Networks and Systems

Prof. V.G.K. Murti
Department of Electrical Engineering
Indian Institute of Technology – Madras

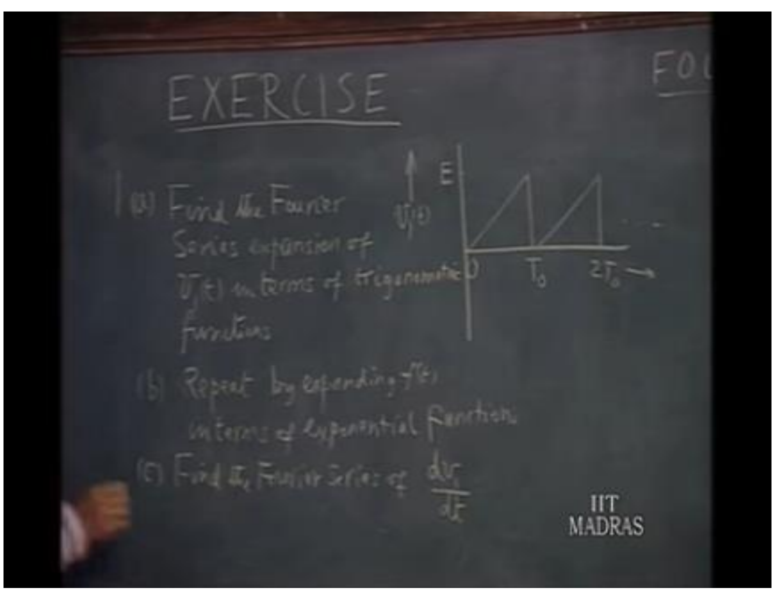
Lecture-28

Exercises on Fourier Series

(Refer Slide Time: 00:13)

I should now, like to give set of 6 problems to constitute an exercise for you to work out covering the topics that we have discussed so far, on the Fourier series.

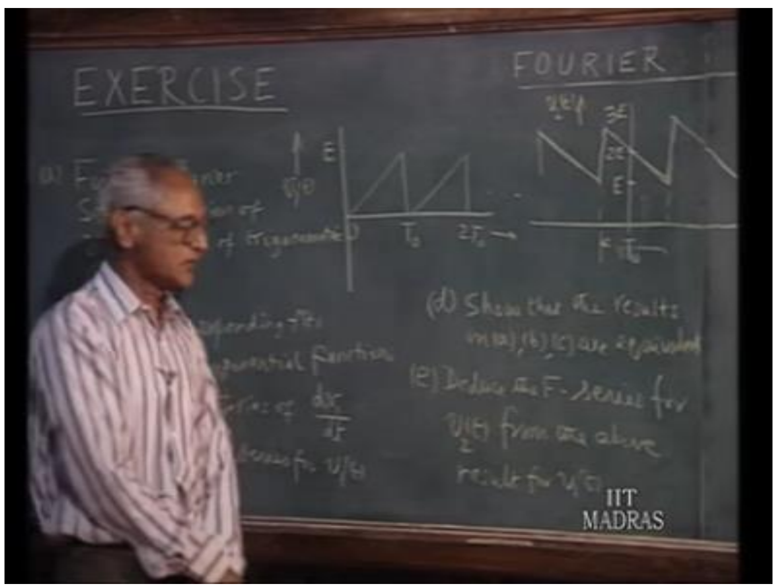
(Refer Slide Time: 00:32)



First problem, Consider a wave like this triangular wave; this is the voltage wave v1 of t. So, this is the triangular wave saw tooth wave as it is called with an amplitude e and period t not. Find the Fourier series expansion of v1 of t in terms of trigonometric functions. That is, in terms of a not a n and bn repeat this by expanding f t in terms of exponential functions, that is; in terms of cn c.

Find the Fourier series of the derivative of this function d v 1 by dt. So, once you take the derivative you should observe the slope is constant here and the slope is constant here. But at this point you have impulses, because there is a certain jump e to 0 e to 0. So, it will have you will have also impulses deal with find the Fourier series d v1 by dt and hence the series for v1 of t. So, you do the find the Fourier series of v1 t by 3 different methods.

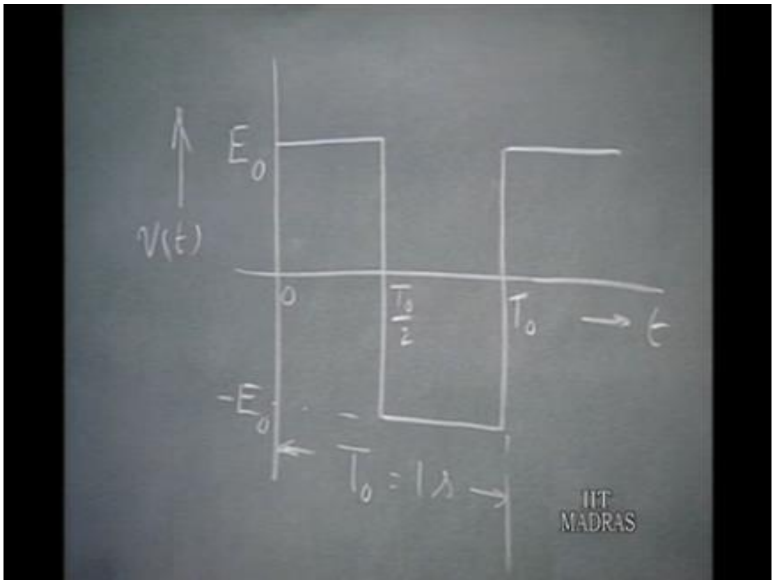
(Refer Slide Time: 03:43)



D: Show that the results obtained in abc are equivalent. Compare them and show that they are all equivalent. E: you take another waveform like this, this is e, this is 3e and this is 2e and the period is t not as before. That means, the wave form is this. This I call v2 of t deduce the Fourier series for v2 of t from the above results for v1 of t.

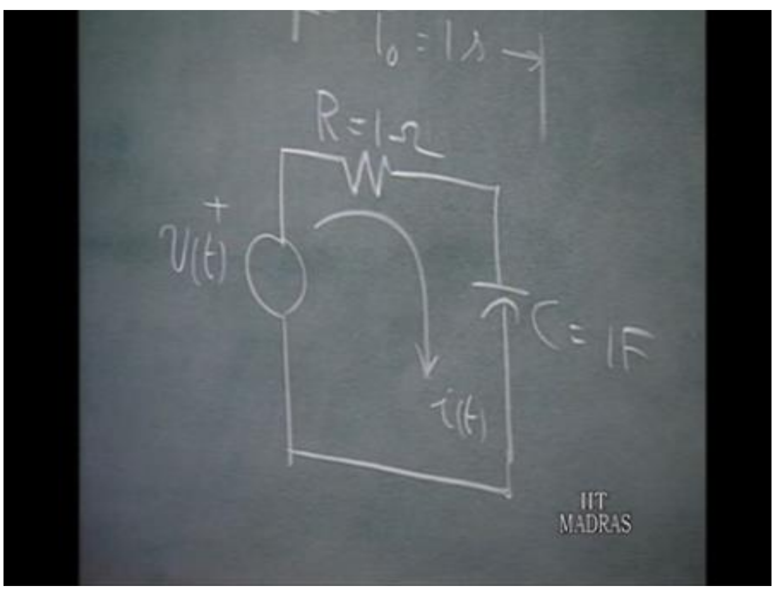
In other words what we expect here is, you do not find the Fourier series for v 2 of t directly. You some where relate v 2 of t with v 1 of t. Since we know, the Fourier series for v 1 of t you use the information to calculate the Fourier series for v2 of t that is the first example.

(Refer Slide Time: 06:10)



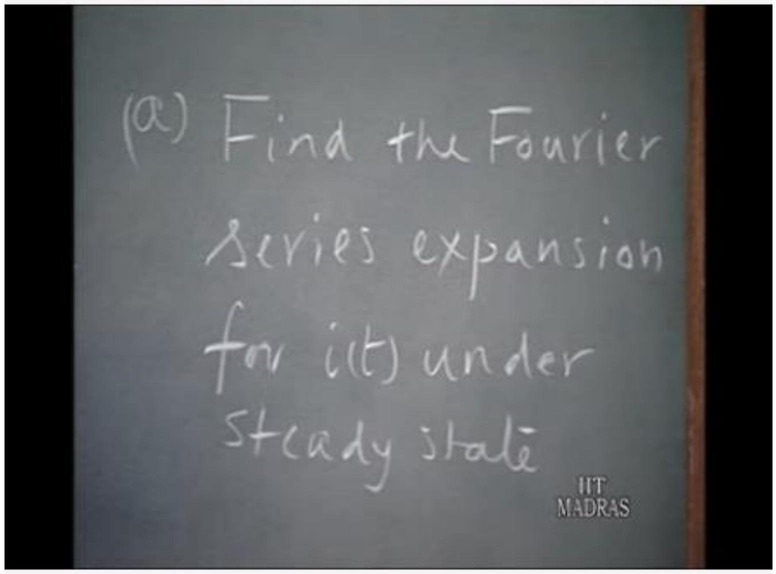
Problem number 2. You have waveform like this e not minus e not period t not take it to be 1 second this is of course, t not upon 2 this is t not 0 time t this is voltage v of t.

(Refer Slide Time: 06:56)



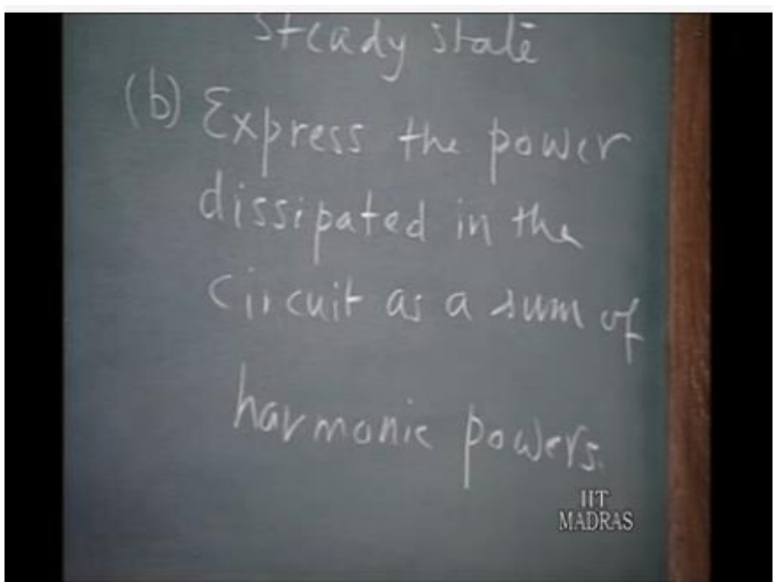
Now, if a voltage adding this type of waveform v of t is given to an rc circuit containing a resistance of 1 ohm and a capacitor of 1 farads. Let under steady state the current i(t) close to the circuit. So, this type of wave form given to this circuit you are asked to find out,

(Refer Slide Time: 07:29)



Find the Fourier series expansion for i of t under steady state. That is; the first part of the question.

(Refer Slide Time: 08:00)

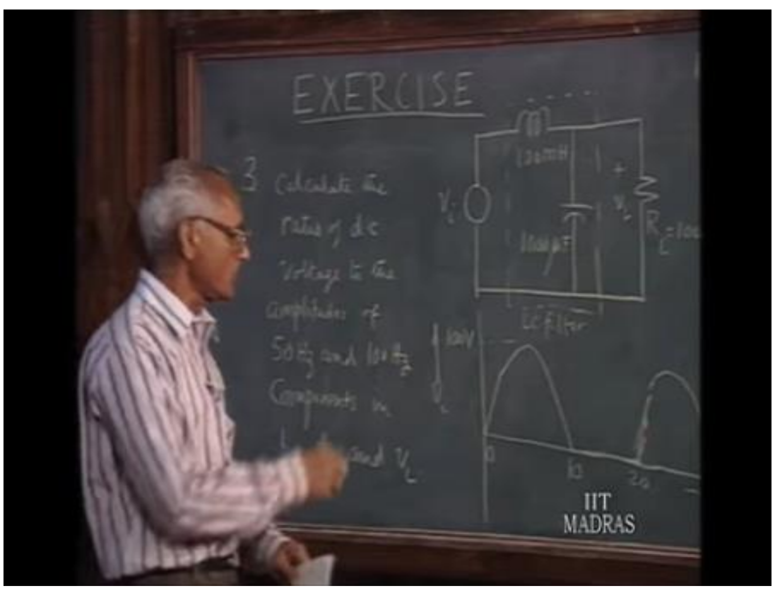


B. Express the power dissipated in the circuit as a sum of harmonic powers. So, the first problem requires to find out the Fourier series expansion for the current i(t) under steady state. You find out the various harmonic components of v of t, find out the component of current at different frequencies given by the source and sum them up and there is Fourier series expansion for i of t. In the second part you calculate you are asked to calculate the power as the sum of harmonic powers you have 2 ways of doing this.

You know the various harmonic components of the current; therefore, knowing the value of r and the harmonic component of current you can find out the harmonic power for each harmonic. Alternately you know the Fourier series expansion for v of t, you know the Fourier series expansion for i of t.

So, you can associate the voltage component at particular frequency, the current component at the same frequency find out the angle between these 2 and find out the harmonic power for each 1 of the harmonics. So, find out the summation express this as sum of the various harmonic powers and that will be the total power this is problem number 2.

(Refer Slide Time: 09:41)



The third example let us consider circuit where we have an input voltage. Which is half wave rectify sine wave like this, with an amplitude of 100 volts this is vi 0, 10, 20 this is t in milliseconds. That means; this 50 cycle's wave from which we generate half wave rectified sine wave 100 volts.

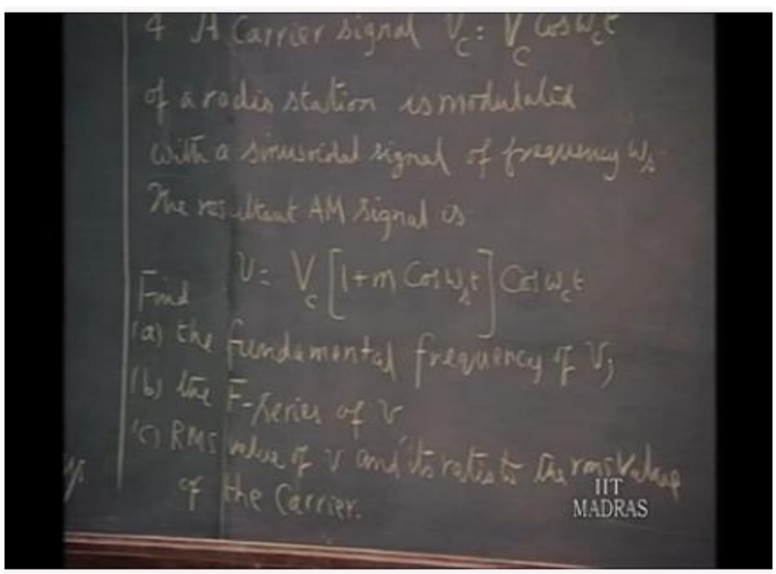
This is the input voltage and we like to have a filter l c filter here and try to supply the load resistance rl of 100 ohms as pure as dc is possible. So, this is the output voltage vl .So, vl should be the dc voltage the extent possible all the ripples should be attenuated. We have a filter which consists of an inductor of 100 millihenries. Let us say capacitor of 1000 micro farads and the input voltage is this.

Calculate the ratio of dc voltage to the amplitudes of the 50 cycles and 100 hertz components in both the input voltage vi and the output voltage vl. So, you have the input voltage you have the dc 50 cycles or hundred cycles and so on.

We want to calculate dc components of the input voltage and the same quantities in the input voltage and you should show, that the 50 cycles or 100 cycle components are attenuated considerably in relation to the dc when you go to the output.

So, this is an lc filter, we have earlier in the class workout example with an rc filter. So, that this is you will see that this will be more effective filter than the rc filters.

(Refer Slide Time: 12:50)



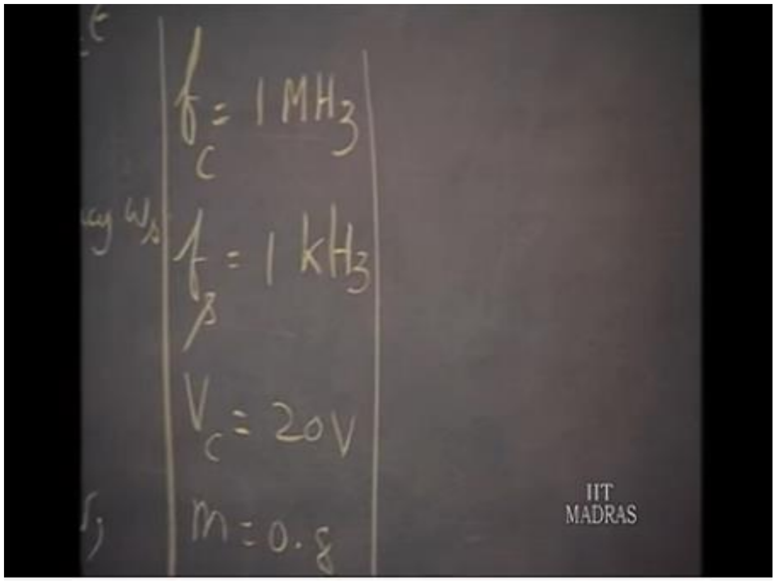
Fourth example: The carrier signal vc is VC cos omega ct of a radio station is modulated with a sinusoidal signal of frequency omega s radius per second. The resultant amplitude modulator, this is AM means; amplitude modulator signal is v equals to vc 1 plus m cos omega s t.

Where omega s is the sinusoidal signal of frequency omega s and this is cos omega ct. So, this is the amplitude modulator signal b. Now, you are asked to find out. a: the fundamental frequency after all this is the periodic wave form because you have got only cosine term with different frequency repeats itself.

What is the fundamental frequency of the amplitude modulated wave, find the fundamental frequency the amplitude of the am wave v. b: the Fourier series expansion of v. Now, if you are clever enough you could see that you do not have to do any major integrations etc here.

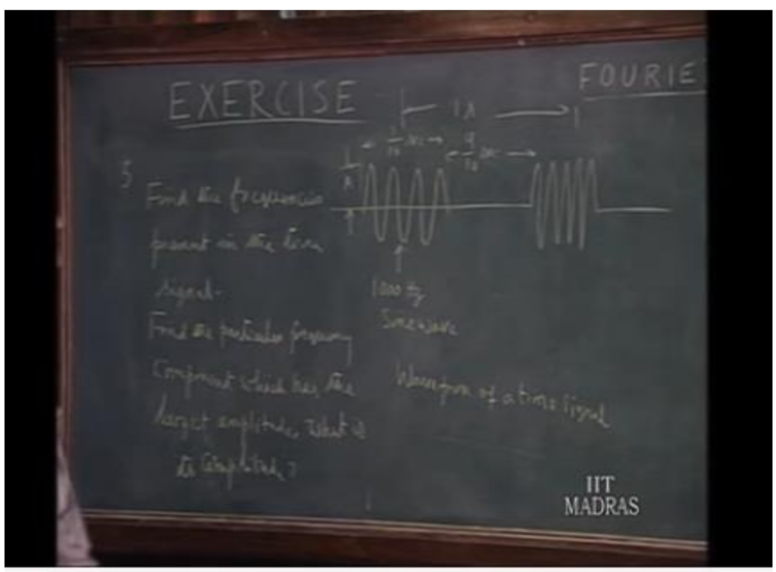
You can simply your work by combining the cos term suitably and you can do this. c: rms values of v and its ratio to the rms value, rms value of v and its ratio to the rms value of the carrier rms, value of the carrier is vc up on root 2.

So, you find the rms value of the amplitude modulated wave and see what proportion it does to the rms value of the carrier these are the 3 things you have to find out in this particular problem. Ah 1 minute, 1 minute, 1 minute. Here, I have to give you some additional data for problem 4 fc.
(Refer Slide Time: 16:53)



You can calculate numerically 1 megahertz, that is; the carrier the signal frequency you take it as 1 kilo hertz vc to be 20 volts that is this and m as point 8. So, this is the numerical data which you can use to work out the details of problem 4. Okay, you often heard, the time signal that is broadcast by a radio station immediately before the news pip pip that type of thing.

(Refer Slide Time: 17:57)



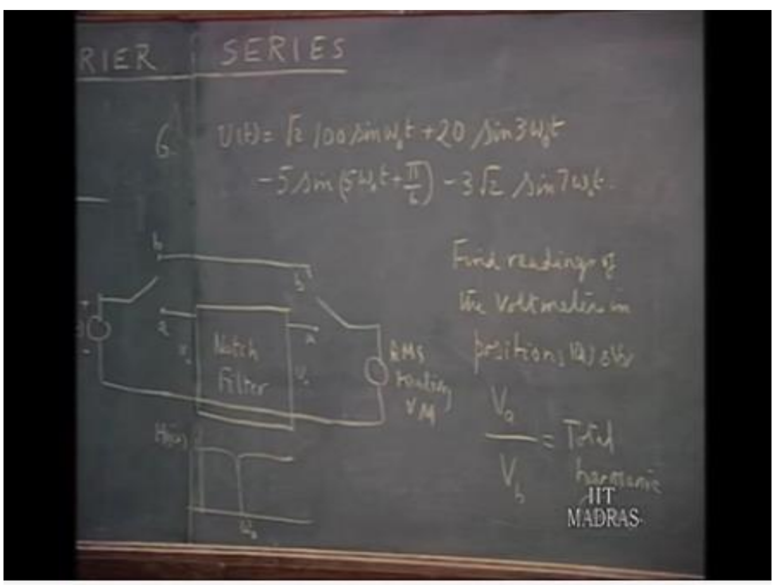
So, the wave form the signal will have something like this like this. So, you have the burst of 1000 hertz signals you take just a data as a typically. Suppose you have thousand hertz sine wave which is maintained for 1 tenth of the second. And this is repeated every

1 second; that means, this is blank period 9 by 10 seconds and this is repeated and this is 1000 cycle pure sine wave.

This is the wave form of a time signal. So, you are required to find the Fourier series analysis of this. So, in particular you are asked to find the frequencies present in the time signal with this waveform. Find the frequency component, particular frequency component which has the largest amplitude and what is its amplitude?

So, if you take amplitude of the sine wave as a. What is the amplitude of the component which has the largest amplitude? Now, in doing these analysis just give a hint you can put your origin, at convenient location and assume that you have the continuous sine wave. If you multiply this by the periodic pulse style you can get this.

So, sinusoidal multiplied by periodic pulse style will give this. So, you use that information to find out the Fourier series for this.
(Refer Slide Time: 20:56)



Lastly it is often interest for us, to know the rms value of the various harmonics. As proportion of the total rms value. This example, illustrates an instrument. Which measures this? Suppose, we have v of t which is root 2 times 100 sin omega not t plus 20 times sin 3 omega not t minus 5 times sin 5 omega not t plus 5 up on 6 minus 3 root 2 sin 7 omega not t.

So, we will stop with that up to the 7th harmonic. Now, we have an instrument which incorporates. What is called a notch filter will explain the characteristic of notch filter in a moment. So, this periodic non sinusoidal signal is given to this notch filter it can be connected to the notch filter.

So, have the switch here which can be thrown to either a or b. So, likewise here is switch can be thrown either b or a. And this is connected to an rms reading volt meter. What this notch filter does is? This is the output v not and this input vi. Suppose, it has a frequency response function h of j omega of this notch filter is it allows all frequencies, but cuts off any desired frequency.

So, if this amplitude is 1. If this frequency is omega x or omega not in our case that particular frequency omega not is attenuated fully. If you introduce a sinusoid with that frequency you get 0 outputs. All other frequencies come out unscathed, without any amplification or attenuation.

So, that is the characteristic of the notch filter this appears to be a notch that is why it is called notch field. So, we have notch filter like this. So, what happens is if you put the switch on b then the meter here volt meter reads the rms value of the input signal of v of t.

On the other hand, if you put switches on a both these on a the fundamental is cut out. And the meter reads the rms value of all the other harmonic components right. So, you have 2 readings in a and b. Find readings of the volt meter in positions a and b. In position b it is vb is suppose the rms value that is the total in position a.

Suppose, you put this a va corresponds the rms value of the various harmonics, vb will be the reading corresponds the rms value of the input voltage. The ratio va to vb is said to be the total harmonic content of this voltage. And suppose, this assumed to be sinusoidal this total harmonic content is said to be the total harmonic distortion of the sinusoidal.

So, this meter as such is called distortion analyzer and this is used to find out. How waveform which is supposed to be sinusoidal how it gets started by contamination with various harmonics. So, you have suppose an amplifier system you feed input sine wave the output also the amplified version must be a pure sine wave, but sometime the amplifier into this distortion and you like to measure.

What is the amount of distortion and that is caused by a device like amplifier which may have some non-linearities.So, this particular instrument which is called harmonic analyzer a distortion. Analyzer measure the harmonic distortion this is called distortion analyzer. So, this distortion analyzer is essentially an instrument which measures the rms value of the various harmonics.

As the ratio of the rms value of the composite wave or usually in practical cases this harmonic this rms value of the total wave very nearly the rms value the fundamental. So, we can say it measures the rms value of the various harmonics, as a fraction of the rms value of the fundamental as well.