Networks and Systems

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Lecture-27
Additional Properties of Fourier Series

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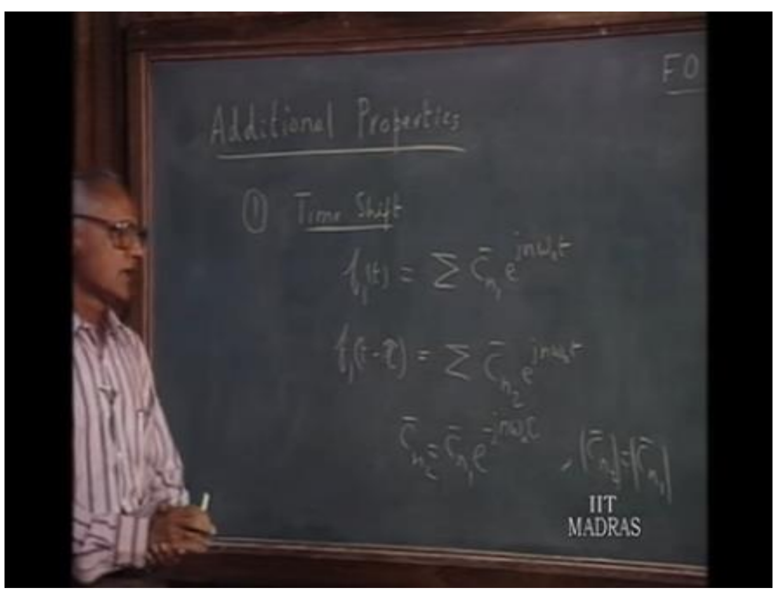

Just discuss 2 additional properties of the Fourier series suppose, we have a function f1 of t and it is fourier series is this with cn 1 and the coefficients. Now, if this function f of t is shifted in time translated in time there certain amount. Let us say, f1 t minus tau it is the same function, but the origin is shifted it is delayed by an amount it equal to tau second.

Then, it can be shown that the corresponding Fourier series for this will be such that cn2 is cn1 multiplied by e to the power of minus j n omega 0 tau which immediately, shows that the magnitude of cn2 is same as the magnitude of cn1 how are the face is different, the angle is different, which means that the magnitude spectrum of f1 of t and the magnitude spectrum of f1 t minus tau will be the same.

The phase; however, is decreased by amount proportional to the frequency and example of this we have seen earlier, but when you took a square wave with the origin at 2 different places and you could express square wave in terms of cosine functions and sin functions.

So, this is the important property that the constitution of the various harmonic component they proposed at the various components will not depend up on where we place the origin.

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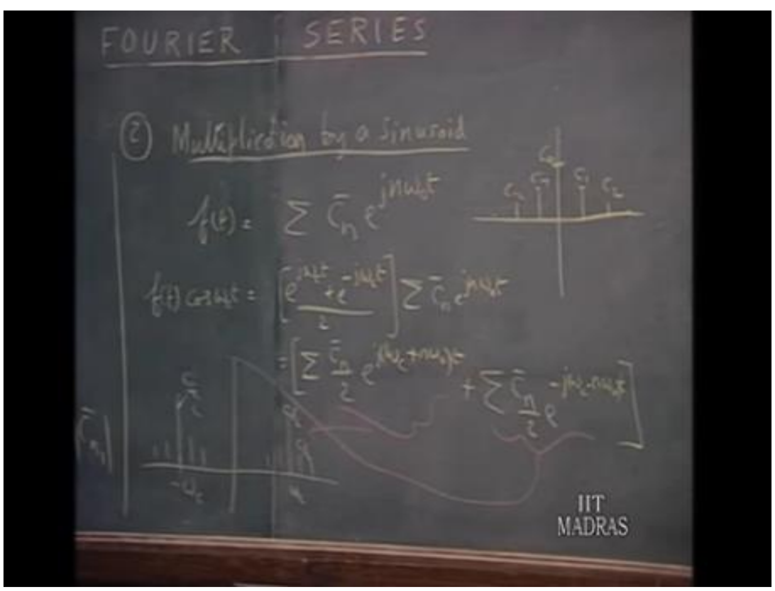

The second property relates to suppose, you have a periodic function you multiply by a sinusoid again let, f of t be the periodic function and this may be the Fourier series for that the spectrum for this, this is c 0 c1 c2 c minus 1 c minus 2. Now, I would like to ask is: if f of t is multiplied by cosine omega c t then we can say this is: after all cosine omega c t e to the power of j omega 0 t plus e to the power of minus j omega c t divided by 2 and you multiply this by cn e to the power of j n omega 0 t.

The result is you get 1 expansion cn up on 2 e to the power of jn omega 0 as i may write omega c plus omega 0 omega c plus n omega 0 plus another group of terms cn by 2 e to the power of minus j omega c minus omega not. So, you have frequencies omega c plus n omega 0 n of course, ranges from minus infinity to plus infinity and another set of terms frequencies omega c minus minus omega c minus n omega not.

So, in other words, if look at the spectrum here, centered around omega c you have various harmonic components this will be c 0 by this is c1 up on 2 etcetera and centered around minus omega c you have second set of component this is again c 0 up on 2. So, this portion corresponds to this and this portion corresponds to this.

So, what it means; is this: spectrum gets divided into 2 parts 1, part is shifted to omega c and other part is shifted to minus omega c it has been centered around the origin centered around plus omega c and minus omega c.

This is done in communication and instrumentation. When you want the information contained of the signal it has been centered around dc value you like to be shifted to a more convenient frequency for purpose instrumentation or communication and in that contest this is: set to be the carrier frequency because it carries the information content associated with f of t you can use this information as an example.

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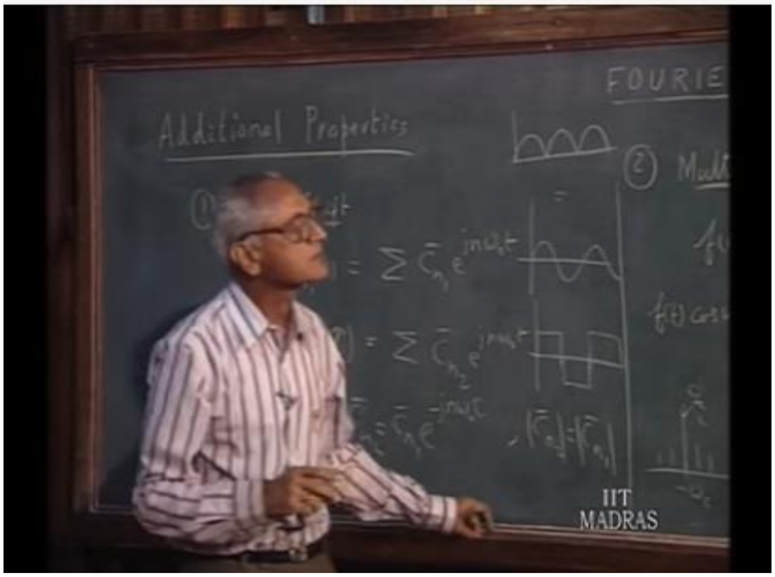

You can think of if you have a full wave rectify sine wave you can think of this as the product of a sin term and a product of and the square wave. Suppose, you multiply these 2 out you can get this therefore, to find the Fourier series for this you can use this property and then find the Fourier series, i will leave this as an exercise to you to sum up we have discussed, at length the various properties of Fourier series.

How the series can be obtained, the characteristic of the series and how it may be used to analyze circuits excited by periodic, but not sinusoidal wave forms we derived a little expression for calculating various harmonic components. This is the process which is refer to harmonic analysis, if the function f of t is given in the form of analytical expression we can carry out this harmonic analysis analytically the wave we have done in the various examples.

However, if the function is available in graphical form with no analytical expression on hand available with you then we can use numerical methods and computers can also be used software programs can be developed to give the various harmonic components. Once you have the wave form prescribe or it can be captured by means of discrete data points experimentally.

We also have instruments available do this harmonic analysis. The categories of instruments: 1 is called the harmonic analyzer, where you feed the signal to the harmonic analyzer and tune it to get the amplitude of magnitude of each harmonic components 1 at a time and read it out and a meter.

The other class of instruments are known as spectral analyzers where, if you feed the signal you will find the entire magnitude spectrum displayed on the screen this is called a spectrum analyzer. The ideas of Fourier series or the harmonic analysis of periodic signals, can be carried over to signals, which are not periodic they are called a periodic signals and this is the subject will take up next.

When we talk about the Fourier integral concept with this, we conclude our discussion, on the Fourier series in the next presentation, I will give you the set of examples as an exercise for you and also we include elaborate, it in demonstration and some of the concepts we have discussed.