Networks and Systems
Prof. V.G.K. Murti
Department of Electrical Engineering
Indian Institute of Technology – Madras

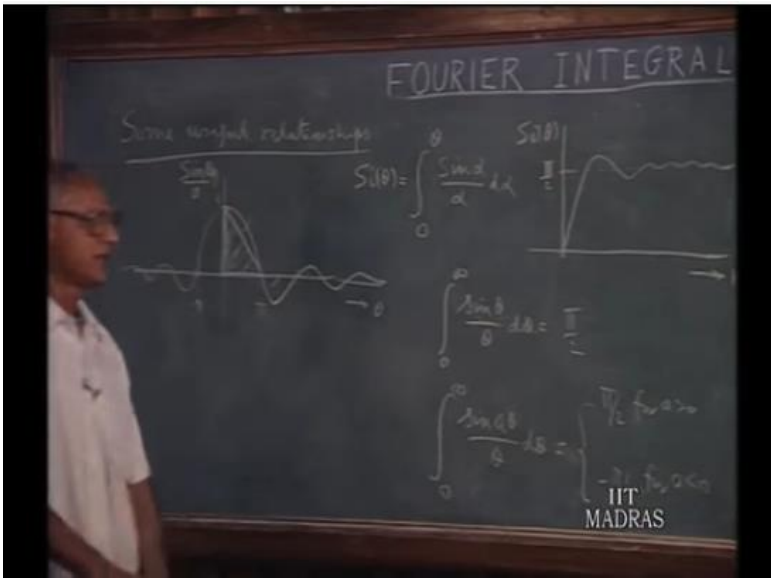
Lecture-38
Helpful Relationships for Inverse Fourier Transform

(Refer Slide Time: 00:12)

So far, we have been talking about signals with a finite energy content and signals which are absolutely integrable over the infinite range of time from minus infinity to plus infinity. You might recall, that I mentioned earlier that we can extend the concept of the Fourier transform to signals which are not absolutely integrable over the infinite range and in particular I mentioned that signals like u of t a dc term, sin omega c t cos omega c t.

Such functions also admit Fourier transforms provided we use impulse functions. Which are justified from the theory of distributions in mathematics? We would now, like to familiarize ourselves with some of the operational rules involving such functions. We will not go in deep mathematical details of this we will just assume some results and use them to derive the Fourier transform of signals, whose energy content is not lesserly infinite or whose is not finite.

(Refer Slide Time: 01:34)



So, to start with we like to have some useful relations. We have discussed earlier sin theta by theta. This will have a variation like this, with an amplitude 1 and this will be pi and this is minus pi. We are interested in the integral such a quantity. Suppose we say 0 to theta of say sin alpha by alpha d alpha, such an integral is referred to a sine integral of theta.

This is given in the symbol si of theta; sin integral of theta sin alpha by alpha from 0 to theta. And the variation of this is pronounced as sine integral of theta versus theta would be something like this; asymptotically it reaches the value which is equal to pi by 2. So, this is the relationship which we assume, that means if you start from this point and go on integrating it reaches a value pi by 2.

That means, the integral the area under this curve starting from theta equal to 0 to theta equal to infinity will be pi by 2. It is easy to remember this particular value because if you take this triangle, this is pi this is 1, the area of the triangle is pi by 2. So, the area of the triangle is pi by 2 and that is also the area under the curve from 0 to infinity.

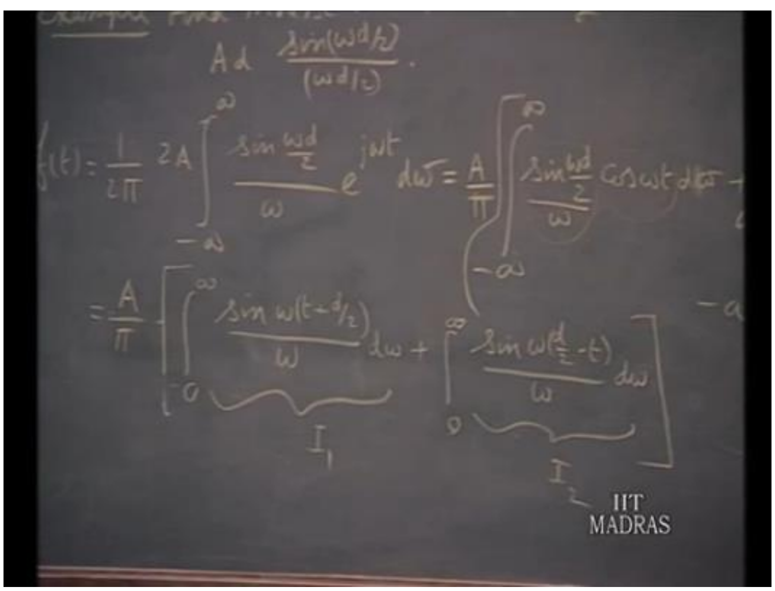
Which means that, the rest of the area; this area, this 1 and this 1 all add up to 0.The positive areas are cancelled out by the negative areas and therefore, the areas the curves sin theta by theta from 0 to infinity becomes pi by 2. So, we have the relation; therefore, 0 to infinity of sin theta by theta d theta equals to pi by 2.

Now, sometimes we like to have instead of theta we have sin a theta by theta d theta 0 to infinity. Now, if a is positive then certainly we can say d of a theta by a theta then this same relation as this. And the limits are not interchanged therefore this will also be equal to pi by 2.

Because a theta can be replaced by another variable x, but if a is negative then no doubt d a theta by a theta a can be cancelled out. But if a is negative instead of plus infinity you have minus infinity. And you know, sin theta by theta is a even function of time, but you are integrating the negative direction; therefore, it will be minus pi by two.

So, the result is this value pi by 2 for a greater than 0 equals minus pi by 2 for a less than 0. Or in other words, if you extend this sin theta sin integral theta in the negative it will be like this. And it reaches minus pi by 2 eventually.

(Refer Slide Time: 06:05)



This is the relationship which is useful for us in our work. Just illustrate this particular rule. Let us consider a example once again. Find inverse Fourier transform of A d sin omega d upon 2 by omega d up on 2. We know this results, we know that sin omega d up on 2 by omega d up on 2 times A d is the Fourier transform of pulse function of amplitude a and width d centered around the origin.

But we like to find out this independently. So let us do this, so f of t would be 1 over 2 pi and i take out this d by 2 outside; therefore, it will be 2 A by t 2 A by 2 A. Integral from minus infinity to plus infinity of f of j omega which is sin omega upon 2 divided by omega e to the power of j omega t d omega.

That is, the inverse Fourier transform of this function. This i can write as a upon pi minus infinity to plus infinity sin omega d up on 2 cos omega t dt d omega plus minus infinity to plus infinity sin omega d by 2 sin omega t d omega and j sign in front j sign in front. Now, this also omega by continue to have here.

What you observe here is, as far second integral is concerned sin omega d divided by omega this is an even function of omega, this is the odd function of omega. And you are integrating between symmetrical limits from minus infinity to plus infinity. So, the contribution of this integral from minus infinity to 0 would be exactly equivalent and opposite to the contribution that arises from 0 to infinity.

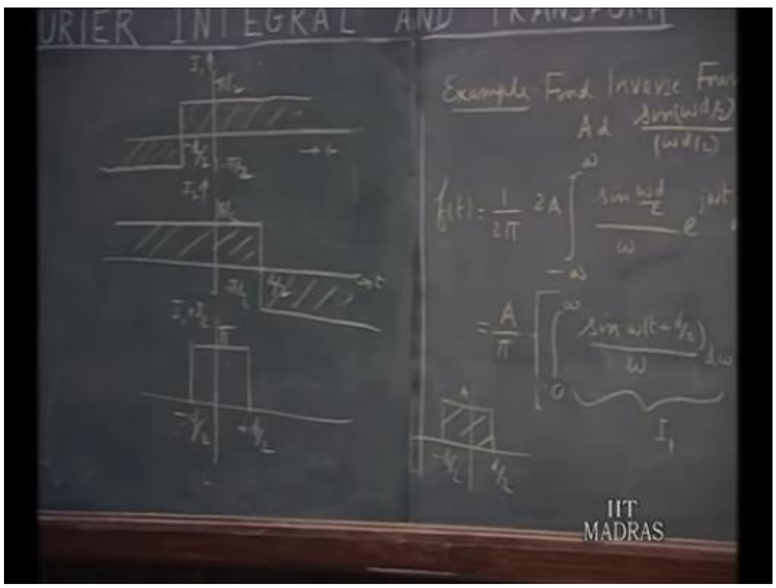
So, since this entire function is an odd function of omega and you are integrating between symmetrical limits this goes to 0. So, what we have left with is only this and this can be written further as you have sin theta and cos theta; therefore, you can combine these 2 as sin alpha plus beta plus sin alpha minus beta type of formulations you can make.

Therefore, I can write this as a by pi and then you have 2 also will come because you are going to express this as sin alpha plus beta and sin alpha minus beta. So, you have sin of omega t plus d by 2 divided by omega, that is; d omega and minus infinity to plus infinity of sin omega d by 2 minus t by omega d omega d.

So, you have 2 such terms. Now, we can also observe here first of all before going further. This is an even function of omega, this is an even function of omega; therefore, whatever integral you get minus infinity to plus infinity would be twice the value of the integral which would have got if you extended if you integrate this from 0 to infinity. So, i can write this further as 4 a upon pi then 0 to infinity and 0 to infinity here, also that also, I can write this here;

So, I will do that. Now, I think this should be corrected first of all since I am doing the integral 0 to infinity. I should have 2 a upon pi, but sin omega t d by 2 sin omega d by 2 minus t will give me 2 times sin omega d up on 2 cos omega t. therefore, I must divide by 2 further; therefore, finally this will become a upon pi. So, a upon pi this integral from sin omega t plus d up on 2 by omega and sin omega d by 2 minus t omega.

(Refer Slide Time: 12:33)



Suppose I call this integral i1 and this integral i2. And let us see, what their values would be for different values of t. For this i make use of this relation here, sin a theta by theta d theta will be pi by 2 for a positive and minus pi by 2 for a negative. Here, if i plo t i1, the value of i1 we have sin omega something by omega d omega something very similar to this and instead of a i have t plus d upon 2.

So, as long as t plus d upon 2 is positive it is pi by 2 as long as t plus d by 2 is negative it is minus pi by 2. That means this value of this integral will be like this is pi by 2. This is minus d, this is t and this is minus pi by 2. So, as far i1 is concerned it will have positive value of pi by 2 for t greater than minus d. So, that t plus d by 2 is greater than 0 and as long as t plus d by 2 is negative it has minus pi by 2 that is the value of i1.

Value of i2 again the same formulation should be used we have sin omega by omega d omega but omega is multiplied by d by 2 minus t, look at this expression again, sin a theta by theta d theta is pi by 2 for a positive minus pi by 2 for a negative. So, as long as d by 2 minus t is positive then it is pi by 2; otherwise, it is negative that means, as long as t does not exceed d by 2.

Then it is going to be pi by 2; otherwise, it is minus pi by 2. This will be the relation that you get finally as far as i2 is concerned. So, what you have got for when you add i1 plus i2 now, no longer need this. So, if you add i1 and i2 add up these 2 things then this positive area will be cancelled by the negative area here and here so we have left with a pulse like quantity which will have a value of pi by 2 and pi by 2 this is pi by 2 this is minus pi by 2.

Therefore, this is pi and this last from minus d by 2 to plus d by 2. I should have put here, because t plus d by 2 and minus d by 2. I should have written here minus d up on 2 and this also minus d upon 2. Therefore, this pulse extends from minus d upon 2 to plus d up on 2 and therefore, this is the value of i1 by i2; therefore, you multiply by a by pi f of t would be a pulse of width a lasting from minus d by 2 plus d by 2.

A result which would have expected from this. So, what we have really done is we know, in the forward direction this is the pulse this is the Fourier transform. We demonstrate it validity in the reverse direction making use of the relation of this sin integral as the arguments goes to infinity. Illustrated, this because we may like to use the this type of formulation later on in our further work.