**Networks and Systems**
**Prof. V.G.K. Murti**
**Department of Electrical Engineering**
**Indian Institute of Technology – Madras**

**Lecture-44**
**Comments about Transient Analysis**

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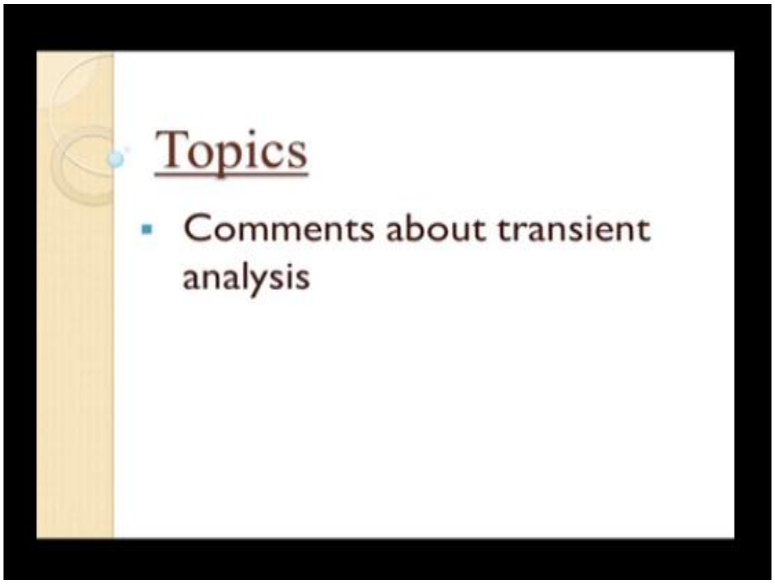


In the last lecture, we saw how the Fourier transform methods can be employed to evaluate the transients in a network. A few comments about these methods: essentially, when we used to find out the transient behavior of a network, it is generally after a switching operation. Let us assume that some excitation function is introduced into the circuit at t equals to 0.

And we wish to find out the response of the network following the switching operation. We should make 2 assumptions first of all let us assume that the network is initially relaxed. That means, it has no initial energy is stored in the network which other words means the capacitors are initially uncharged and the inductors carry no current.

This assumption of the network being initially relaxed does not entail any loss of generality. Because, if you indeed there are initial charges on the capacitors and initial current in the inductors. They can be replaced by equivalent sources and these equivalent sources can be considered to be additional excitations.

The principle of superposition is brought into play to evaluate the responses to the various excitations and adding them up to find the total solution. Therefore, we shall assume that the network is initially relaxed and we are interested in finding over the response of the network following this switching of the excitation at t equal to 0.

A second assumption we will make is that the natural response of the network is such is 1 such that, it dies down the negligible proportion in course of time. This is common to most network that we deal with particular deceptive network and therefore, this is once again does not seriously disturb the generality of this equation.

However, the Fourier methods can also be applied where the natural frequency the network have 0 real part in other words there is sustained natural response oscillation, sinusoid oscillations or a dc component. But the analysis become little more complicated, but we will avoid situation of this type in this course.

That means, we will assume that the natural response decays with time under these assumptions when we have a excitation switched in t equal to 0. We expect the response also to be 0 for up to e equal to 0. Because the network is assumed to be causal and therefore, any excitation is applied t equal to 0 cannot produce a response prior to that.

So, we expect the response to be 0 for up to t equal to 0 and then become nonzero from t equal to 0 onwards for positive t. Now, the Fourier method tells us that any such excitation function can be split up as the sum of tiny sinusoids starting from t equals minus infinity onwards and going up to t equal to infinity.

Now, each 1 of the sinusoids produce a response which can be evaluated using the steady state sinusoid circuit theory methods, that is using the frequency response techniques. Then, all these responses can be added up to find the total response. So, the sum of the steady state responses is what we obtained for the Fourier integral approach.

And it turns out that, the sum of the various steady state responses for the types of networks that we have talked about is also will become 0 up to t equal to 0. So, this is as should be because no response can occur before the application of the excitation and we further observe that, if there are any natural responses terms produced at t equals to minus infinity at the time the excitation starts all the sinusoids starts from t equal to minus infinity.

All these natural responses will decay to negligible proportions by the time t equals to 0 arrives and this is the period which we are interested in. We are interested in finding out the behavior of the network from t equal to 0 onwards. That means, the natural response terms which might have emanated t equals to minus infinity would have decayed negligible proportions, when we come to t equals to 0 onwards.

The sum of the steady state response that we obtain will include transient terms natural responses starting from t equals to 0 that is what we are really interested in. So, the steady state responses will add up to 0 up to t equals to 0. And from t equal to 0 onwards the response that we get the total response that we get include not only the force response but also the transient response or the natural response of the network starting from t equal to 0. That is how it goes.

Now, let us take a second example to illustrate the application of the Fourier theory to a network or system which is characterized by the frequency response. So, let me take filter network and see how the Fourier theory network, Fourier theory can be applied to this network.

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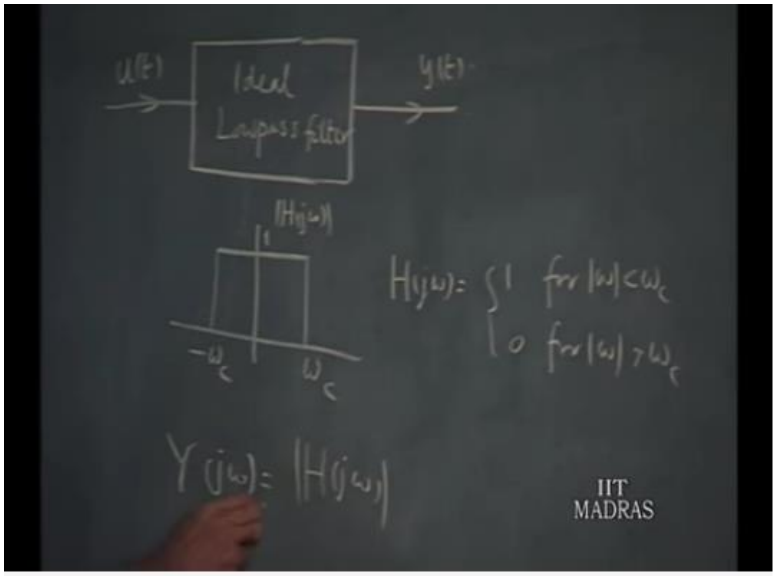


Let us consider an ideal low pass filter and we apply a unit step function as its excitation and we like to find out what is the response is. Now, the ideal low pass filter has a frequency response function like this. So, the frequency response function of the ideal low pass filter can be considered to be 1 for omega magnitude less than omega c for omega magnitude greater than omega c.

That is this is kind of response function that we have now, under these conditions this is the system function of the low pass filter. Therefore, y j omega the Fourier transform of the output is h j omega the system response function frequency response function times the Fourier transform.

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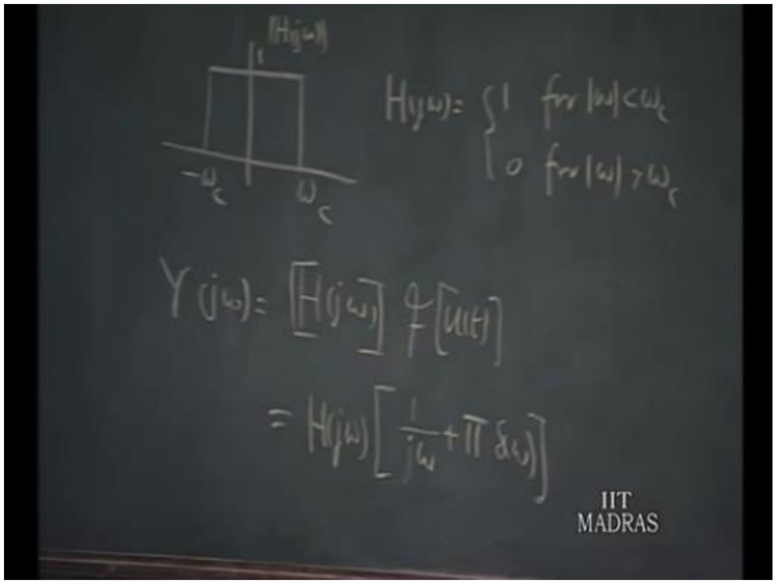


This is h j omega times the Fourier transforms of u of t which is the input function. Therefore, this will be h of j omega times the Fourier transform of the step function is 1 over j omega pi delta omega. So, this is y of j omega, if you find the inverse Fourier transform of this you would naturally get the y of t this is what we are interested in.

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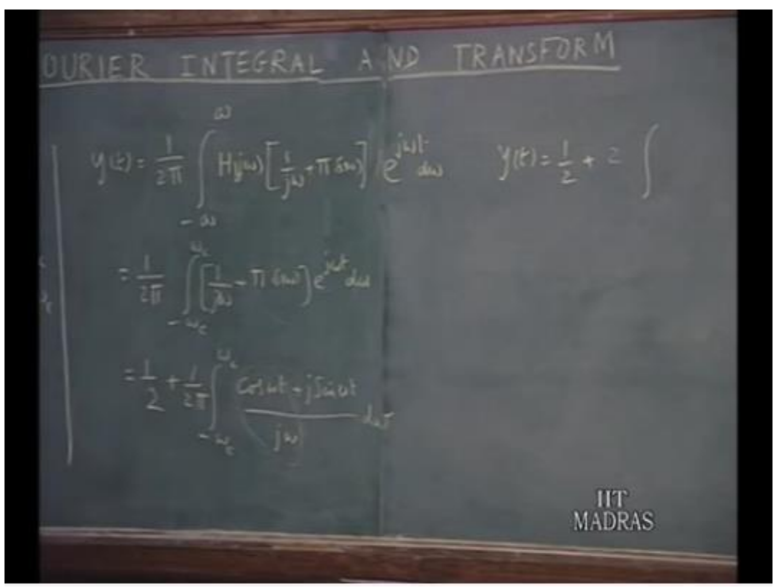


So, y of t is obtained as 1 over 2 pi from minus infinity to plus infinity of h of j omega times 1 over j omega plus pi delta omega d omega e to the power of j omega t d omega. This is the inverse Fourier transform function. Now, we are integrating with respect to omega and we know that h of j omega vanishes when omega magnitude exceeds omega c.

Therefore, we need to carry out this integration from between the limits minus omega c to plus omega c because outside the integral vanishes. Therefore, this minus omega c to plus omega c 1 over j omega plus pi delta omega e to the power of j omega t d omega. Now, we observe as far the second term is concerned pi delta omega is multiplied by e to the power of j omega t dt d omega.

Now, the multiplying factor delta omega we can substitute omega equal to 0 in that. Therefore, this becomes 1 and we are integrating a delta omega over the range which covers omega equals 0. Therefore, that results in pi that means, the result of this portion of the integration will result in pi and therefore, the pi multiplied by 1 over 2 pi and become half plus we have minus omega c to plus omega c e to the power of j omega can be t can be written as, cos omega t plus j sin omega t divided by j omega d omega.

Now, this portion cos omega t divided by j omega is an odd function of omega and we are integrating between the symmetrical limits from minus omega c to plus omega c. Consequently, this is being the odd function and you are integrating between symmetrical limits minus omega c to plus omega c the contribution of this portion become 0.

Therefore, the contribution arises only from sin omega t by omega term and sin omega t by omega t is an even function of omega therefore, i can write this as 2 times 0 to omega c sin omega t by omega d omega. That means, this will become y of t becomes half plus 2 times we must have also 1 over 2 pi from this because 1 over 2 pi times this 1 you must have. So, 1 over 2 pi you must have.

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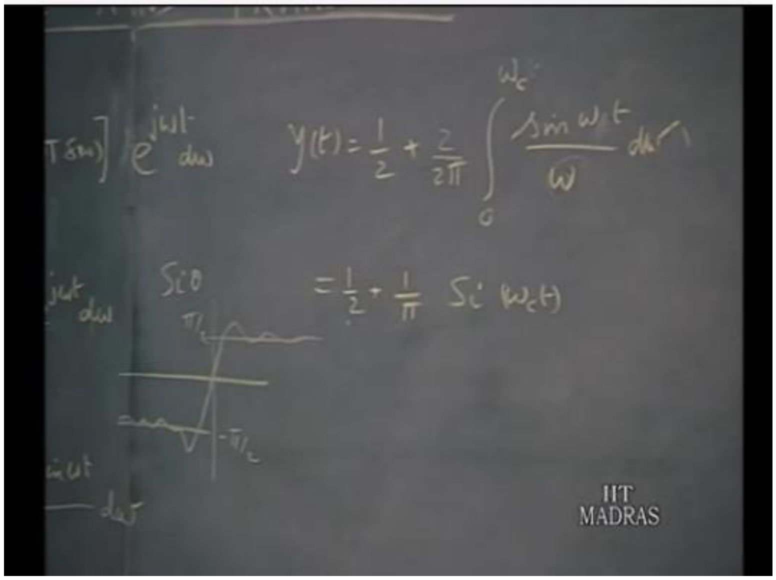


So, 2 times 2 pi 0 to omega c sin omega c t by omega sin omega t by omega t omega. Or this is half of 1 over pi this can be put as sin integral of omega c. Then, after all i can write this i can introduce t here and d omega t here and this can be thought of a sin theta by theta d theta. And once you have this is taken as theta this will become omega ct.

So, this can be put in this form. Now, if you evaluate this you have this is the constant dc term as far sin integral is concerned. You recall the sin integral we have a characteristic like this it goes to pi up on 2 and minus pi up on 2 this is sin integral of theta, which we have thought of discussed earlier.

This is multiplied by pi; that means, this quantity will reach asymptotically minus half of negative values of time and plus half for positive values of time.

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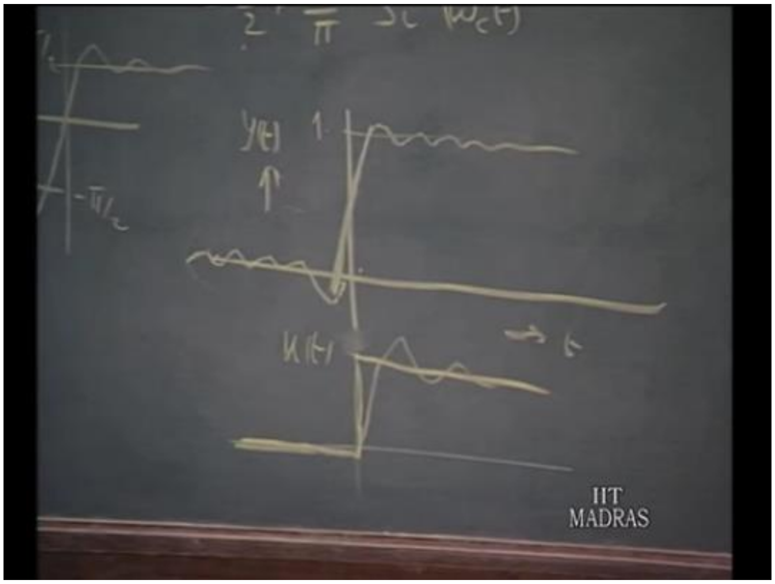


So, if you add to that this half eventually y of t will be like this will be some oscillations like this and eventually, this reaches 1 and this t. So, this will be the response of the ideal low pass filter for unit step input. So, if you have a unit step input here 1 the output will be like this.

And depending up on the cut off frequency omega c for the low pass filter if you make the omega c larger and larger this will become more closer, closer to the step function. That means, the overshoots the oscillation will reduce and this portion of this curve will be steeper and steeper.

So, the more the cut off frequency of the low pass filter; that means, the greater bandwidth of the low pass filter become closer and closer to this. But, interesting thing is you are applying a sin wave like this actually the excitation function is 0 up to this point and you are applying this is your u of t. But the response has is non 0 for negative values of time.

So, even before the excitation is applied there is a response. So, this clearly shows that this is a non causal system. And which means physically this is something which we cannot expect to achieve in the physically and therefore, the ideal low pass filter that we are talking about here this type of characteristic; can only be approximated it can never be realized in practice. Because, we believe that no physical system can give response even before the excitation is applied.

So, this is non causal situation and this is 1 way of showing that an ideal low pass filter of that particular characteristic is not achievable in practice. All 1 head can do is modify this characteristic in such way that, this portion of the response is avoided. That means, all low pass filters that we can build in practice cannot give this type of response. It can only give a response suppose this is the input then only starts giving response on this point onwards may be something like this and this can be avoided.

That means the ideal low pass filter characteristic cannot be obtained in practice. However, my point in showing this giving this example is that Fourier transform methods can be applied to network characterized in terms of frequency response as here and this is very convenient tool in such situations.