Networks and Systems
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Lecture-41
Truncated Sine Wave and Convolution Properties

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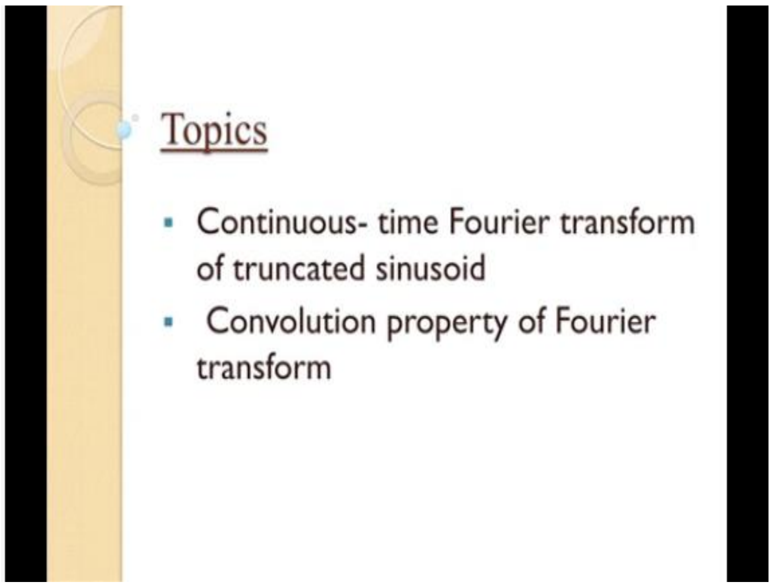

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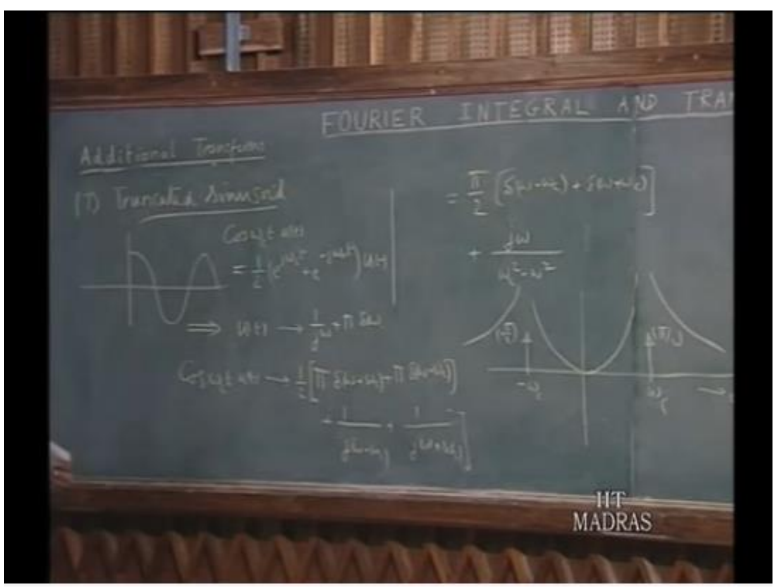

What we are really mean by this is; suppose, we have a function which is only lasting for positive t. So, we are taking cos omega c t u t; that means, for the negative values of time it is cut off. This we are calling truncated sinusoid. To find out the Fourier Transform for this, we recognized that, this is equal to half of e to the power of j omega c t plus e to the power of minus j omega c t times u of t.

Now, that we have the Fourier Transform for u of t, we can find out the Fourier Transform of u of t multiplied by e to the power of j omega c t and e to the power of minus j omega c t. So, this will be; we know that u of t has for the Fourier Transform 1 over j omega plus phi delta omega.

Now, we have u of t multiplied by e to the power of j omega c t which means; instead of omega we have omega minus omega c and delta omega minus omega c. It is also multiplied by half. So, we take that into account. Similarly, u of t is multiplied by e to the power of minus j omega c t.

What it means is; the Fourier Transform of e to the power of minus j omega c t times u of t would be 1 over j omega plus omega c plus pi times delta omega plus omega c. So, on this basis cos omega c t u of t will have the Fourier Transform 1 half of pi delta omega plus omega c plus pi delta omega minus omega c, because of the multiplication by this.

In addition you have 1 over j omega minus omega c plus 1 over j omega plus omega c. You put all these terms together, you can show that this will be equal to pi up on 2 delta omega minus omega c plus delta omega plus omega c t, 2 delta functions plus j omega by omega c squared minus omega squared.

So, the spectrum would be like this; at omega and minus omega c you have delta functions of strength pi up on 2. In addition, you have a continuous variation of the magnitude corresponding to omega over omega c squared minus omega squared. So, something like this we observe that.

We are not taking a pure cosine wave lasting from minus infinity to plus infinity if e to the power of minus j omega t minus e to the power of j omega t dt. If, it had been a cosine wave lasting from minus infinity to plus infinity, there is only 1 signal frequency term that is present therefore, you would have plus pi omega c and minus omega c. This is not a pure cosine wave; it is cut off for negative values of t.

Therefore, it is not a periodic phenomena lasting from minus infinity to plus infinity. Therefore, you have other frequency components, not only the infinite the coefficient density plus omega c and minus omega c plus the continuous band of frequencies with various magnitudes as shown in this.

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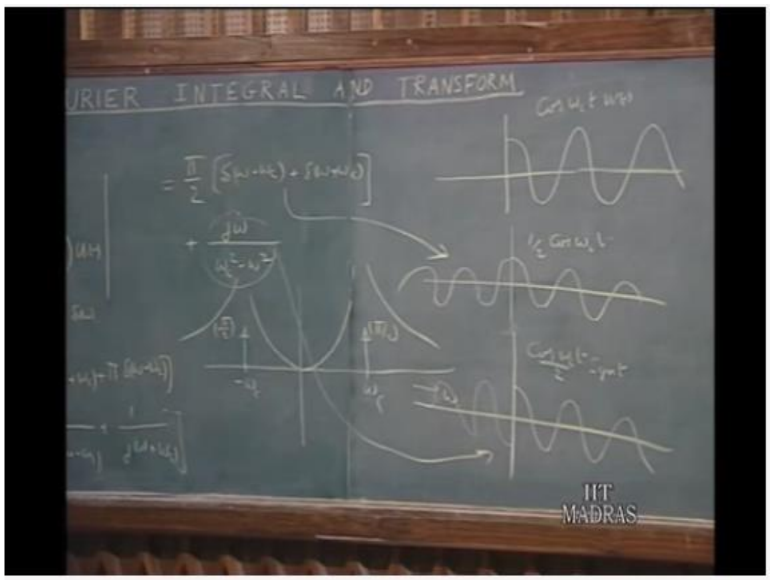

In fact, you can show that, if I have a cosine omega c t u t, you can think of this as the sum of a cosine function which last forever. This is half cos omega c t plus the same function repeating for positive half under by the negative of this occurring for the negative t. So, this is cos omega c t divided by 2 multiplied by the signum function.

So, it is this part which contributes the delta functions. It is this part which contributes to this spectrum. So, the delta part corresponds to this and the continuous frequency. So, that is how it goes. So, a pure cos omega c t u t if, it is a cos omega c t as such without multiplication by u t, it would have been pure impulses at these 2 frequencies.

But, because it is not periodic function in the sense it is lasting from minus infinity to plus infinity, we have the additional portion of the spectrum like this. Similarly, you can derive the spectrum for sin omega c t u t, but we will not do that in this, leave that as an exercise for you.

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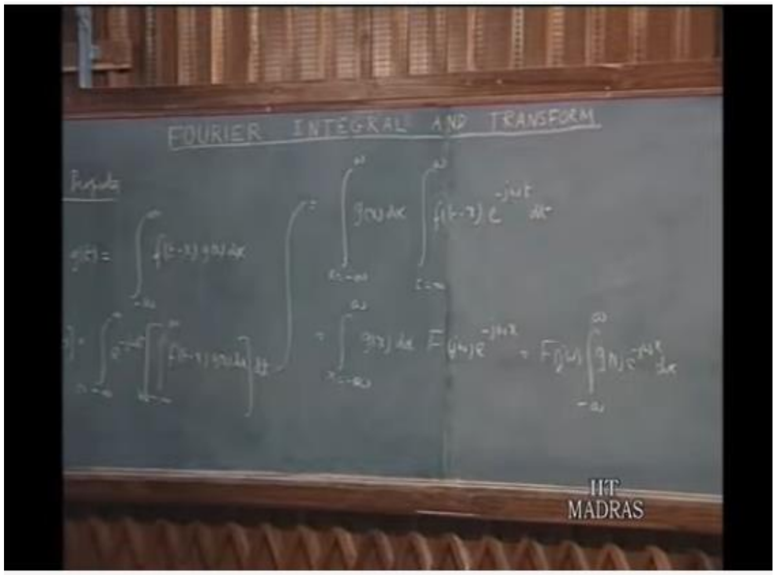

We will discuss 1 additional property in this lecture, which relates the convolution. You recall that the convolution of f of t; 2 time functions f of t and g of t have been defined to be f of t minus x g of x dx from minus infinity to plus infinity. So, this is the function of time and this is defined in this manner; f of t minus g of x dx.

The physical meaning of convolution etcetera we have discussed earlier. Let us try to find out the Fourier Transform of this, in terms of the Fourier Transform of f of t and g of t. So, the Fourier Transform of f t g t convolve g t can be written as; minus infinity to plus infinity of e to the power of minus j omega t times this.

This is the function of time now minus infinity to plus infinity of f of t minus x g of x dx. This is the function of time, the new function of time dt. This is what the Fourier Transform of this would be. This is integration with respect to time t this is integration with respect to x. Now, let us interchange the limits of integration.

This is t from minus infinity to plus infinity, this is x minus infinity to plus infinity. Let me interchange the limits of integration; let us interchange the order of integration.

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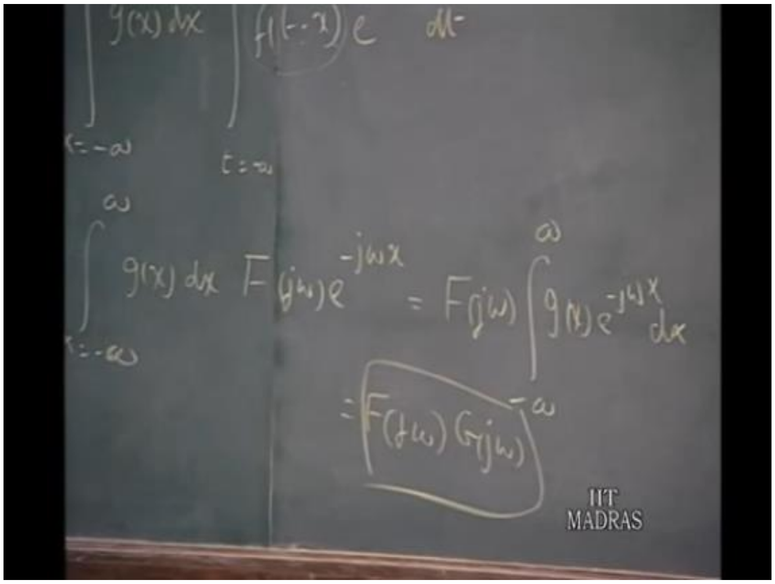

I would like to do integration first with respect to t and then later with respect to x. So, all terms which are independent of t can be put outside. Therefore, I put g x dx and those which are depended on t, I will put inside the first integral which is to be done; f of t minus x e to the power of minus j omega t.

Now, from the property of the translation in the time domain, if f of t has the Fourier Transform of F of j omega, when you have f of t minus x, the Fourier Transform of this we know is F of j omega multiplied by e to the power of j omega x. So, this turns out to be x from minus infinity to plus infinity of g x dx e to the power of F of j omega multiplied by e to the power of minus j omega.

And in this we are integrating with respect to x. So, i can write this as F of j omega can be pulled outside the integration sign and then, you have minus infinity to plus infinity of g of x e to the power of minus j omega x, which is the standard formula for the Fourier Transform of g of t, instead of t, I have put x is the dummy index.

Therefore, this is indeed G of j omega therefore; F of j omega multiplied by G of j omega is the Fourier Transform of this. So, that is the nice result, in the sense that, you have the convolution in the time domain corresponds to multiplication the transform domain.

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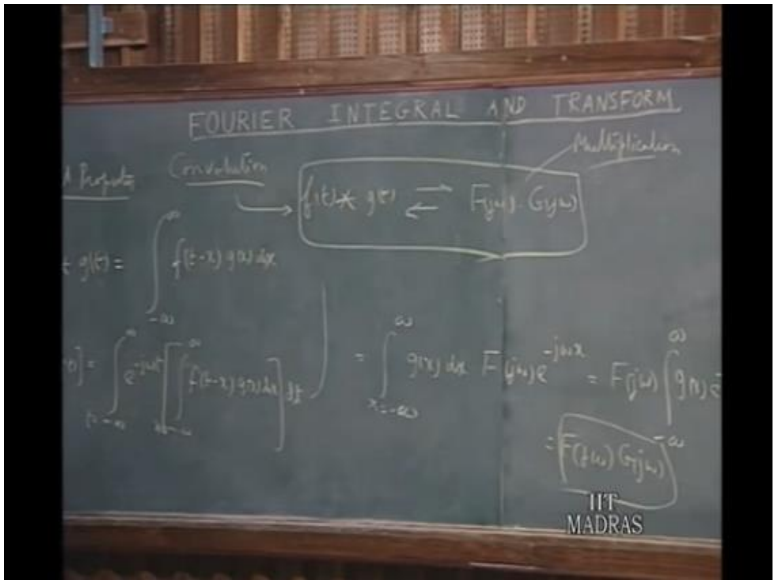

So, f of t convolved with g of t which is complicated integral operation is, carries over in the frequency domain or the transform domain has pure multiplication of the 2 time functions. So, convolution in the time domain corresponds to multiplication in the transform. This has a interesting application in system and network theory which will, come across as go along.

And in the convolution formula here, when you take the, apply this Fourier Transforms; Fourier Transform theory, all functions of time for assumed to be lasting from minus infinity to plus infinity. So, the limits must be taken from minus infinity to plus infinity. This may be different when you go to Laplace Transform later on.

But for the Fourier Transform theory is concerned, the convolution must take minus infinity to plus infinity. And 2 functions which are convolved in time as far the transform domain is concerned, it is equivalent to multiplication in the transform domain. In this lecture, we first of all looked at the Fourier Transforms of periodic functions of time.

To start with a pure dc, will have in the Fourier Transform is spike that is, a impulse at the origin and the Fourier Transform is 0 everywhere else indicating that, concentration of energy is only at 1 frequency that is the, dc. Then, we took up a sinusoidal function of time angular frequency omega 0 and we found in the Fourier spectrum we have, 2 delta functions at plus omega c and minus omega c.

Therefore, the spectrum consists of 2 impulses sitting at these frequencies; plus omega c and minus omega c. That means: there is no energy at all other frequencies. We extended this concept to a periodic function of time, which admits the Fourier Series representation. In the Fourier Series spectrum consists of lines having a finite amplitudes that is, the various coefficients; C n coefficients and this is a line spectrum in the case of Fourier Series.

But in the case of Fourier integral, it is continued to be a line spectrum because, the energy is only at the discrete frequency. But, now since we are talking in terms of coefficients density, the line spectrum consists of series of impulses sitting at the various discrete frequencies. And each strength of each impulse is related to the Fourier coefficients, in the Fourier Series representation by a factor 2 pi.

That means: instead of C 0 in the Fourier coefficient, we have 2 pi times C 0 being the strength of impulse in the Fourier spectrum. After having dealt with, the periodic functions of time, we looked at functions of times at the unit step function or a sinusoidal function, which last only for positive t and then try to look at the Fourier spectrum of this.

We notice that, this is not a pure periodic phenomena, this has an a periodic attached to it. Therefore, you have in this spectrum, concentration of energy at 1 or more discrete frequencies corresponding to the basic character of the time function. If, it is unit step function, the concentration of energy is dc. If, is cosine omega c t u t then, the concentration of energy is plus omega c and minus omega c.

But, since these are not periodic functions, their cut off for negative values of time, you have also a portion of the spectrum, which is continuous, which is representative of periodic function. Therefore, we have continuous spectrum as well as 1 or more lines representing the particular frequencies which are present.

We saw that, in the analysis of such functions, the signum functions play the important role. Signum function is the 1 which, has got minus 1 for negative t and plus 1 for positive t. And we saw a signum function of the Fourier Transform 2 upon j omega. Lastly, we look at the convolution property in the time domain and how it is going over in the transform domain, a 2 time functions f of t and g of t.

When they are convoluted in the time domain, in the transform domain the result in function of time, will be having a Fourier Transform which is, the product of the Fourier Transform of the individual components, which are convolved together. In particular, f t star g t which is the convolution of f of t and g of t, has the Fourier Transform which is, F j omega times G of j omega.

This particular property is very useful in network and system study and we will see later. In the next lecture, we start with an example illustrating the convolution property and we will go on from there