Networks and Systems
Prof. V.G.K. Murti
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture-34
Properties of Fourier Transform (Contd.)

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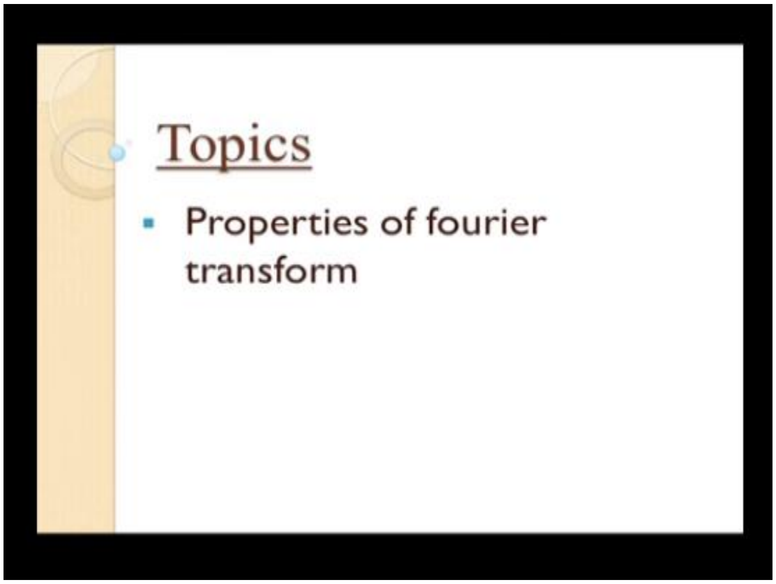

In this lecture, we will continue with a study of the properties of the Fourier transform. We have been talking about this property of translation in time.

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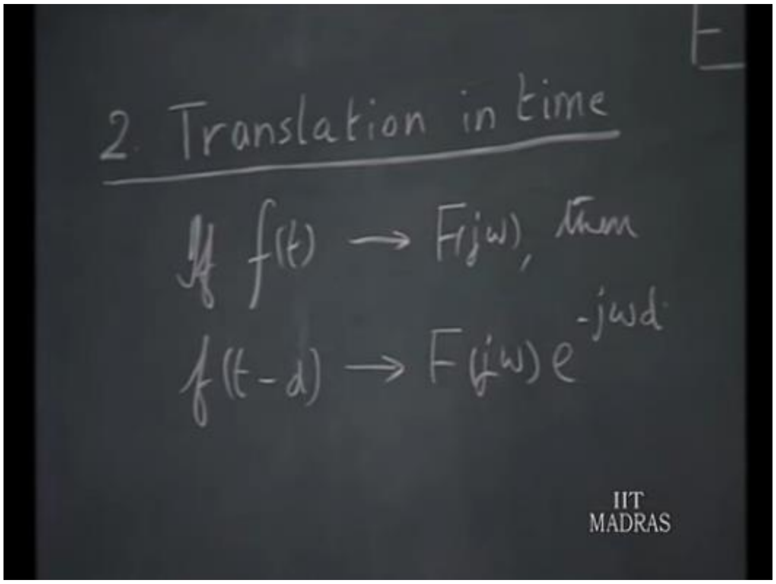

You recall at what you said here is if f(t)has a Fourier transform f j omega then, the same function shifted in time translated in time by an amount d f t minus d will have the Fourier transform. Which is f j omega multiplied by e to the power of minus j omega d and we observed that, the magnitude spectrum of f(t)minus d and f(t)remain 1 and the same. Means only the phase spectrum gets shifted. The phase in fact gets smaller by an amount omega d proportional to omega.

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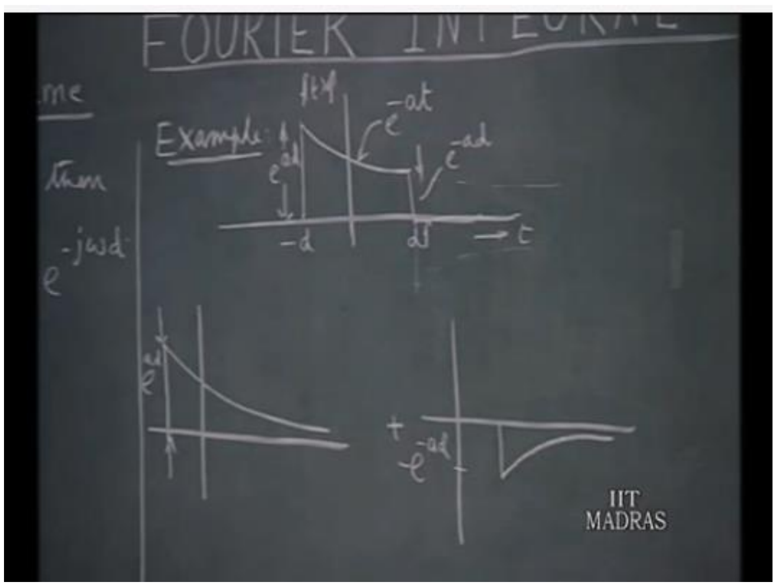

Let us work out an example how this property can be used to find out the Fourier transform of certain signals. Suppose, we take a function which follows the rule e to the power of minus a t for finite duration from minus d to plus d and it is 0 everywhere else. So, that is the function we are talking about.

We can find the Fourier transform by integrating this f(t)multiplied by e to the power of minus j omega d from minus d to d. Alternatively, we can make use of the properties of the linearity of the Fourier transform as well as the time shift property which we have just been discussed. What we can do is this f(t)can be taken the thought of as composed of 2 functions like this plus another starting here decaying like this.

The magnitude of this is of course e to the power of minus a t therefore this will be e to the power of ad. So, that is the amplitude of this and this decays by exponentially as e to the power of minus a t from this point onwards. So, what we are saying is this composite function can be thought of as the continuous function like this it continuous forever.

And then you clip this style by introducing negative exponentially decaying function like this which is given by this. So, now what is the amplitude of this? This must be exactly equal to so that from that point onwards e decays to 0. So this will be e to the power of minus r e to the power of minus ad this is this quantity is e to the power of minus ad.

So, a negative going view form like this will do the trick so we have to find out the Fourier transform of this and the Fourier transform of this and add them up that will give the Fourier transform of the composite wave.

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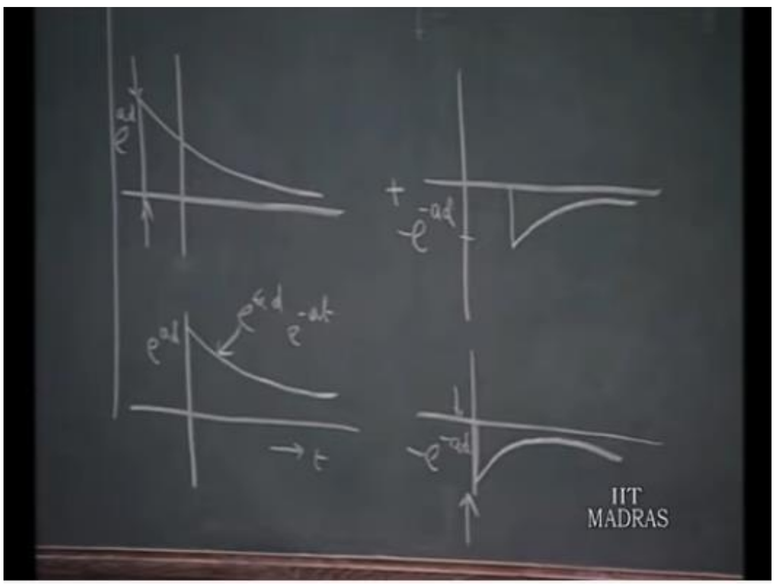

Now, to find the Fourier transform of this what we can do is let us shift this to by an amount d. So that, this is e to the power of ad and this delays e to the power of ad multiplied by e to the power of minus a t and this is exponentially decaying function like this. If you know the Fourier transform of this we can find the Fourier transform of this because, this is translated into time by an amount d. Similarly, as far as this is concerned if you can find the Fourier transform of this with initial amplitude e to the power of minus ad.

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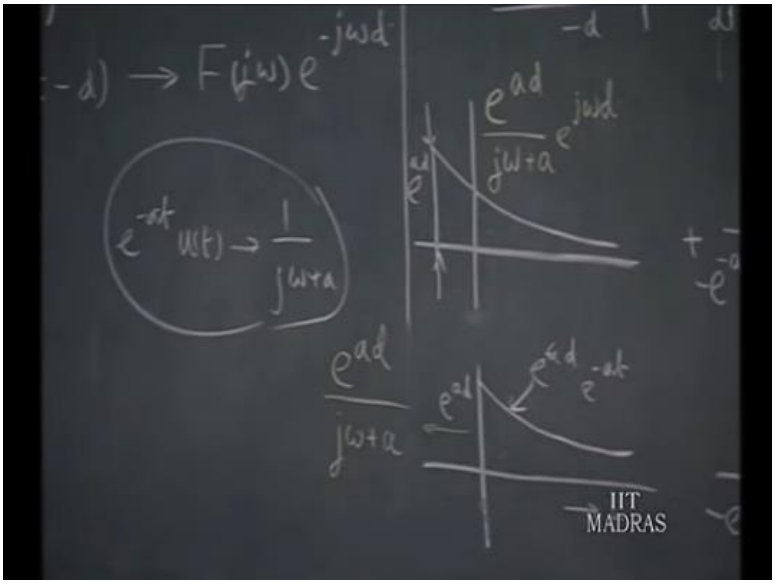

Then you can find the Fourier transform of this and this is a function for which we already have you recall, that we said e to the power of minus a t u t has the Fourier transform 1 over j omega plus a. This is something which we already found so as far as this is concerned, its Fourier transform would be instead of unit amplitude e to the power minus a t 1 at time t equal to 0 it has an amplitude e to the power of a d. So, this Fourier transform of this would be e to the power of ad.

The Fourier transform of this would be e to the power of ad divided by j omega plus a. 1 over j omega plus would be the Fourier transform of exponential decaying function starting with unit amplitude start with but since, this has the amplitude e to the power of ad start with e to the power of ad by j omega plus a.

Now coming from here how do we go to this? This is f(t)this is f(t)plus d because, it is advanced in time. If this is if you call this is f(t)this will be f(t)plus d so we go back this rule, this rules applies incidentally whether d is positive or negative. So, this is f of j omega for this as far this wave form is concerned it is e to the power of ad by j omega plus a multiplied by e to the power of j omega d.

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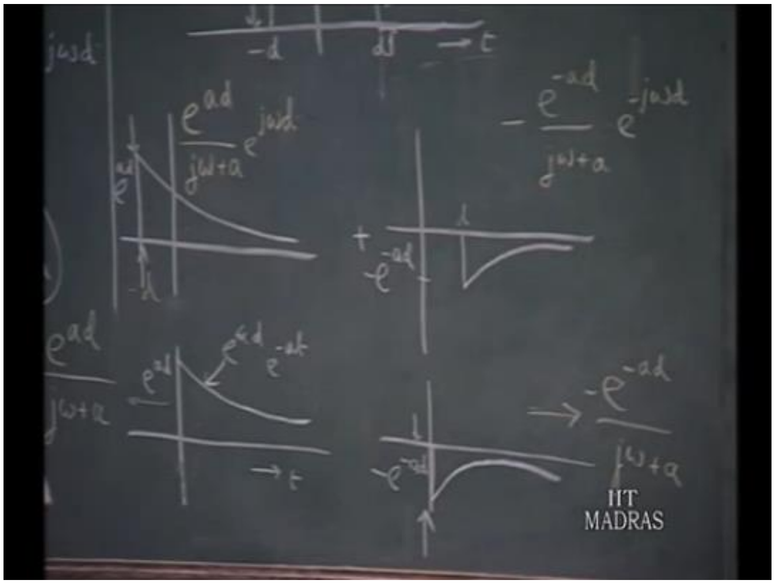

Now, coming to this, this is similar to this except that it has the initial value of minus e to the power of minus ad. Therefore, the Fourier transform of this would be e to the power of minus ad by j omega plus a with a minus sign because, the amplitude starts with an negative. Now, this function is nothing but this except it is delayed by d units. This is minus d here and this is plus d.

Therefore, the Fourier transform for this is minus of e to the power of minus ad divided by j omega plus a multiplied by e to the power of minus j omega. So, we found out the Fourier transform of these 2. So the Fourier transform of this would be the just sum of these because, Fourier transform follow the linearity property.

That is if f 1 of t and f 2 of d a Fourier transforms f 1 j omega and f 2 j omega the Fourier transform of c 1 f 1 t and c 2 f 2 of t f would be c 1 f 1 j omega plus c 2 f 2 j omega irrespective of the values of c 1 and c 2 constants.

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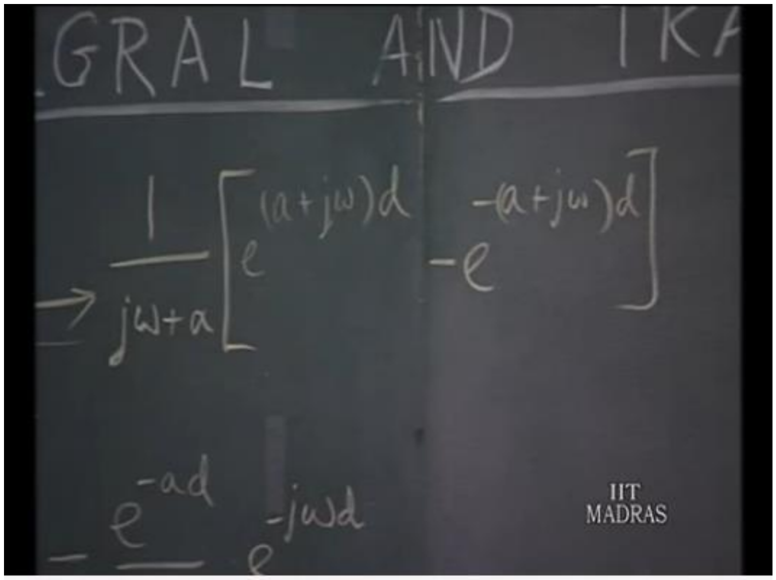

So, the Fourier transform of this now given by the sum of these 2 which of course the common factor j omega plus j. I can keep out side and you have e to the power of a plus j omega d minus e to the power of minus a plus j omega. So, this example tell shows us that, if you recognize the function as composed of a linear combination of functions whose Fourier transforms you already know. We can intelligently get the Fourier transform of this without having to do the integration starting from scratch.

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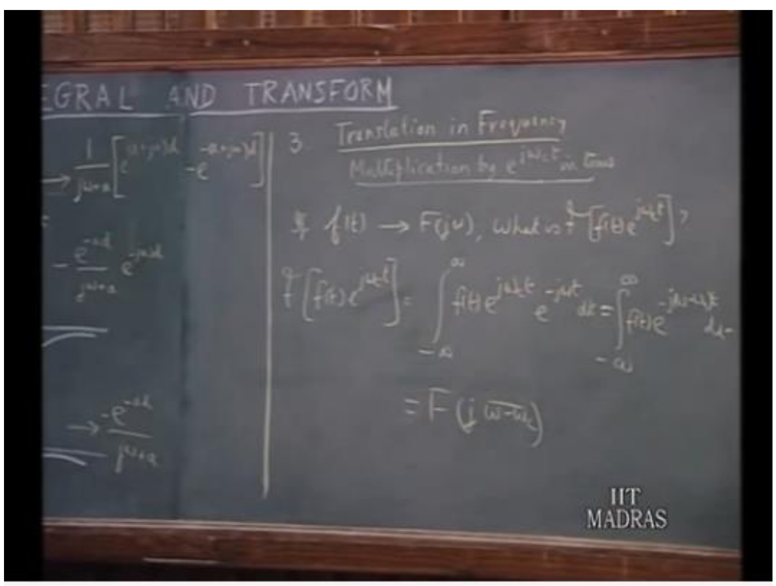

Okay, Let us take up the next property just like you have translation in time we can think of translation frequency which is equivalent to multiplication by e to the power of j omega c t in time. Note what we here, in the property 2 we have translation in time in time axis f(t)goes to f(t)minus d. In the transform domain f of j omega gets multiplied by the exponential quantity.

In the similar fashion what we have here is if you multiply in the time e to the power of j omega c t the Fourier transform get shifted in the frequency axis. What happens is this, if f(t)has the Fourier transform f j omega. Then what we would like to know is, what is the Fourier transform of f(t)multiplied by e to the power of j omega c t.

This is the question which we like to ask I multiply f(t)by the exponential function e to the power of j omega c. And would like to know what its Fourier transform is going to be. So, let us do this the Fourier transform of f(t)e to the power of j omega c t by definition is minus infinity to plus infinity of f(t)e to the power of j omega c t multiplied by the e to the power of minus j omega t dt.

And this straight away, seen to be minus infinity to plus infinity of f(t)e to the power of minus j omega minus omega c. Now, if you add f(t)e to the power minus j omega t dt this would have called it f of j omega. All we have now is instead of omega we have got omega minus omega c. So, it is almost same of the defining relationship of the Fourier transform of f(t)except that instead of omega we have got omega minus omega c.

So, this obviously will be f of j instead of omega we have got omega minus omega c which means that, the Fourier transform of this is f of j omega. This is the same Fourier transform except that the spectrum gets displaced by an amount of omega c. So, whatever value you take at a particular value omega takes the same when omega minus omega c takes the particular value; the function behaves the same values.

So, this is again an important relation which will be useful in the contest of modulations of signals, amplitude modulation of signals. Where f(t)is really multiplied by instead of e to the power of j omega c something like cosine omega c t or sin omega c t we will take this up we show that presently.

But, you notice the duality between the property of translation in time and the translation frequency. When you are translating the function of time then, the transform domain the transform gets multiplied by an exponential term. If you are translating into the frequency domain as we have done here then, in the time domain it gets multiplied by e to the power of j omega c t.

So there is the duality between the transform domain and the time domain. Now, let us carry this illustration of this let us take up a case where f(t)is multiplied by 0 e to the power of j omega c t but cos omega c t.

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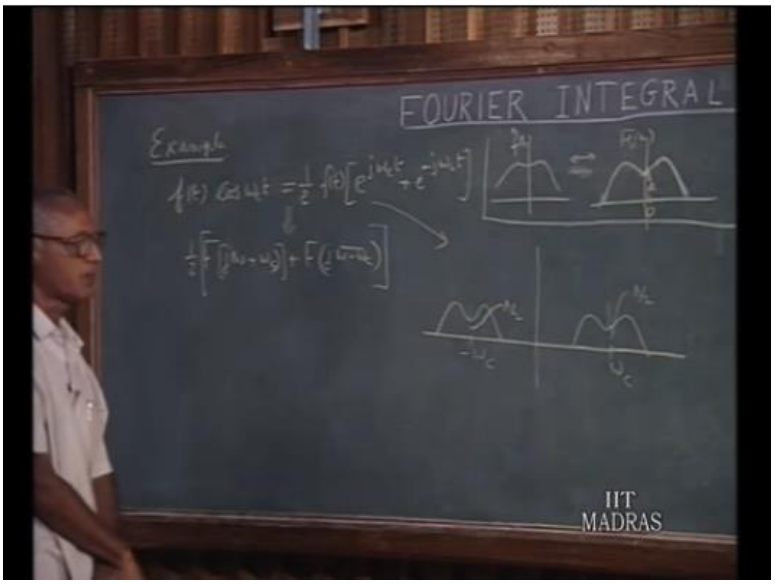

As an application of this property let us take this example, where f(t)is multiplied by cosine of omega c t, we would like to find out its Fourier transform. Now, we can express this of course as 1 half of f(t)multiplied by e to the power of j omega c t plus e to the power of minus j omega c t because, cosine omega c t is expressed as e to the power of j omega c t plus e to the power of minus j omega c t divided by 2.

So obviously then we have 2 terms: half f t e to the power of j omega c t, half f t e to the power of minus j omega c t. We observed in this property that if we have f(t)e to the power of j omega c t then, the Fourier transform is f of j omega minus omega c. If you have minus omega ct this would have been omega plus omega c.

So, we use those 2 properties and then find the Fourier transform of this. So, the Fourier form of this would be 1 half of f of j omega plus omega c and f of j omega minus omega c that is what we have got. So, what is the implication of this? Let us see, suppose I have f(t)and the corresponding Fourier transform Fourier spectrum let us say it is like this it is amplitude spectrum which is symmetrical.

Something like this suppose, we call this just for the sake for reference let us call this a, this is 0 these 2 format Fourier transform pair. Now, when you are talking about f(t)cos omega c t. The spectrum for this now consists of 2 sections: 1 is half f of j omega plus omega c, another is half of f of j omega minus omega c.

This portion is now centered around omega c and then it will have the same spectrum is reproduced here except that it is centre around omega c instead of the origin. Therefore, it will have something like this if this is this would be a by 2 because it is half of that. Similarly, f of j omega plus omega c is the spectrum which is similar to this except that reduced by the scale factor half and it is advanced by an amount omega c.

Therefore it will be centered around minus omega c, so it will be like this. So, this is the spectrum of this 1 and what is the how this spectrum is related to original spectrum. We can say that the original spectrum gets split into 2 half equal half and 1 half shift forward and sit around omega c instead of 0. The other half is shifted in the reverse direction and centered around minus omega c instead of 0.

That means, the essential shape of the spectrum remains the same except that has been centered around the origin in the frequency domain it is centered around omega c and minus omega c. And this is common place operation in communications and the amplitude modulations. Where, information is contained of the signal is purposely shifted from the instead of being centered around the d c it is shifted to a more convenient location of the frequency axis .

It is shifted to be centered around plus omega c or minus omega c this is called the carrier frequency. This is done in the interest of 2 things: 1 first of all the for the transmission of the information it will be more convenient the information is centered around a higher frequency rather than the d c. And secondly the same channel communication channel more effectively used by having several items of information in a more convenient location.

Instead of that is you can have another information channel here, another information channel here. And therefore, it makes for more effective communication in a given channel because of the reasons it is convenient to modulate a signal f(t)by multiplying by trigonometric function of this type. These details of this of course will have in your communication theory but, as far as we are concerned we just want to find out an application of this property of the Fourier transform. Now, let us take up the next property.