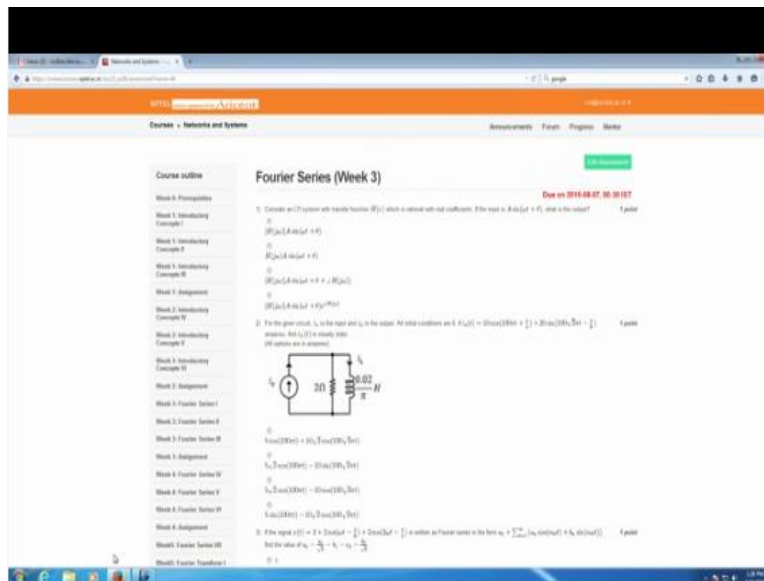


Networks and Systems
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Hints for Assignment 03

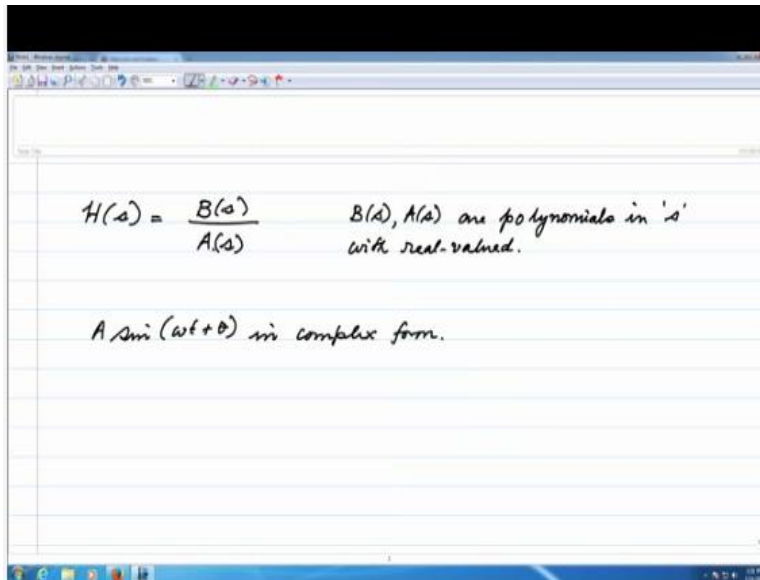
Hello, welcome to the short video in which I am going to give hints for solution to week 3 assignment. So, this is a short video in which we are going to give hints for each of these questions.

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So, let us look at the first question here, the first question is an LTI system with transfer function h of s which is rational and real valid coefficients are given and the input is a $\sin \omega t + \theta$ and what is the output? So, this is the question and you are given the 4 choices.

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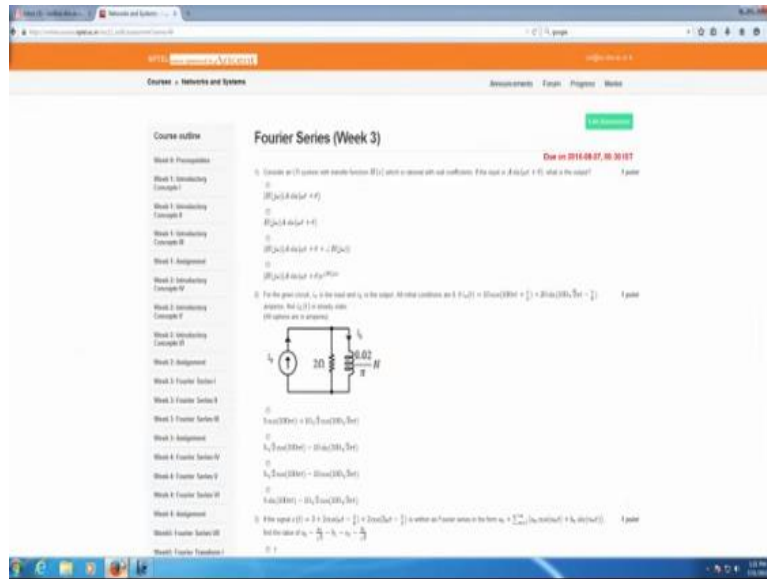
The first important point to notice here is h of s is a rational transfer function which means that h of s is of the form b of s over a of s , where b of s and a of s are polynomials in s and the other property that is stated is if the coefficients are real value and then the input given is $A \sin(\omega t + \theta)$ and you are asked to find out what the output is going to be.

So, this is 1 of the very first concepts that has been explained in Unit 10 Week 3 Fourier Series 1. So, in that lecture Professor V. G. K. MURTI gives you explanations on how to do this and what you need to do is you need to write $A \sin(\omega t + \theta)$ in complex form and then use the fact that $e^{j\omega t + \theta}$ is an Eigen function or an Eigen signal.

To a system with rational transfer function LTI system and then expresses the output in terms of input times of complex constant the other thing that you need to use is the fact that the system has real valued coefficients. Which means that the frequency response has certain properties namely the magnitude response is even and the phase response is odd using that fact and the fact that the input consists of 2 complex Eigen signals.

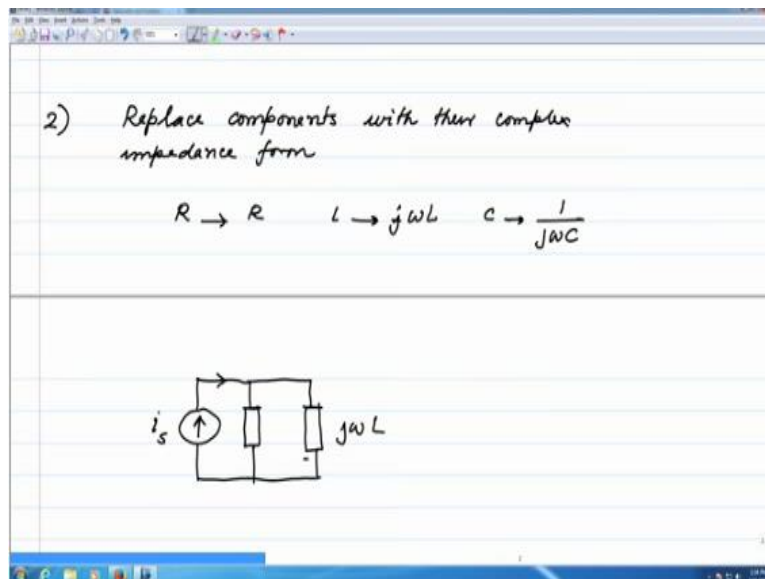
You will be able to get the output for each of these complex exponentials and then combined them and get the final form. So, this is the way you need to proceed with question 1.

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Now, let us move on question 2, question 2 you are given a circuit which is driven by a current source which has 2 components in parallel a resistance and an inductor and the input current source is sinusoidal. 2 components are there 100 cosine, 10 cosine, 100 pi t plus pi by 4 plus 20 sin, 100 root 3 pi t minus pi by 6 and you are asked to find i_L of t in its steady state and it is also given that all initial conditions are 0.

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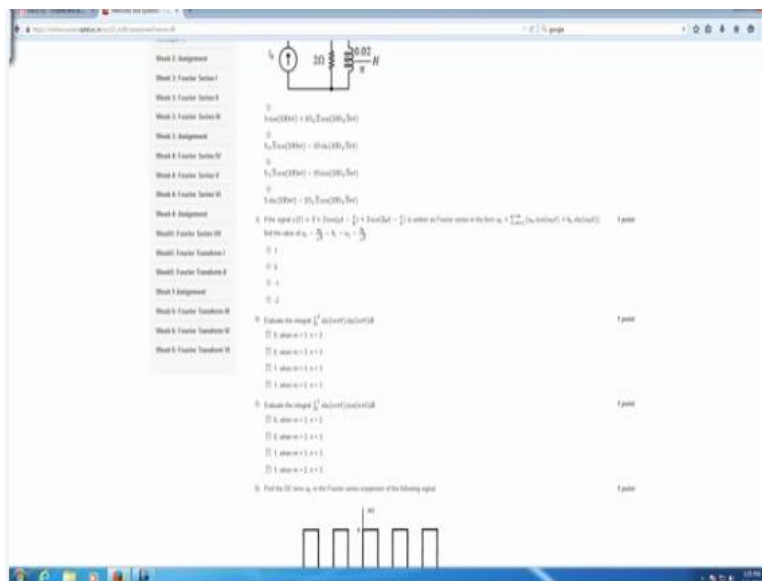


And the way to do question number 2 is you need to replace all components you need to replace components with their complex impedance form which means r is replaced by r , l is replaced by $j\omega l$, c is replaced by $1/j\omega c$ and remember ω is the frequency of the input. Now, in this particular case the input consists of 2 components 1 components frequency is 100π , the second components frequency is 100 times root 3 times π .

So, what you need to do is, you need to consider the circuit replace by r by itself and replace the inductor by $j\omega l$ and then use current division property. So, this is is and you need to use the current division property and find the current in the inductor and you need to do this for each input component.

Namely you need to do it for the first component with frequency 100π you also need to do this for the second component with frequency $100\sqrt{3}\pi$ and since the circuit is linear, the final answer is the addition of the answers that you get from each individual component So, that's the how you need to proceed with question number 2.

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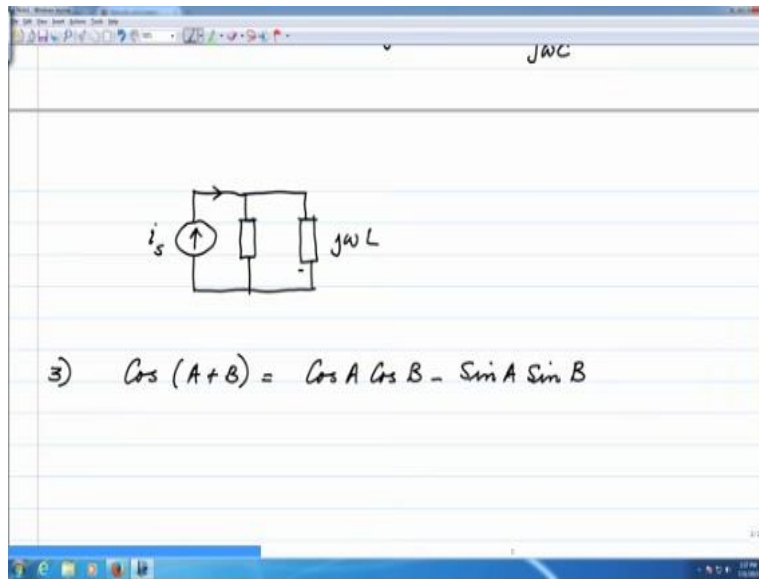


Let us now move on to the next question. In the next question you are asked to find the value of a_1 by root 3 minus b_1 minus a_2 minus b_2 by root 3 and you are given a signal x of t which is $3 + 2 \cos \omega t - \pi$ by $6 + 2 \cos 2 \omega t$

minus pi by 3 and this is the periodic signal and this is expressed in the form of Fourier series in particular it is expressed in the trigonometric form and you need to find the value of this expression.

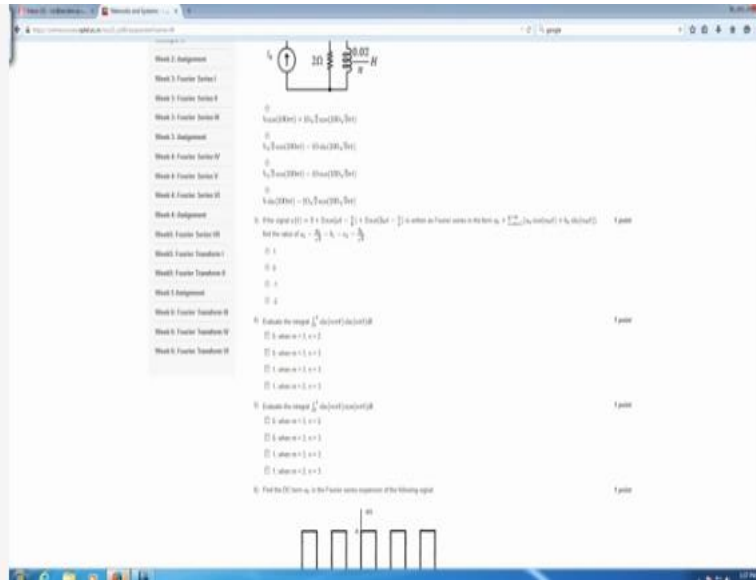
Which means you need to first express the given signal x of t in the trigonometric form as given here.

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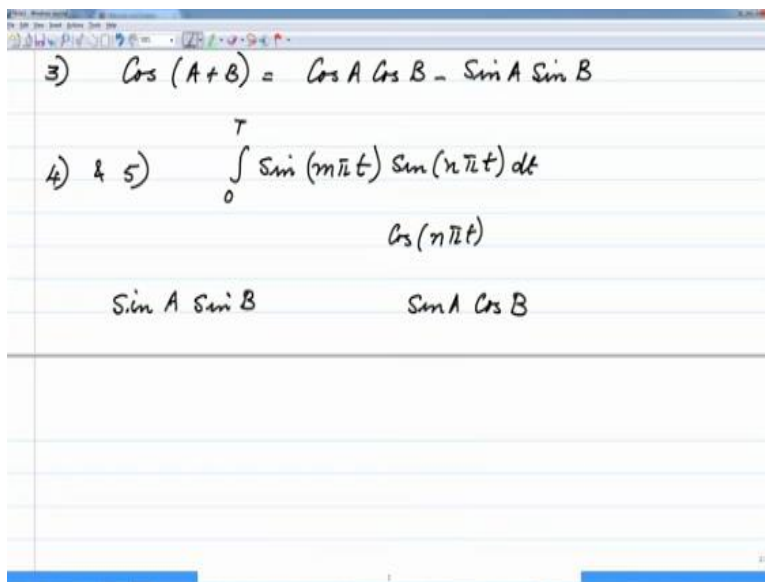
So, this question involves taking cosine $a + b$ and writing it as $\cos a \cos b$ minus $\sin a \sin b$. Once you do this for each of these terms and then collect all the similar terms together you will get an answer or you will get an expression that is in the standard trigonometric forms identify all the coefficients a_0 , a_1 , a_2 , b_1 and b_2 and then use those values in the expression given there and simplify.

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Now, let us move onto the next question. So, this question number 4 right by the way these things evaluating the Fourier series is covered in unit 11week 3Fourier series lecture 2 "An Evaluating Fourier Series Coefficients" at the title of the lecture. If you listen to that you will be able to follow the questions and understand the questions we are doing right now and solve them.

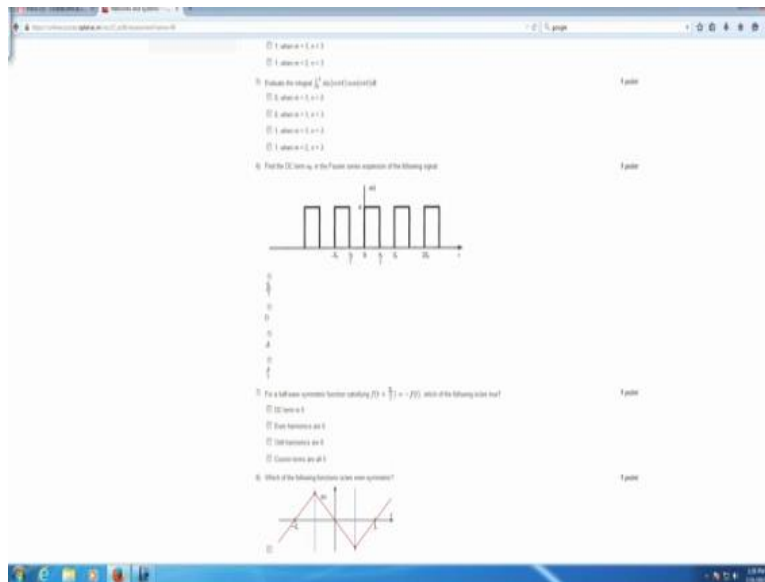
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Now, let us look at question number 4 and 5. These are very similar you are asked to evaluate integrals of the form $\int_0^T \sin m \pi t \sin n \pi t dt$ and for the second for the fifth question so of sin for the second term you have cosine $n \pi t$ and this is a pretty simple to evaluate these 2 integrals.

And the steps that you need or you need to know how to express $\sin a$, $\sin b$ for the first question and then that is when I said for the fourth question and you need to know the value of $\sin a$, $\cos b$ and $\sin a$, $\sin b$ is half \cos difference minus \cos sum and $\sin a \cos b$ is half of \sin sum plus \sin difference use these 2 formulas and then break this single term inside the integral into sum of 2 terms and then evaluate the integral and simplify you will get the answers to questions 4 and 5 they are very similar.

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Let us now go to question 6, question 6 you are asked the find the dc term a_0 not in the Fourier series expansion of the waveform given. Which is a square pulseis amplitude a between 0 to $T/2$ and from $T/2$ to T the amplitude is 0 and this fund basic fundamental period repeats itself every T not seconds and you are asked to find the Fourier series coefficient a_0 .

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4) & 5) $\int_0^T \sin(m\pi t) \sin(n\pi t) dt$

$\cos(n\pi t)$

$\sin A \sin B$ $\sin A \cos B$

6) $a_0 = \frac{1}{T} \int_0^T x(t) dt$

And as you recall from the lecture the formula for a not is given by 1 over t 0 to t x of t dt. So, apply this definition to the function that is given there and the integral is very simple and you will get the answer immediately. So, this is calculating the dc coefficient or dc value of this periodic signal given in question 6.

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The screenshot shows a quiz interface with the following content:

- Question 4: $\int_0^T \sin(m\pi t) \sin(n\pi t) dt$
 - 1. $\sin(n\pi t)$
 - 2. $\cos(n\pi t)$
 - 3. $\sin(m\pi t)$
 - 4. $\cos(m\pi t)$
- Question 5: $\int_0^T \sin(m\pi t) \cos(n\pi t) dt$
 - 1. $\sin(n\pi t)$
 - 2. $\cos(n\pi t)$
 - 3. $\sin(m\pi t)$
 - 4. $\cos(m\pi t)$
- Question 6: Find the DC term in the Fourier series expansion of the following signal.
 - 1. $\frac{1}{2}$
 - 2. $\frac{1}{4}$
 - 3. $\frac{1}{8}$
 - 4. $\frac{1}{16}$
- Question 7: For a half wave symmetric function satisfying $f(t) = -f(t - T/2)$, which of the following is true?
 - 1. DC term is 0
 - 2. Even harmonics are 0
 - 3. Odd harmonics are 0
 - 4. None of the above
- Question 8: Which of the following functions is not even symmetric?
 - 1. $\cos^2(t)$
 - 2. $\cos(t)$
 - 3. $\cos^2(t) + \sin^2(t)$
 - 4. $\cos^2(t) - \sin^2(t)$

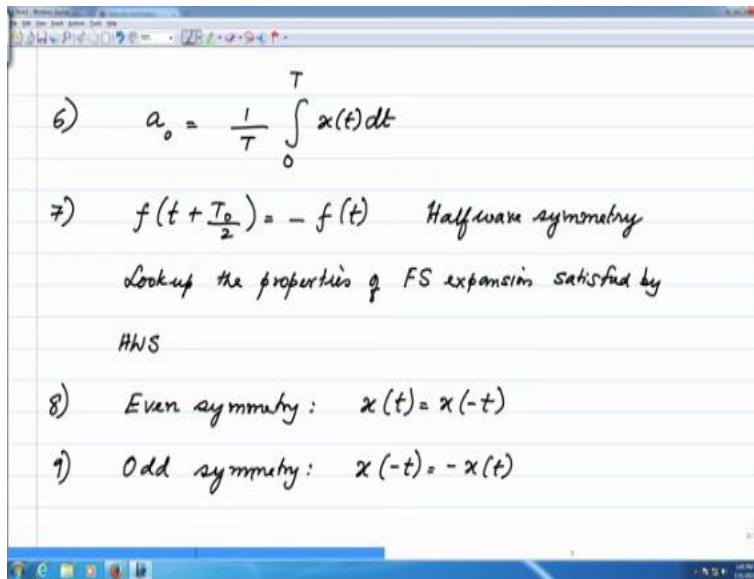
The waveform plot for Question 6 shows a periodic signal with a period of 4. The signal is zero for $0 \leq t < 1$ and $3 \leq t < 4$. It has a constant value of 1 for $1 \leq t < 2$ and a constant value of 2 for $2 \leq t < 3$.

Let us now move on to question number 7. You are given a waveform with half wave symmetry and as you know half wave symmetry is a function that satisfies $f(t) + f(t - T/2) = 0$ and you are asked which of the following is slash or true. So, you need to pick all the choices that apply and this is discussed in unit 11 week 3 Fourier series lecture 2 and the title of the lecture is "Symmetry Conditions" there Professor

V.G.K. MURTI talks about what are the implication in terms of Fourier series components present in a signal that satisfies half wave symmetry.

And if you follow the lecture carefully the answers to this question all the properties satisfied by half wave symmetry are clearly mentioned there and you should pick up all the terms that apply for the general half wave symmetry.

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So, question number 7 f of t plus t not by 2 equals minus f of t . So, this is Halfwave symmetry and you need to look up the properties of harmonics or rather the properties of the Fourier series expansion satisfied by Halfwave symmetric wave forms. So, this is pretty straightforward here listen to the lecture and you will be able to identify which are the properties that apply and you can choose the right ones.

Questions 8 and 9 relate to symmetry of waveforms. Question 8 talks about which of the following functions given or even symmetric and question 8 is same set of wave forms you need to find out which ones are odd symmetric and even symmetry by definition is a function x of t satisfying x of minus t .

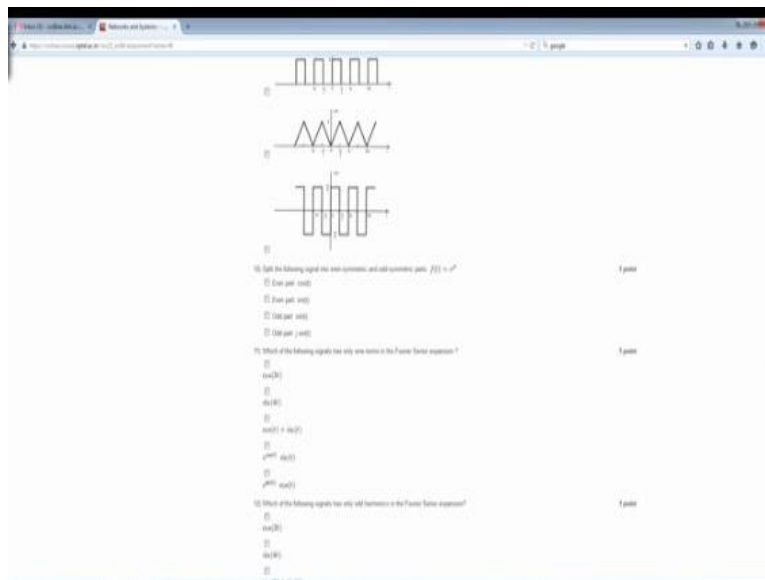
So, look at the 4 waveforms given find out which of the waveforms satisfy x of t equals x of minus t . For example x of 1 should be equal to x of minus 1 or x of t not should be

equal to x of t not for all t not right. So, that is easy to identify and the pictorial interpretation of x of t equals x of t that is a function satisfying even symmetry is basically you take the function reflected about the y axis.

So, if the negative time portion is a reflection around the y axis of the positive time signal then the signal is even symmetric. So, this should be easy to identify by looking at all the waveforms. And question number 9 you are asked to identify odd symmetry, odd symmetry is x of t is $-x$ of t .

Again the pictorial interpretation of this is if you reflect about the y axis and the reflection is negative of the positive portions of the signal then the signal is odd symmetric around the origin. So, that is the mathematical way of stating that is x of t equals $-x$ of t . Again, look at all the waveforms that are given find out which ones appear to be the negative portion must be not only reflected but also sign change so this is odd symmetry.

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Moving on to the next question, this is question number 10. You are given a general signal say f of t and then you are asked to find the even and odd symmetric parts in this particular case the signal f of t is $e^{j\omega t}$. Now, again this is a question of applying the basic definition here right.

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10) $x(t) = x_e(t) + x_o(t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \Rightarrow x_e(t) = x_e(-t)$$
$$x_o(t) = \frac{x(t) - x(-t)}{2} \Rightarrow x_o(-t) = -x_o(t)$$
$$x(t) = \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2}$$

Conjugate Even: $x(t) = x^*(-t)$
Conjugate Odd: $x(t) = -x^*(-t)$

So, any signal x of t can be written as a sum of x_e of t plus and x_o of t that is x_e of t its even part x_o of t is its odd part. x_e of t is nothing but x of t plus x of minus t by 2 right and you can immediately see that x_e of t equals x_e of minus t . Therefore this is even part and x_o of t which is the odd part of x of t is nothing but x of t minus x of minus t by 2 and you can immediately see that x_o of minus t is minus x_o of t .

So, this indeed is even symmetric the even part x_e of t is indeed odd symmetric and if you add these 2 you should get back x of t again it is a easy to verify that x of t is indeed x of t plus x of minus t by 2 which is the even part plus x of t minus x of minus t by 2 this is the odd part and if you simplify you get back x of t and we saw that the individual components are the even and the odd parts respectively.

And take $e^{j\omega t}$ write this as cosine plus j sign and then find the simplified value of x of t plus x of minus t by 2 that should give you the even part and x_o of t should be x of t minus x of minus t by 2 that should give you the odd part. One remark I would like to make in this context is you should know the difference between the even part and conjugate even part okay.

So, this is a new definition suppose a signal we want to check whether it is conjugate even, What is the definition of the conjugate even? Conjugate even means x of t is x star of minus t . So, compare this with the earlier definition of even a signal is set to the even if x of t equals x of minus t . So, that was even but conjugate even is if x of t equals x star of minus t then it is conjugate even. Similarly, if a signal x of t equals minus x star of minus t then it is said to be conjugate odd.

So, these are 2 different definitions and typically we have applied the conjugative even definitions to complex signals and the earlier given definitions to real valued signals. But is not the case that conjugate even and conjugate odd apply only to complex signals. Both even and odd and conjugate even and conjugate odd can be applied to either real signals or complex signals.

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The image shows a handwritten slide with the following content:

$$\begin{aligned} \text{Conjugate Even: } & x(t) = x^*(-t) \\ \text{Conjugate Odd: } & x(t) = -x^*(-t) \end{aligned}$$

$$f(t) = e^{jt} = \text{Conjugate even} + \text{Conjugate odd}$$

Now, as an exercise take the same signal f of t equals e power jt and then write this as a sum of conjugate even plus conjugate odd. Express it as its conjugate even part and find the conjugate odd part of e power jt and notice the difference between expressing it as even plus odd part.

So, this conjugate even and conjugate odd part is not part of the assignment is a concept that you need to know and these are the definitions and apply these definitions to the

same example that is given and notice carefully the difference between decomposing it into its real and even and odd parts versus conjugate even and conjugate odd parts.

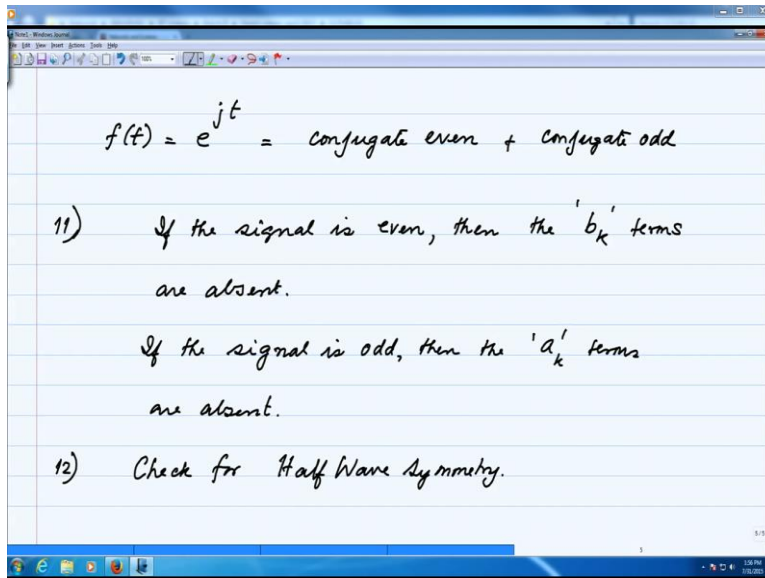
Now, as you might have seen in the lectures even and odd symmetry signals they are Fourier series expansions satisfies certain properties. That is if you have an even signal is Fourier series coefficients satisfy certain properties, if a signal is odd again some other properties satisfied and the properties of even and odd signals that property is tested in question number 11.

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So, in question 11 you are asked which of the following signals has only sin terms in the Fourier series expansion and you are given various choices and you need to pick up all the choices that apply.

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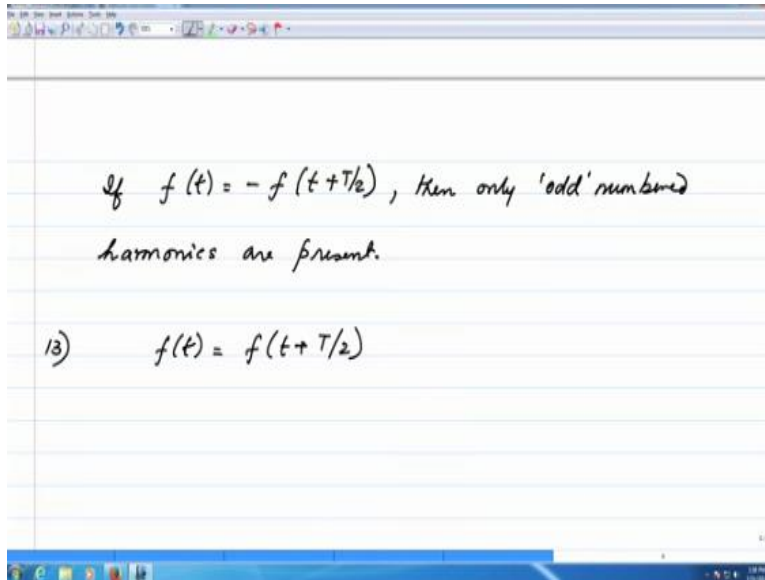


And what you need to recall from the lecture. If the signal is even then the b_k terms are absent recall b_k terms are the coefficients of the sin terms in the trigonometric expansion. Similarly, if the signal is odd then the a_k terms are absent and recall a_k terms are the terms a_0 which is the dc term and a_1 through infinity which are the terms corresponding to the cosine in the trigonometric expansion.

Therefore, question number 11 which tells you which of the following signal has only sin terms is another way of seeing check for any symmetry that the signal might have, does a signal have odd symmetry if it does then it has only sin terms. So, this is what you need to do for question number 11.

Let, us now move onto question 12, which of the following signal has only odd harmonics in the Fourier series expansion. So, this is another kind of symmetry that a signal might posses that has implications on, what kind of harmonics are present? In this particular case you need to check for half wave symmetry, check for half wave symmetry. So, this is discussed in unit 12 week 3 Fourier Series Lecture 3, the title of the Lecture is "Symmetry Condition Examples". So, you need to check for half wave symmetry.

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And if $f(t)$ equals minus $f(t + T/2)$ then only odd harmonics are present. Therefore, the question which of the signals has only odd harmonics of Fourier series expansion is the another way of stating check for half wave symmetry. If the signal has half wave symmetry then it will have only odd harmonics in the Fourier series.

Thirteenth question again relates to half wave symmetry except that this half wave symmetry is even symmetry. And check for the property that is satisfied in such a case as mentioned in the video lectures 1 way of looking at this is, this is thinking about the Fourier series being periodic with period t by 2. However, it is also useful to think of this has been periodic with period t and then seeing what harmonics are present.

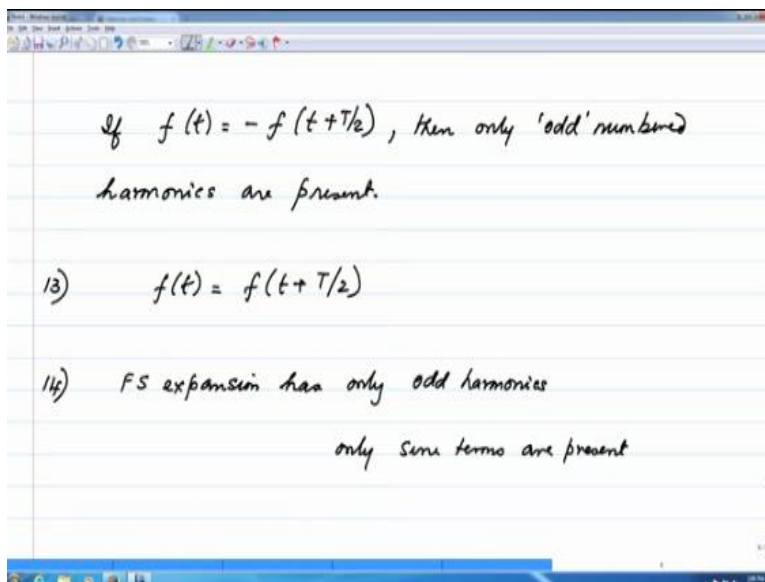
For example, if you consider simple sin wave and if you rectify it you will get a signal that satisfies this property provided the rectification is full wave. In such a case you will find that if you want to analyze this signal based on its harmonic content but you want to relate this with respect the original signal it is useful to think of the period to be still t and then think of the full wave rectified signal as being having half wave even symmetry.

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Lets move on to question number 14, in question number 14 what you are given is a partial waveform of periodic signal. The period is t not and you are given information from 0 to t not by 4 and you are also given the fact that the full wave form has only odd harmonic sin terms units Fourier series expansion and then you are asked which of the following could represent the full waveform.

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That is the Fourier series expansion has only odd harmonics and not only that; only sin terms are present. So what this says is if only sin terms are present it has to be an odd symmetric sequence, if only odd harmonics are present it must satisfy half wave

symmetry. So, combine these 2 facts and then complete the waveform and choose the one that or choose not only 1 as many wave forms are satisfies this condition.

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Moving onto the last question, you need to refer to unit 12 week 3 Fourier series lecture 3 and this is write it at the beginning of the lecture titled "Applications Network Analysis". What you are given here is, you are given here a wave from that is periodic with period T not and then as you know this can be represented in terms of a Fourier series and what you are asked in this particular question is you need to find the value of $a + b$ where a is the term multiplying 1 over π and b is the term in which the coefficient of $\sin 100 \pi t$.

This is very similar to the problem that has been worked out in Professor V.G.K. MURTI'S lectures right at the very beginning. If you watch that this is a very minor variant of that question you should easily be able to find what the value of $a + b$ is. So, this concludes the short video on hints about solving the assignment that is due next week.

So, please I mean look at all the problems carefully worked more they are not very difficult but they are very nicely capture all the concepts that have been explained in the video lectures. Thank You.