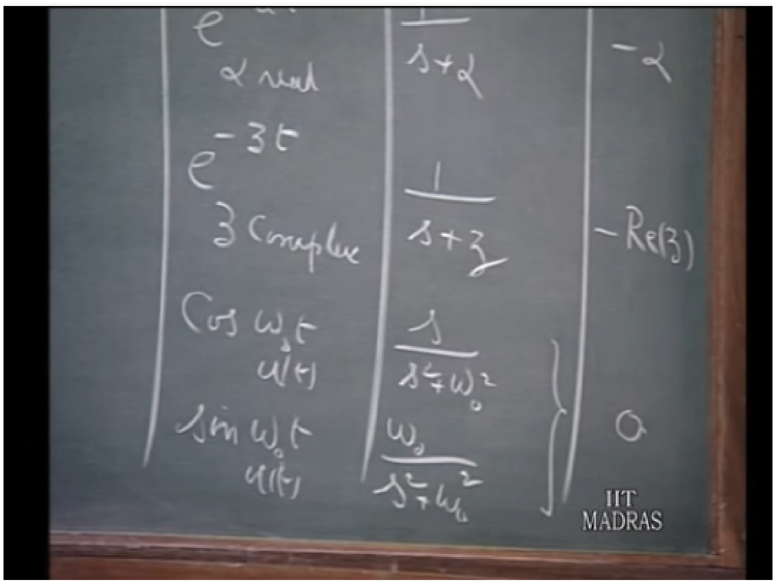

You, do not confuse this omega the real imaginary part of s, we can as well write cos omega not t can write this as s over s omega not squared and sin omega not t this is of course, always this u t continues with us because, we are assuming this function to be 0 for negative values of time. This will be omega not over s squared omega not squared and the abscissa convergence for this is 0. So that is what we are having for the sin function and the cosine function which are truncated at t equal to 0 that means: the sin and cosine exists only for positive values of time.

Now, we can represent the poles and zeros of this F of s in the complex frequency plane. For example, for this these are all rational functions of time. Therefore, the poles and zeros exist for this. As far as delta t is concerned it has either poles or zeros. As far u of t is concerned there is a pole at the origin. As far as e to the power of the Laplace Transform e to the power of minus alpha t is concerned 1 over s plus alpha. Therefore, there is a pole at the minus alpha.

Forget about the z taking these you have, for cos omega not t and sin omega not t, there are 2 poles at plus omega not and minus omega not. These zeros will depend up on the cosine function and the sin function may be. And now, you observe that the abscissa convergence in all these cases. This is 0 the abscissa convergence is 0. Therefore, the region of convergence is the region to the left of the right most poles. In all these cases you will observe that the region of convergence which is defined by, the abscissa convergence is the region of convergence to the right of the right most poles.

In this case this is there are only 2 poles that means: entire region to the right of this pole is the region of convergence. Entire region to the right of this is the region of convergence e, entire region to the right of this is the region of convergence e that means: the region of convergence is defined by the extreme pole the right most poles that, you are having for the particular function, more about this take up in the next lecture.

To summarize what we have done today is: we said that Fourier Transforms of certain time functions which grow exponentially do not exist and to take care of such situations, we can think of introducing a convergence factor e to the power of minus sigma t. So, instead of finding out the Fourier Transform of f of t, we can think of the Fourier Transform of f of t e to the power of minus sigma t.

And then try to find out the Inverse Fourier Transform and introduce cancel out the e to the power of minus sigma t which we introduce in the first place. So, both these formulas which we have let them Laplace Transformation formula become, we do not have to introduce e to the power of minus sigma t artificially.

If, you introduce a new variable s which is sigma plus j omega; so the Laplace Transformation evolves from such considerations and we have F of s which is given as f of t e to the power of minus st integrated from t from 0 to infinity and Inverse Fourier Laplace Transform is 1 over 2 pi j c minus j infinity to c plus j infinity of f of s e to the power of t ds. And this integration is along the vertical line in the region of convergence.

The region of convergence is defined by: the half plane where real part of s is greater than the particular value the abscissa convergence, the abscissa convergence depends up on the particular function that we have already on hand. And we then took up the consideration of Laplace Transformation of areas important time functions.

In particular we found out the Laplace Transformation of the impulse unit impulse function at the origin which happens to be 1 itself. The Laplace Transformation of the unit step function which is 1 over s a particularly simple relation and e to the power of minus alpha t u t, the Laplace Transform of that is 1 over s plus alpha. And then cos omega not and sine omega not t gives reduce to simple Laplace Transformations of this type: s over s squared omega not squared and omega not squared omega not over s squared plus omega not squared.

We will continue this discussion in the next lecture, by enlarging the class of functions for which, define the Laplace Transformation and also, we will look at some of the important properties of the Laplace Transformation and such.