Networks and Systems
Prof. V.G.K. Murti
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture-37
Energy Considerations II

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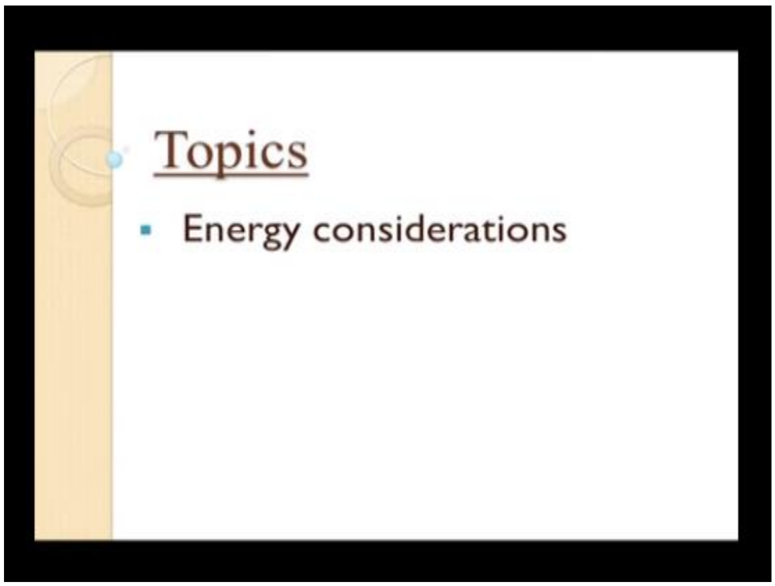

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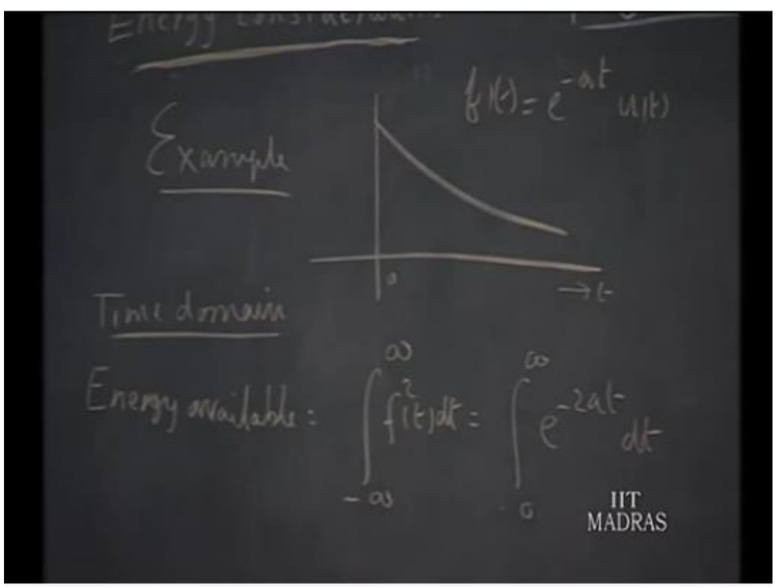

Take this example; we have f of t e to the power of minus a t u t. Now, we like to calculate the energy content in the signal energy available from the signal in 2 ways. Time domain doing with the time domain energy available integral from minus infinity to plus infinity of f square t dt. That is; minus infinity to plus infinity of e to the power of minus 2 a t dt. But this lower limit is not correct because f square t exists only from t equal to 0 to infinity; therefore, we have to write substitute 0 to infinity e to the power of minus 2 a t dt.

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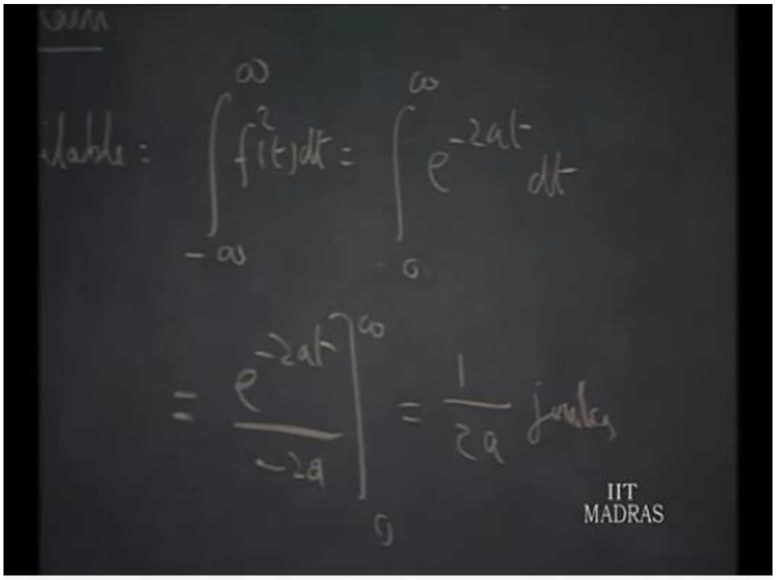

Therefore, this will become e to the power of minus 2 a t divided by minus 2 a evaluated between 0 and infinity, but the upper limit this is going to be 0 because are taking to be a the positive quantity and the lower limit of this is 1 this will become 1 over 2 a so 1 over 2 a joules is the total energy available from the signal.

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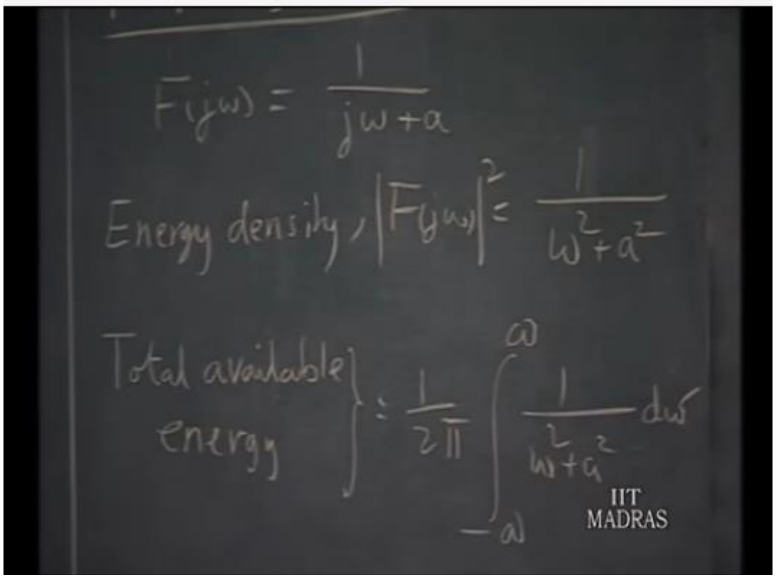

Let us work out, in frequency domain f of j omega is 1 over j omega plus a, but we are talking in terms of energy therefore energy density is f of j omega magnitude squared is 1 over omega square plus a squared so many joules cycles per second. So, the total energy available from the signal is the 1 over 2 pi times the integral minus infinity to plus infinity 1 over omega squared a squared d omega.

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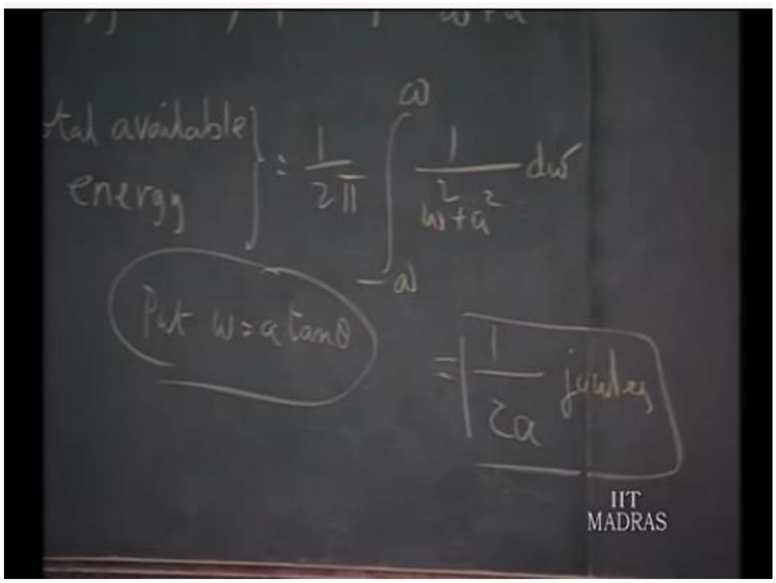

And this can be shown by substituting omega put omega as a time tan theta. And carry out the integration and you can show this is indeed 1 over 2 a, which is the exactly the result that we have obtained earlier.

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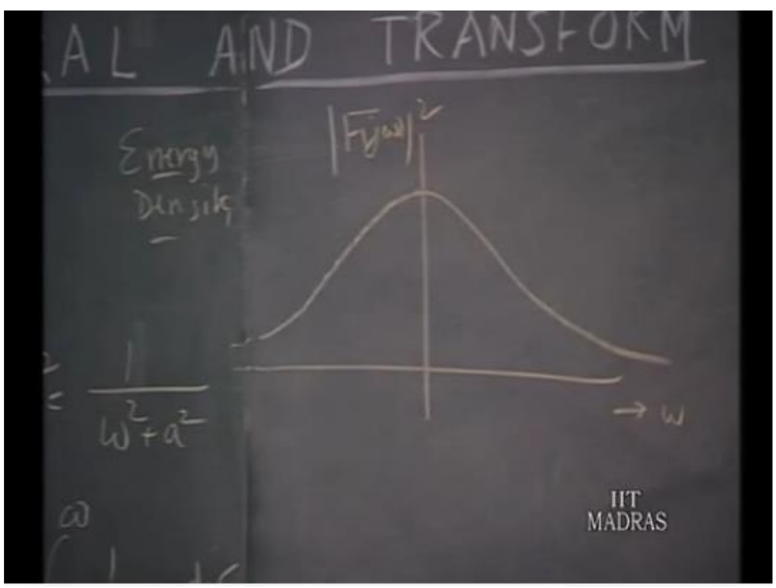

If you plot f of j omega squared versus omega, the energy density is a function of frequency omega. You will get a curve like this. So, this is the energy density. So, this gives us an idea where the energy is concentrated at what frequency band there is more energy. Now you can see, there is a lot of energy in the low frequency band centered around the D. C.

But higher frequency the energy is somewhat less. This additional information is obtained through the Fourier transform analysis as compared with time domain analysis.

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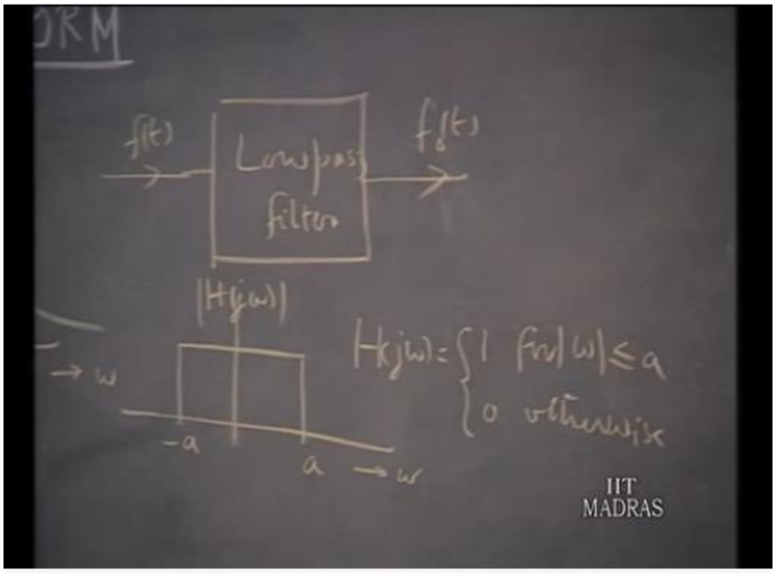

Let us see, how this such a concept can be made use of in practice. Let us assume that, this particular f of t which we had the exponential signal is put through a low pass filter. And we get output signal f 0 of t this low pass filter has a characteristic like this. H of j omega magnitude a low pass filter essentially allows all frequencies up to a certain point then shuts off all other frequency components.

So, let us imagine that a low pass filter here has a cutoff frequency equal to a and minus a. In other words in the spectrum that is available for the given f of t all frequency components having a magnitude omega greater than a become 0. That means, H of j omega we are talking about the transfer function or the system function relating f 0 of t and f 0 f t is equal to 1 for omega less than or equal to a and 0 otherwise.

So, essentially what we are doing is, this low pass filter is such cuts off all frequency components beyond certain value a.

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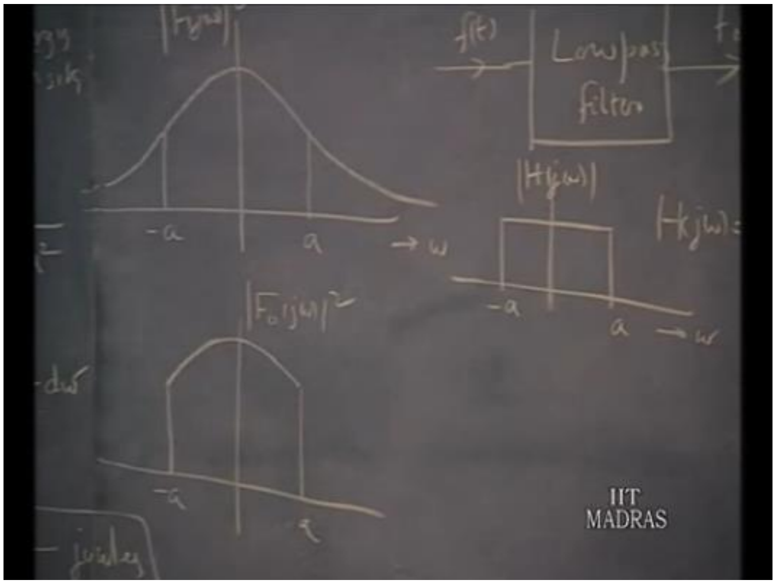

So, if this is a, this is minus a, as far the output is concerned f 0 of t. The corresponding energy density f 0 of j omega squared would be the 1 which contains frequency components within this band only. That means, it will have only this minus a plus a the rest of the terms are being cut off. And so, suppose I want to know what is the energy available from f 0 of t? Then you have to take the same function but integrate between minus a to plus a rather than from minus infinity to plus infinity.

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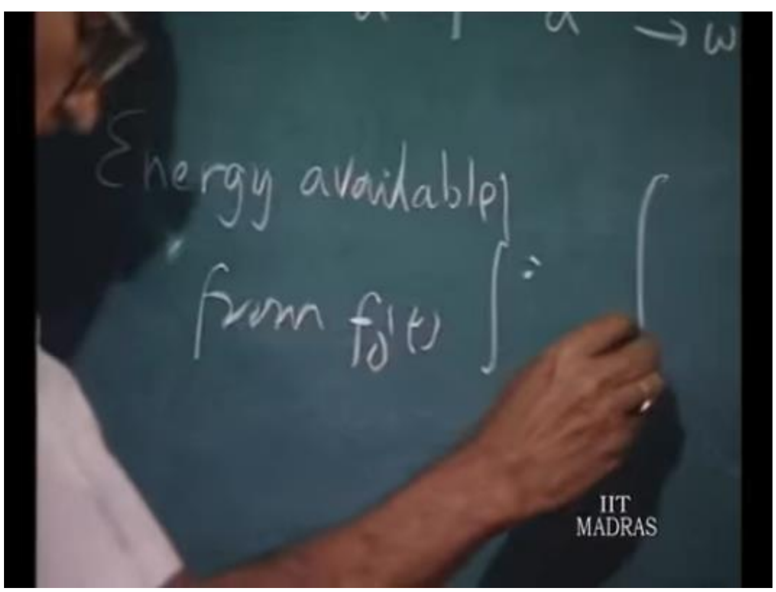

So, we have the energy available from f 0 of t equals the integral of 1 over omega square plus a square. That is the energy density into d omega over 2 pi the integral now, from minus a to plus a instead from minus infinity to plus infinity.

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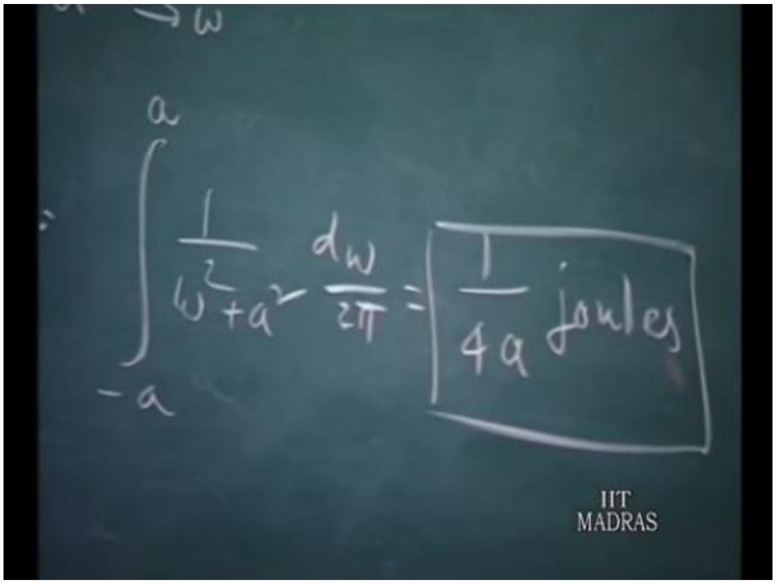

And then, making use of the standard substitution that omega equals to a time theta this can be shown to be 1 over 4 a joule.

It means, that out of the total energy available from the input signal ft which is 1 over 2 a joules 1 half of that energy resides in the band, frequency band omega from minus a to plus a. So, since when signals are transmitted through linear system. Like a low pass filter and these systems are characterized quite nicely in terms of, the system function H of j omega.

And if you like to find out, the properties of the signals that come out the system or the response of the system then, frequency analysis in this manner trans out to be very convenient tool because filters are like this characterized in terms of the frequency response. Not in terms of their time response and therefore, the energy available from the f 0 t can be more conveniently to handle in terms of frequency characteristic of the signal in terms of the spectrum as shown here.

If we wanted to the same analysis in term of time domain, then it would be much more difficult because you have to characterize the filter. In terms, of the time response which can be done but it much more complicated. So, these examples illustrate how we can utilize the energy concepts of also in finding out the characteristic of various signals.