Networks and Systems
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Lecture-36
Energy Considerations

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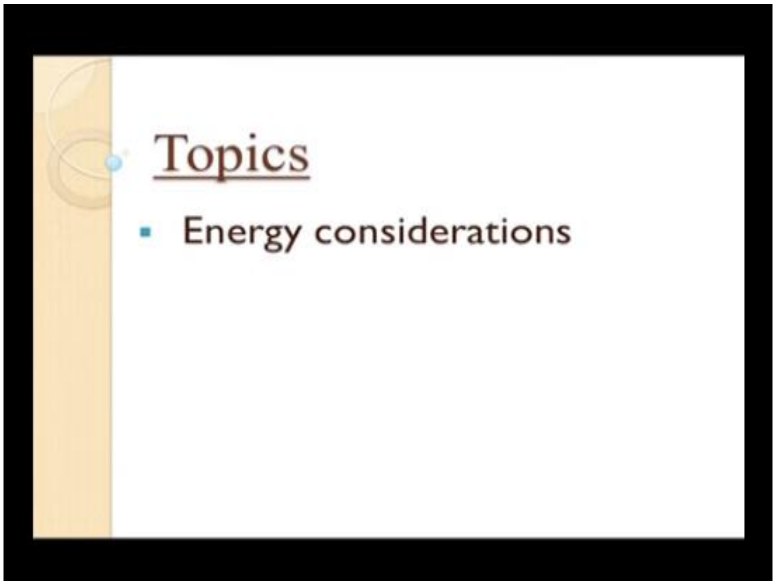

In this lecture, we will continue our discussion of the various properties of the Fourier transform. To start with let us consider aspects related to energy.

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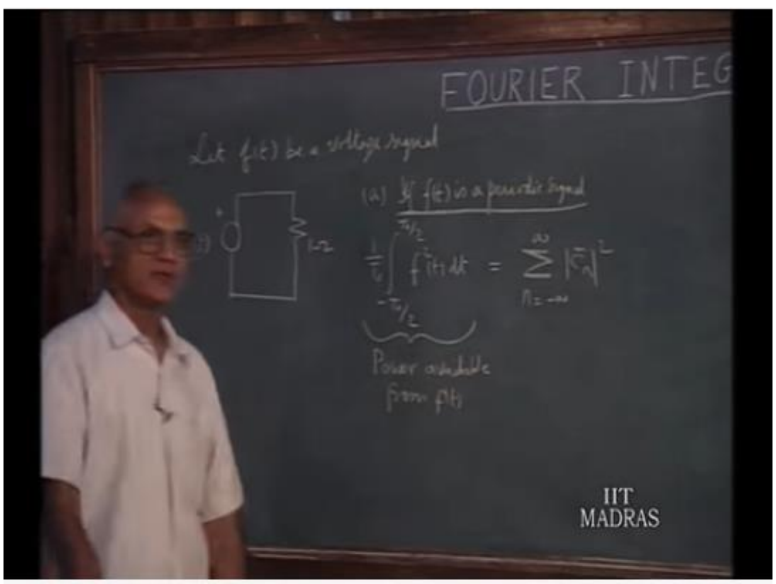

Suppose f of t be a voltage signal. Now, let us imagine that such a voltage is given to 1 ohm resister. Then if f of t is a periodic signal then the power dissipated in the 1 ohm resister is from minus t up on 2 to plus t up on 2 of f square t dt. The average value 1 over t 0 this is called the power dissipated in the signal or we often refer to power available from f of t.

So, irrespective whether the signal is a voltage signal or something else, normally; we refer to this as power available from f of t of course it is quite appropriate if f of t voltage signal. Even otherwise, we can refer to this as a power available from f of t. And this is a periodic signal and we saw that this quantity power available from the signal can also be obtained from the Fourier coefficients as n from minus infinity to plus infinity of cn magnitude square.

This is the relation which we already discussed when talked about the Fourier series expansion of the periodic signal.

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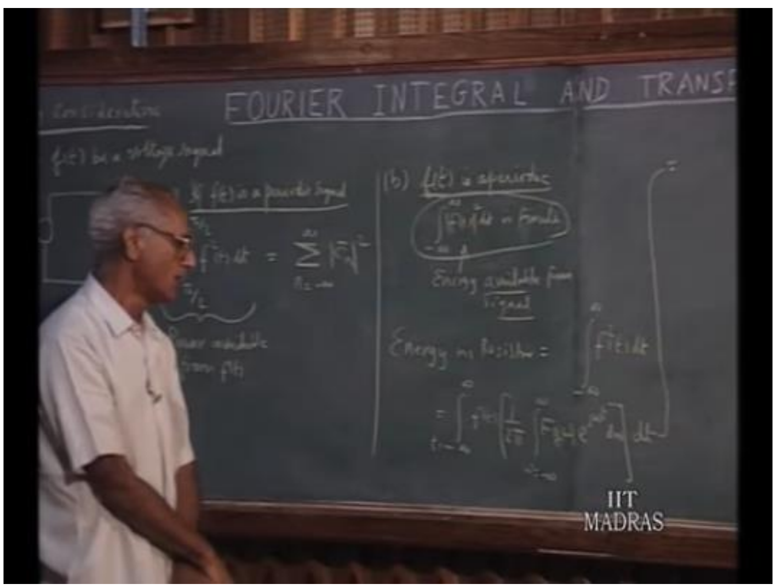

Now, what we like to see is how the concept can be extended in the case of f of t which is not periodic, f of t is a periodic. And we also assume that it has got a finite amount of energy that in the sense that minus infinity to plus infinity of f of t magnitude dt is finite. This is the restriction which we already imposed on finding the Fourier transform of the various signal that we have considered so far and we also assume that this is also true.

So, this is so, then when you divide by t 0, when t 0 goes to infinity then the power become 0. So when we talk about a periodic signals of this type where the square of the integral is over the infinite band of frequencies infinite time is finite then there is no meaning in talking about power because power become 0.

So, all that we can do in such cases is only calculate the value of the integral f square t dt and that has the dimensions of energy because we are not dividing by t 0. So, this is energy in the signal is finite is non 0. Energy available from the signal so, when we talk about any periodic a signals it would be more convenient for us to talk in terms of energy available from the signal therefore we will have the this particular heading for this topic energy consideration.

So, in the case of periodic signals we talk about energy available from the signal where we are talking about periodic signals, we talked about power available from the signal. So, we like to see what is the energy available from the signal in terms of the Fourier transform. So, the energy that is dissipated in the 1 ohm resister is given by this.

So, that energy in 1 ohm resister, energy dissipated in the 1 ohm resister is minus infinity to plus infinity of f square t dt. And this we like to write as minus infinity to plus infinity t is running from minus infinity to plus infinity f of t and other f of t or use the inverse transform formula. Which is 1 over 2 pi omega from minus infinity to plus infinity f of j omega e to the power of j omega t d omega.

This is the expression for the f of t so, instead of writing f square t i multiply f of t by the another equivalent expression for f of t times dt. So, this has got 2 integrals 1 is on time domain and 1 in the frequency domain. What I like to do is interchange the order of integration. That means, instead of integrating the omega domain first and later on the time domain. I like to reverse the role i like to integrate the expression in terms of time domain first and then frequency domain later.

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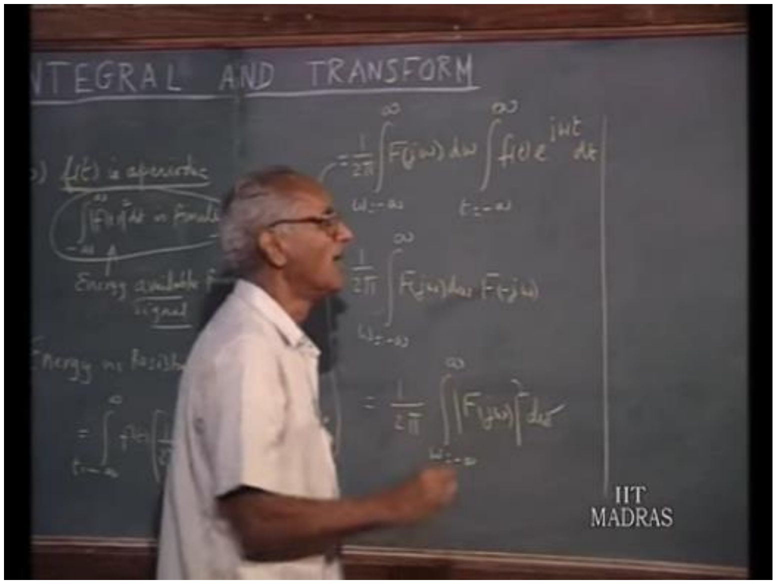

So, if I do that then i want to integrate in the frequency domain at the second stage. So, I put outside and all terms involving omega which do not involve time can be brought outside 1 over 2 pi come out of course, f of j omega will come and the rest of the terms are, f of t e to the power of j omega t and of course d omega is also outside. And you have dt f of t e to the power of j omega t these are the terms that are left.

So, in the time integration t minus infinity to plus infinity I have f of t e to the power of j omega t dt. Now let us, look at this if it had been f of t e to the power of minus j omega t dt it would have been straightaway f of j omega. But, instead of minus j omega t we have plus j omega t. Therefore, this whole expression now become f of minus j omega so this can be written as 1 over 2 pi omega from minus infinity to plus infinity f of j omega d omega and this quantity is f of minus j omega.

And we already observed that for real functions of time your f of t is a real function of time, f of minus j omega is the conjugate of f of j omega. So therefore, this will become 1 over 2 pi omega from minus infinity to plus infinity of f of j omega magnitude square d omega. So, the power available, energy available from the signal is given in time domain in this fashion and frequency domain in this fashion.

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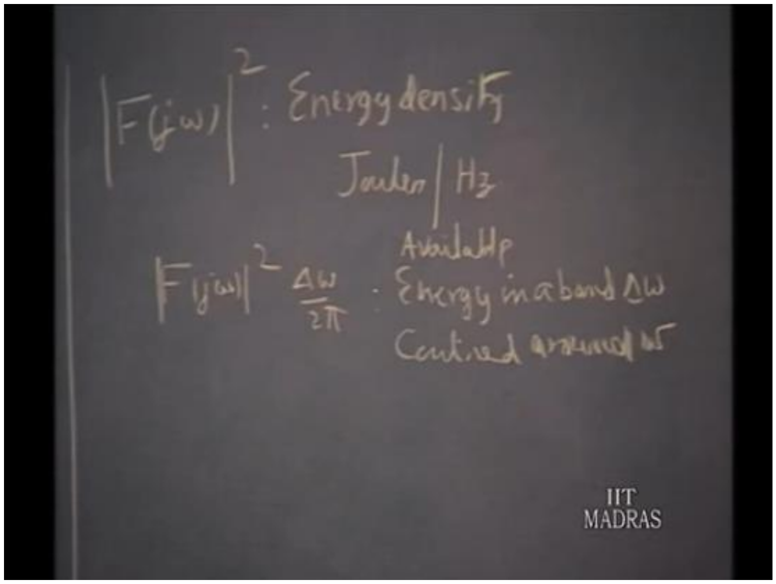

Now, f of j omega squared magnitude squared can be thought of as a energy density. What it means is; around the band, around omega. If we consider a small band of frequency delta f, f j omega squared time delta f is the energy concentrated in that band centered around omega. So, this energy density is expressed in terms of joule per hz.

So, in a small band of frequency centered around omega this will be f j omega squared magnitude squared df which is delta omega by 2 pi is the energy available. Available energy in a band delta omega centered around omega. So, if you take a small band of frequencies centered around omega, small delta omega this is the energy that is available all we have to do is; therefore, to find the total energy.

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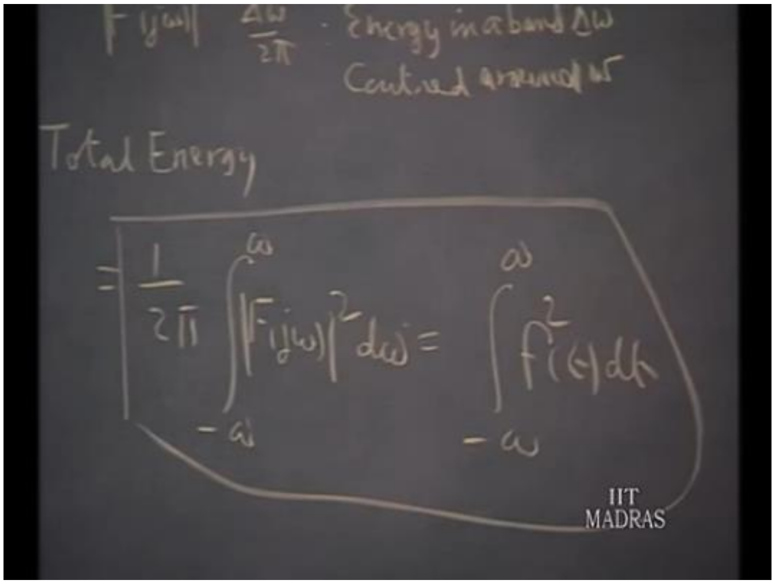

We have to sum up all such energy say quanta in different bands and this become this integral, that becomes 1 over 2 pi minus infinity to plus infinity of f of j omega magnitude square. So, this is an interesting relation that f of t this of course is, minus infinity to plus infinity of f square t dt. So, you can calculate the energy in time domain minus infinity to plus infinity f square t dt.

That is the energy that is called energy available from the signal f of t. You also calculate the energy available from the signal frequency domain in this fashion. f j omega squared is the energy density, energy available per cycle per second. So, you have to multiply by frequency d omega by 2 pi integrate over them whole range of frequencies you get the total energy available from the signal.

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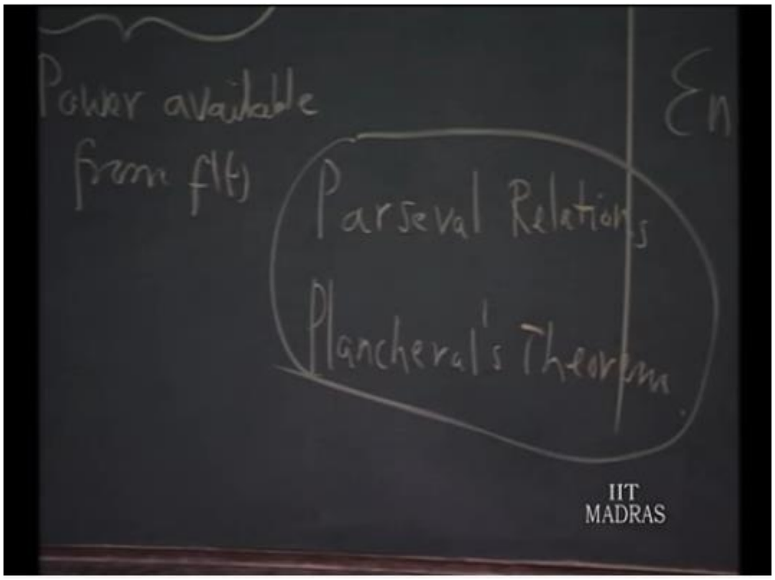

These relations, relation of the periodic case and this relation for the periodic case are usually referred to Parseval's Relation. It is also case refer to Parseval's theorem particularly in the case of periodic case. So, Parseval's Relation in the case of, a periodic signal is in terms of the Fourier coefficients in the case of a periodic signals in terms of integral; involving the energy density f of j omega magnitude squared d omega.

To summarize discussion here, when you talk about a periodic signal with finite amount of energy, then you cannot talk about the power available from the signal which is going to 0. Because, the base is t 0 is going to be infinity the average power over a complete time period is 0 we can talk about only energy and the total energy available from the signal can be computed in 2 ways;

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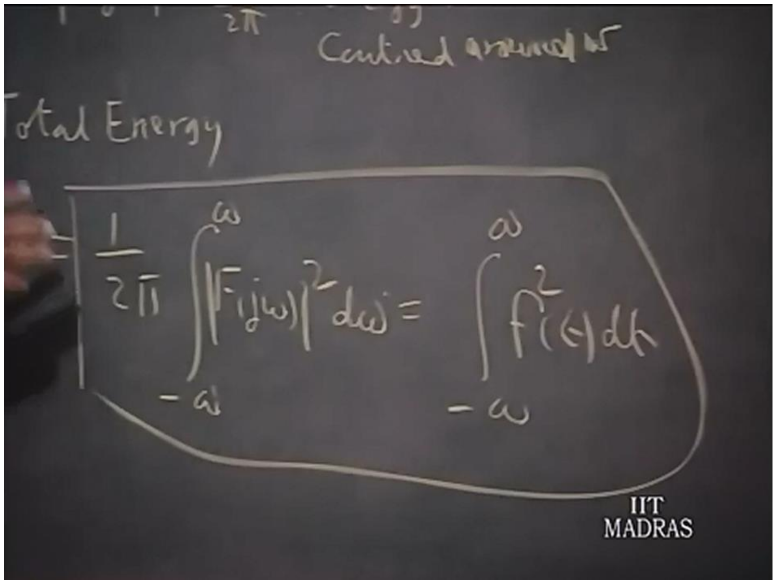

One in time domain this fashion and 1 in the frequency domain. The meaning of this is, if this is a voltage signal this is given to 1 ohm resister. This is the energy dissipated in that 1 ohm resister. You can calculate it either in time domain, frequency domain the case may be. Now, let us work out an example to illustrate the particular concept.