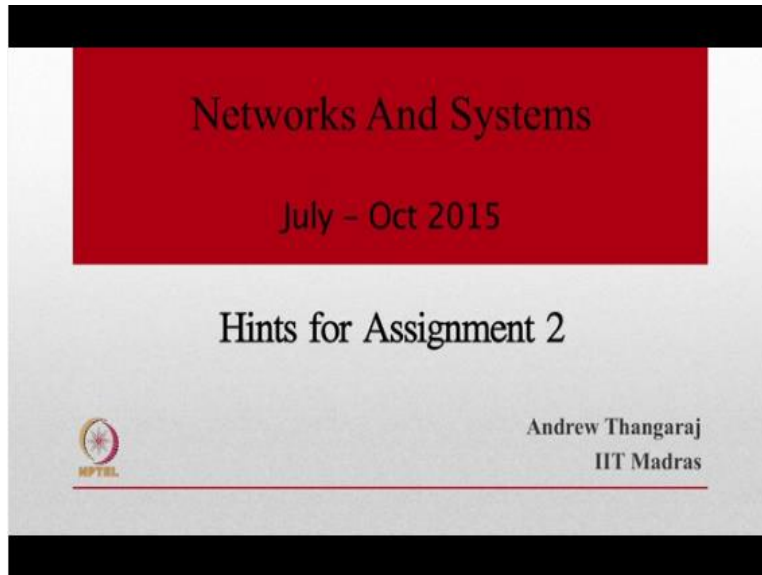


Networks and Systems
Andrew Thangaraj
Department of Electronics Engineering
Indian Institute of Technology – Madras

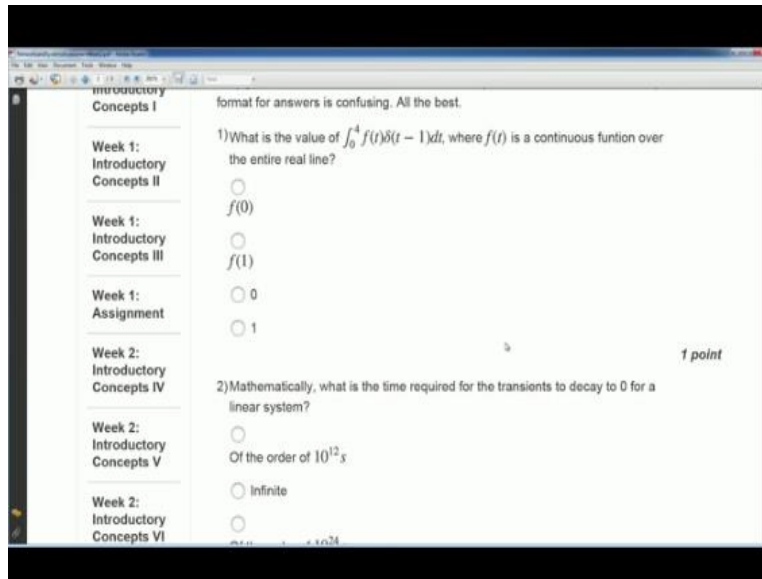
Assignment-2
Hints for Assignment 2

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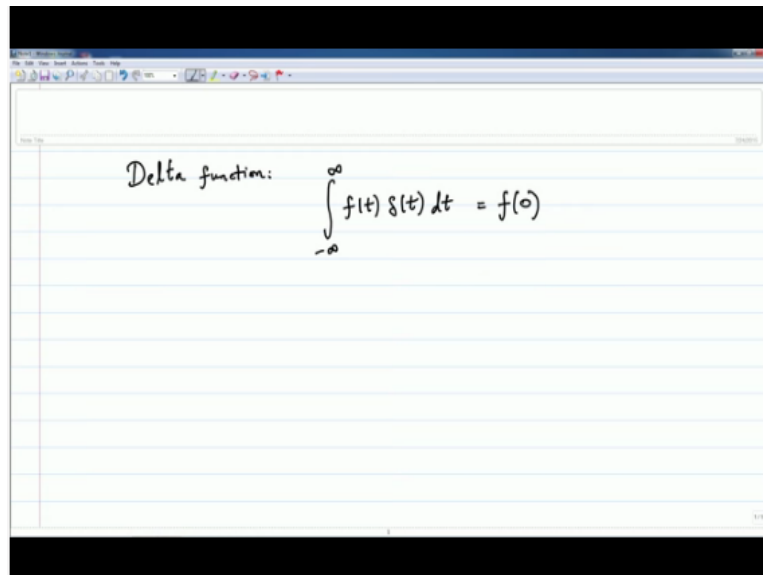
Hello, everyone, this is a short video on assignment 2 and the purpose of this video is to provide you some hints and help in solving assignment 2. Assignment 2 at this point is currently open you can still solve it and given that the course is becoming slightly more intense and more involved. We thought it will be beneficial for you if we can provide some hints and how to go about solving the questions in assignment 2 so this lectures for that purpose. Okay, so let's begin.

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So, if you look at the first question in assignment 2, it involves the delta function and the delta function is really a quite complex idea if you want to define rigorously and carefully and use it so what most electrical engineers do in practice is to have a rough idea of what the delta function is and how to use it. In fact, how to define the delta function is an interesting question to ponder about different books define it in their own way.

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And we believe the best way to define delta functions is to do the following if you want to define a delta function. The dirac delta function the best thing to do is to define it using this property you say for any continuous function f of t of t delta of t dt equals f of 0 .

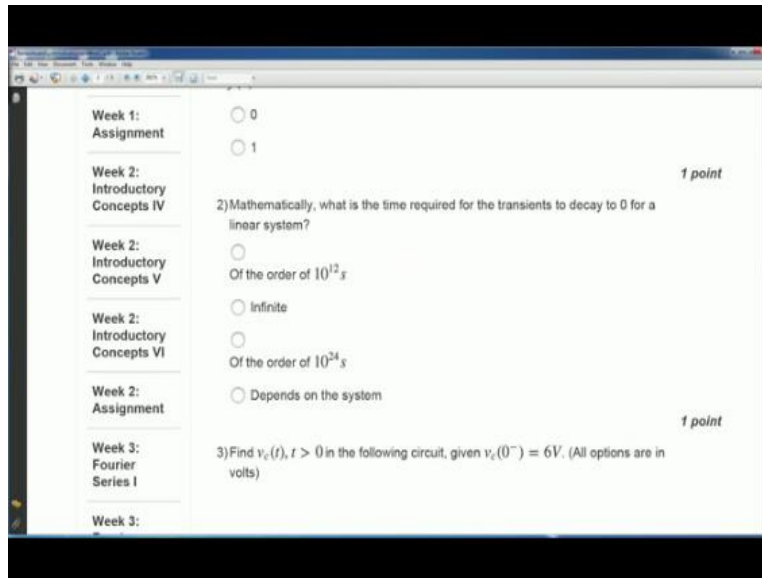
Okay, so this is known as the shifting property of the delta function and this turns out to be a kind of a decent way to define it and this is the most important critical property of the delta function that I keep using repeatedly in many applications.

There are many dangerous definitions you will see in the text and it's good intuitively to think of the delta function in various different ways, but if you want to be rigorous about it it's very hard to define. So, a good strategy is to have to be aware that the delta function can lead you into the wrong path very easily, but nevertheless it's a very powerful thing to use in electrical engineering particularly we use it quite often in a formal way we use it and most of our formulas work with the delta function treated more or less in a normal way.

So, just be aware of it if you want to know something deep you can maybe read more complicated definitions from various math textbooks, but for us this property is quite crucial and in fact the first question is a very simple application of this property and you will get the answers using that okay.

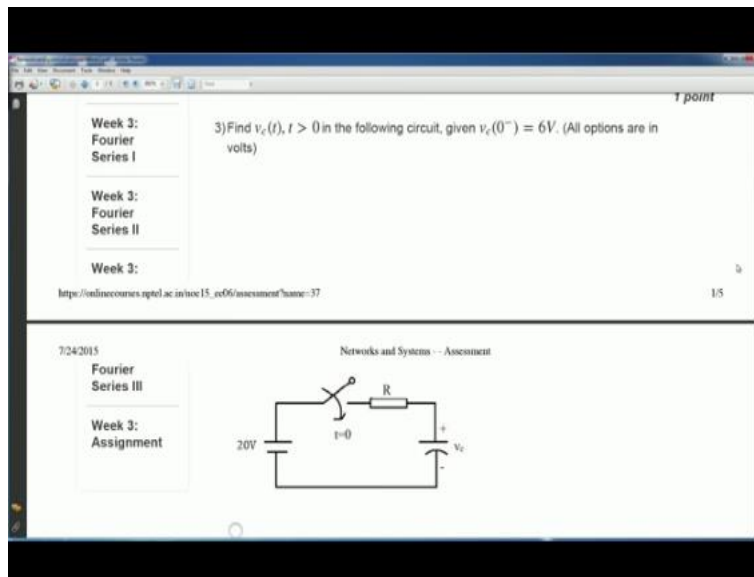
So that's all I want to say about the delta function and I think the lectures cover it quite well hopefully you will see more of it as we go along okay. So, if you move on to the other questions the second question is once again an interesting and simple question.

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It's an intuitive question it is mathematically what is the time required for the transients to decay to 0 for a linear system okay. So, this is a mathematical question not a practical question and its linked to the following question you have this decaying exponential e power minus t okay mathematically when does a power minus t become equal to 0 so more or less that's the question that's being asked here think about it and ponder about it and you might get an get an answer okay. So that is the hint for the second question.

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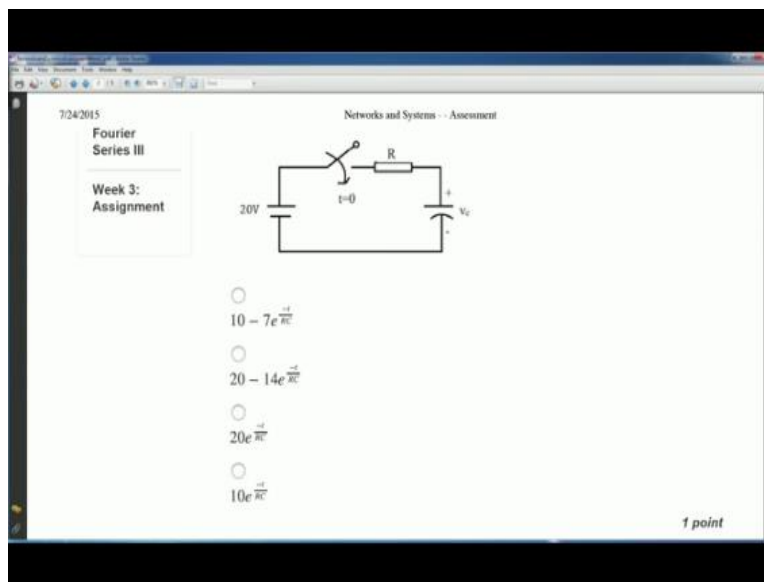


And as you go through look at the third question from the third question on you will start getting circuits okay. So, there will be a circuit and some initial conditions and something

would happen at time 0 you will have to write, you will have to solve the circuit in some form and answer a question.

For instance, this question says there is a battery, there is a switch and there is an rc circuit this resistor r and a capacitor c okay and the questions says v_c of 0 minus the voltage across the capacitor just before the switch got turned on was 6 volts and the battery is 20 volts etc, and you supposed to find v_c of t which is the voltage across the capacitance as a function of t for t greater than 0 and write the answer.

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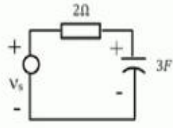
The screenshot shows a quiz interface for "7/24/2015 Networks and Systems - Assessment". On the left, it says "Fourier Series III" and "Week 3: Assignment". The main part of the screen displays a circuit diagram with a 20V DC source on the left, a switch in the top branch, a resistor R, and a capacitor C on the right. The switch is shown closing at $t=0$. The voltage across the capacitor is labeled v_c . Below the diagram are five radio button options: $10 - 7e^{-\frac{t}{RC}}$, $20 - 14e^{-\frac{t}{RC}}$, $20e^{-\frac{t}{RC}}$, and $10e^{-\frac{t}{RC}}$. The text "1 point" is visible in the bottom right corner of the question area.

So, this requires a solution from your experience you might know this $e^{-\frac{t}{RC}}$ will show up in the answer and what exact form it will take I can find by knowing the initial and final conditions. So, if you know the initial and final condition for the voltage across the capacitor you can plug it into each of these answers and you will get the answer right way. So that's the basic hint you can use the initial and final conditions for the voltage across the capacitor and you will get the answer okay.

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1 point

4) In the following circuit, if the voltage across the capacitor is $v_c = (10r(t) - 2r(2t - 1))$ V, what is $v_s(t)$? (All options below are in Volts)



The circuit diagram shows a single loop containing a voltage source v_s on the left branch, a 2Ω resistor on the top branch, and a $3F$ capacitor on the right branch. The voltage source v_s has its positive terminal at the top. The capacitor has its positive terminal at the top.

$10r(t) - 2r(2t - 1) + 60u(t) - 24u(2t - 1)$
 $10r(t) - 2r(2t - 1) + 60u(t) - 12u(2t - 1)$
 $20r(t) - 4r(2t - 1) + 60u(t) - 12u(2t - 1)$
 $20r(t) - 4r(2t - 1) + 60u(t) - 24u(2t - 1)$

The question number 4 is again another circuit and once again all circuits once solves by writing the loop equation or a node equation and using the basic equations connecting the current and voltage across the capacitor or an inductor okay. So, if you have an inductor L the voltage across the inductor is $L \frac{di}{dt}$ right where i is the current through the inductor okay so that is the first definition of course for a resistor the property is very simple voltage across is r times i okay Ohms Law.

For the capacitor, the current is $C \frac{dv}{dt}$ okay. The voltage across the capacitor is v then the current is $C \frac{dv}{dt}$ you use these equations and of course for the voltage source the voltage remains a constant okay. So, you use these equations relating the current and voltage of every circuit element and then you write node equations or loop equations and you solve them. That's the way you solve circuits

Here is a simple example it says in this circuit the voltage across the capacitance is given as a function of time they are asked to find v_s of t okay so now you can use what I told you and then you can write down this expression the voltage is given in terms of the ramp you might want to use properties of the ramp carefully and get your answer okay so that's the hint for question number 4.

Write a loop equation and use the equations relating the current and voltage of circuit elements you should be getting the answer okay. There are also other things you can do to eliminate some other wrong choices if you want to do that that's fine okay.

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5) In the following figure, what is the differential equation relating i_1 and v_s ? Use the symbols D for $\frac{d}{dt}$, D^2 for $\frac{d^2}{dt^2}$ and D^{-1} for $\int_{-\infty}^t (\cdot) dt$. Then, simplify the node/loop equations using these symbols.

The circuit diagram shows a voltage source v_s in series with a resistor $R_1 = 1\Omega$ and a switch that is closed ($t > 0$). This is followed by a parallel combination of an inductor $L = 4H$ and a branch containing a resistor $R_2 = 2\Omega$ and a capacitor $C = 0.5F$ in series. Currents i_1 and i_2 are indicated in the diagram.

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$\frac{d^2 v_s}{dt^2} = 2 \frac{d^2 i_1}{dt^2} + \frac{di_1}{dt} + \frac{i_1}{4}$
 $2 \frac{d^2 v_s}{dt^2} + \frac{dv_s}{dt} + v_s = 6 \frac{d^2 i_1}{dt^2} + 5 \frac{di_1}{dt} + i_1$

So, the fifth question is actually quite a complicated question as you can see we are given 3 points for it and it is very difficult I mean I can tell you even now if I start solving this problem if I start writing how to equations to solve this it will take quite a while I need to do some fair amount of algebra before getting to the answer.

So, the question number 5 is a difficult question, but the method is exactly the same you have a circuit with circuit elements and there are some there are 2 loops here and you have to write loop equations of mind be mindful of how the loop equation show up and solve for the for the unknown.

So here i_1 and v_s have to be related v_s is the voltage across the voltage source and i_1 is the current in the first loop and you have to relate i_1 and v_s and its possible you have to write down the equations and there is a hint that's been given use some symbols for d by dt d square by dt squared and then d inverse for the integration and you simplify you will get to the answers.

I am not saying its very hard, but nevertheless the algebra involved is a bit intense so you write 2 loop equations and solve it you will get you will get to the answer okay. So, this is a slightly more difficult problem it carries 3 points

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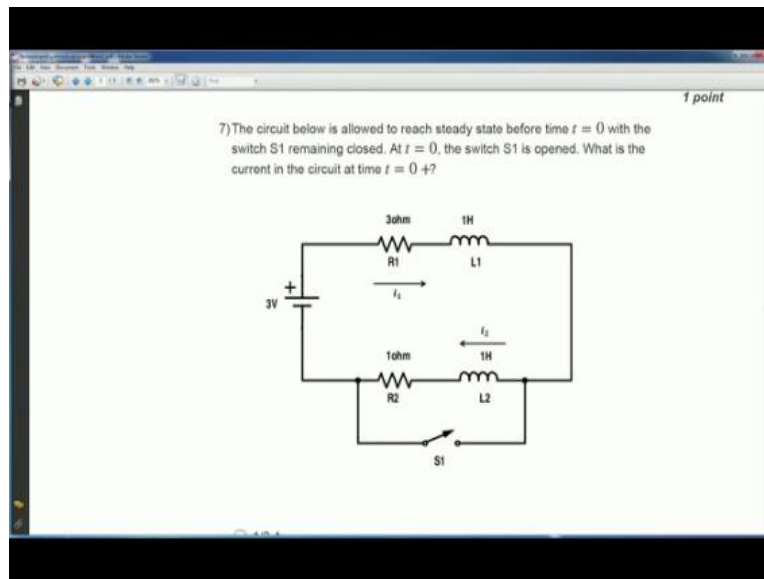
- A differential equation: $\frac{d^2 v_2}{dt^2} = 6 \frac{d^2 i_1}{dt^2} + 5 \frac{d i_1}{dt} + \frac{i_1}{5}$ (3 points)
- Problem 6: "To a discrete time LTI system whose input-output relation is $y[n] - 4y[n-1] + 4y[n-2] = x[n]$ the input 2^n is given for $n \geq 0$. The initial conditions are to be set to 0. Find the output of the system at $n = 3$." (1 point)
- Problem 7: "The circuit below is allowed to reach steady state before time $t = 0$ with the switch S1 remaining closed. At $t = 0$, the switch S1 is opened. What is the current in the circuit at time $t = 0^+$?"
- Circuit diagram: A 3V DC source is connected in series with a 3ohm resistor (R1) and a 1H inductor (L1). Current i_1 flows clockwise through the resistor, and current i_2 flows clockwise through the inductor.

So, the sixth 1 is the discrete time system so the reason why we put in this question is to just show you in 1 way how easy discrete time systems are compared to continuous time systems when you try to solve them. So, if you look at this equation and it says the input output relation is y of n minus 4 times y of n minus 1 plus 4 times y of n minus 2 equals x of n and the input 2^n is given for n greater than y equal to 0.

So, what does that mean x of n is 2^n okay so that's the meaning of the input and then the initial conditions are 0. So, what does it mean y of minus 1 is 0 y of minus 2 is 0 and so on okay. So, now you have to find the output of the system at n equal to 3 okay so that's it you put plug in n equal to 3 here you will get something and then plug in n equal to 2 plug in n equal to 1 plug in n equal to 0. You will get to the answer okay.

So, you just repeatedly use this equation you can find the answer without any problem. So, the discrete time system at least for the initial part just solving for a few inputs and outputs, it's quite easy and its very simple substitution okay

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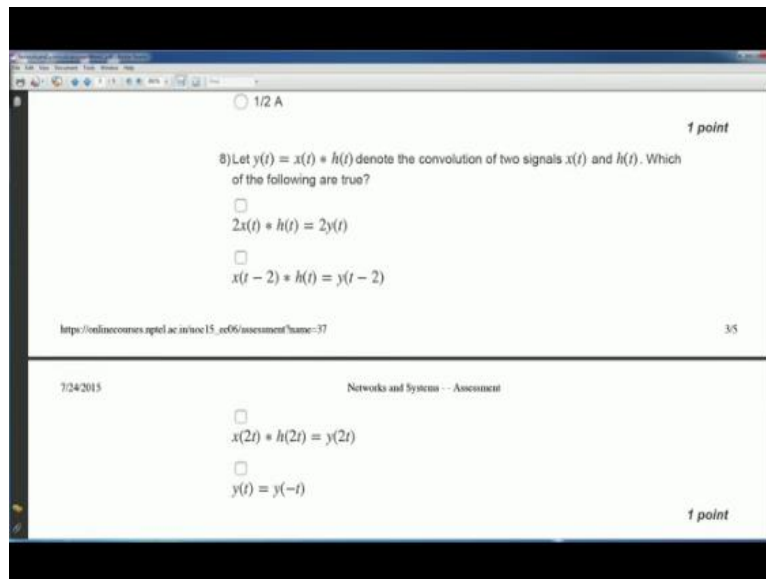


So, the seventh question involves a little bit more complex ways of solving circuits. So in this problem, a switch remains closed for a long time and then its opened, so its closed for a long time so the 1H inductor will have some current flowing through it sometime and then suddenly there is in a loop another inductor is introduced after t equal to 0 okay.

There is detailed lecture in the in the lessons about such things inductor loops and capacitor loops and how to go about doing them its there will be some discontinuities you have to you have to think of the think of that method very carefully this question is very similar to the method that was used this question is very similar to 1of the solved examples in 1of the lectures.

So, you go back and look at the lectures closely you will see what to do is not a particularly hard, but it's an it's an interesting problem and hopefully you will be able to get to the answer okay. So, this is this involves node equations, but you have to pay particular attention to the initial conditions what happens from 0 minus to 0 plus there will be some impulsive scenarios and you have to pay attention to that and you get to the answer carefully okay. So, that's the hint for question 7.

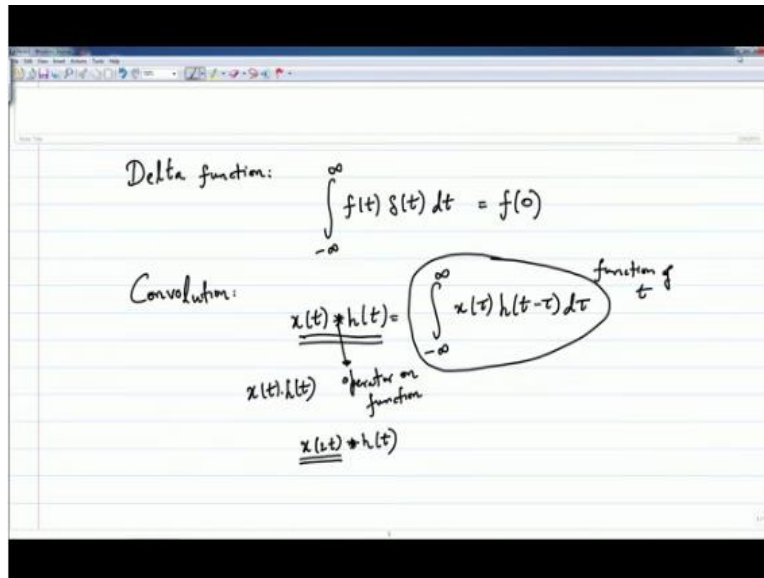
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And question 8 is on convolution okay so, in electrical engineering particularly in systems I cannot emphasize the importance of convolution more okay. It is 1 of the most crucial operations that I needs to study in various forms you need to be very comfortable with the basic notion of convolution, how to convolve 2 signals various properties of it how to think about it all these things are very critical.

So, I would suggest doing as many problems as possible in convolution so that becomes really, really familiar with it. So, this question asks you to check various things and a word about the notation in convolution I is to be very careful when I write y of t is x of t convolved with h of t . It's slightly complicated you have to think of x of t as a function for all t and you have to think of h of t as a function for all t . Now convolution is an operation, which involves these 2 functions and then these 2 functions for all t and solves for the answer y of t okay.

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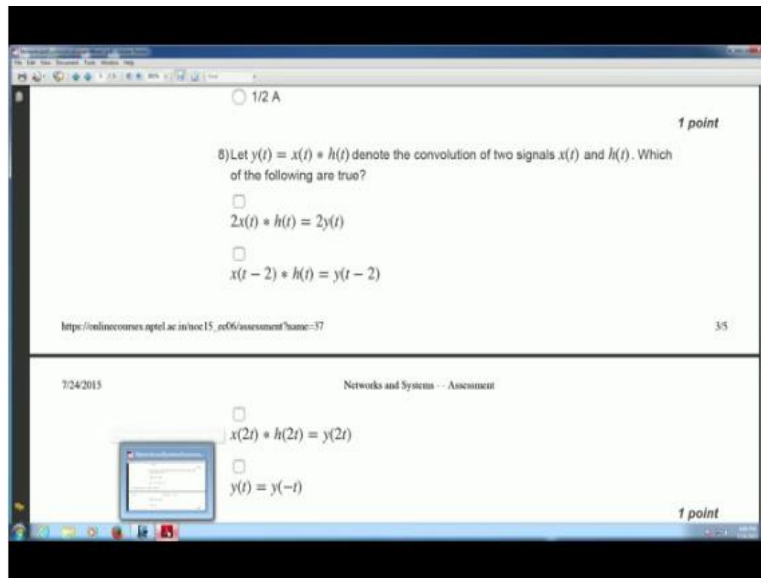


So, let me emphasize let me write up something for this just to tell you what is how does this works so look at convolution okay. So, if you want to write x of t convolved with h of t could write in 2 ways let me write it 1 way minus infinite to infinity x of tau h of t minus tau d tau.

Okay, so normally if you take an operation so, for instance if I write an operation I am sorry let me write it once again write an operation x of t multiplied with h of t okay. Now, if you write an operation x of t multiplied by h of t to find the result of this operation at time t I only need to know the value of x at time t and value of h at time t. Okay, only values are time t you take those 2 values multiplied you will get the answer, but convolution even though we write it like that x of t h of t convolved it is not enough if I know values of x only at t and values of h only at t.

No way, you cannot find the convolution that way this is just a short hand notation to denote this complicated integral and this integral as you can see uses the values of h and the values of x for all time okay. So, this notation is very misleading particularly when you write x of t h of t it looks like you are using the values of x and h only for t, but that is not true you are using the values for all t you are treating this as a function of t a full entire function of this.

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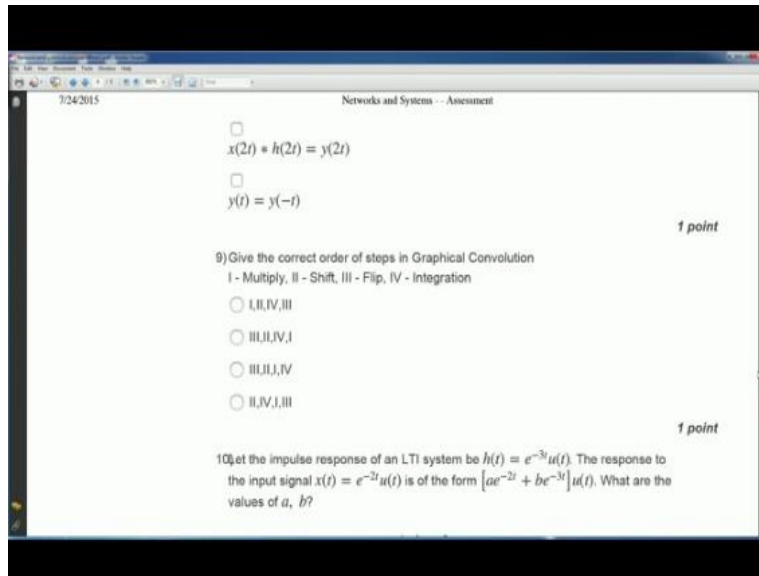
So, this star is more like an operator on functions. Okay, the entire function x of t is operated with a entire function h of t by convolution to get this answer okay. So, what you get here, is also a function of t right okay so, when I say x of $2t$ convolved with h of t what does that mean i have this function x of $2t$ i have to take that entire function and convolved with h of t .

Okay, so sometimes it can get a little murky as to what I mean so you have to you have to be careful in how you how you put this thing in and evaluated so this question in particular will test your understanding of how functions are written etcetera.

So, it tells you if you if you look at the first option for instance if you convolve the signal 2 times x of t with h of t what will you get? Will you get the signal 2 times y of t that's question of the first the first part of this eighth question is asking similar question part 2, 3 and 4.

Okay, the fourth one is the little bit different it says is it always true that y of t is equal to y of minus t is it always true that y of t is even symmetric if you just convolve 2 signals is it true that you will get a even symmetric signals and think about it and you will be able to answer some of these questions okay.

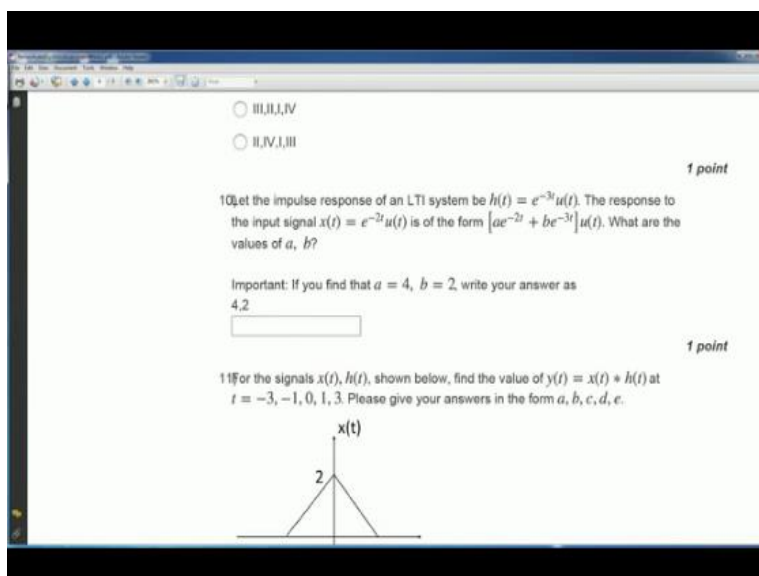
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Now ninth is an it's once again a simple question on convolution, but this is something you have really master you should know when you convolve 2 signals what should you do? There is a series of steps that you have to do you have to do something first and something next and something third and something fourth.

There are 4 steps in that and the 4 steps are multiplication, shifting, flipping and integration okay. In what order, you do them is the question and if you know the definition you can look at closely and you get to the answer okay. So, the tenth question okay.

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So, the tenth question is a kind of illustration of why impulse response, why LTI property, why convolution all of them are really, really crucial now if you take a system you want to describe the system. I was mentioning in 1 of the previous lectures that the input and output relationship is a complete description of the system.

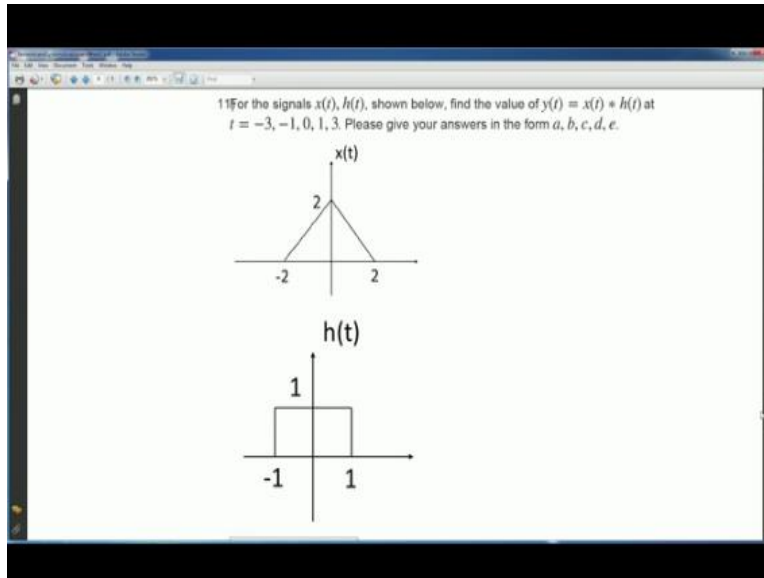
How do you find y of t for an input x of t , so, quite often the system itself you may not be able to analyze all the things that actually happen in the system and finally you may not be able to write down y of t is a function of x of t that might involved really complex arithmetic and all kinds of calculation?

I was just mentioning in the previous circuit, which had a 2 loops and you have to find i_1 is a function of v_s and you do see how hard it is to do those manipulations so what is desirable is some nice property of the system that you can exploit and come up with very simple descriptions of the entire system and as it turns out impulse response is 1 such powerful idea for LTI systems.

So, if your system is linear and time invariant if I tell you what the impulse response of the system is, you can find the output y of t corresponding to any input x of t and that is done by convolution okay. Y of t is x of t convolved with the impulse response h of t . This is the critical and crucial idea and why we study LTI systems why LTI systems are so popular for analysis and all that okay.

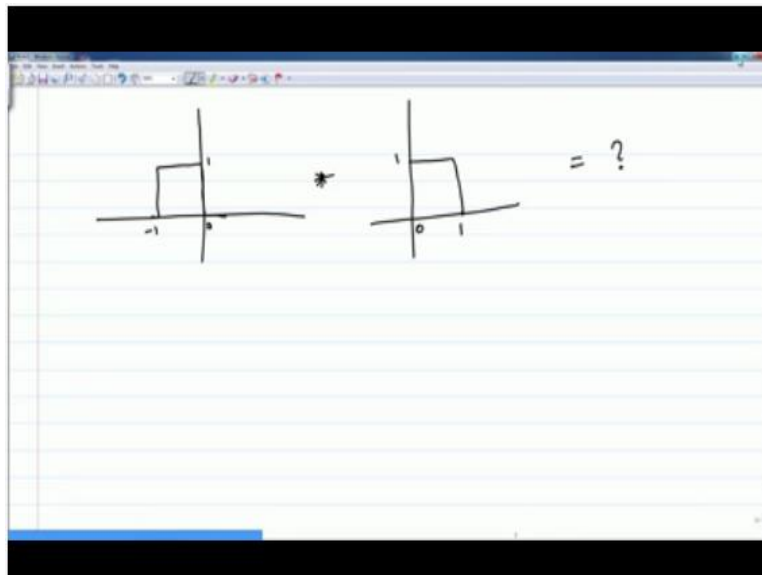
So, this is the crucial and if you look at question number 10 again it describes an LTI system through the impulse response says that an LTI system whose impulse response is like that and if you give any other input signal you have to perform the convolution and get to the answer okay. So, it is quite nice to see how this is done.

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And question number 11 is practice for convolution okay so there are signals define there and you have to do the convolution and you have to do the 4 operations in a certain sequence and you get the value and be asked to evaluate the convolution at minus 3, minus 1 0 1 and 3 and you will do it carefully get to the answer and I believe this should be okay enough just as a motivation for you if you are interested in slightly more difficult convolution.

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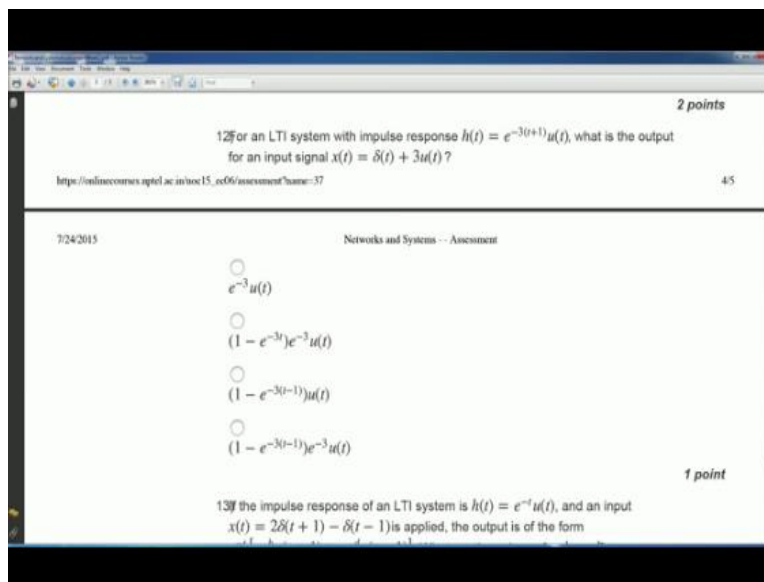


I would urge to you to try the following okay. So, this is a I should say is a slightly more difficult convolution can you convolve these 2 signals 1 guy is like this is 1 from minus 1 to 0 okay its a square pulse is 1 from minus 1 to 0 0 elsewhere. You might to want to

convolve this with this signal which is again 1 but from 0 to 1 okay so if you convolve these 2 guys what will you get okay.

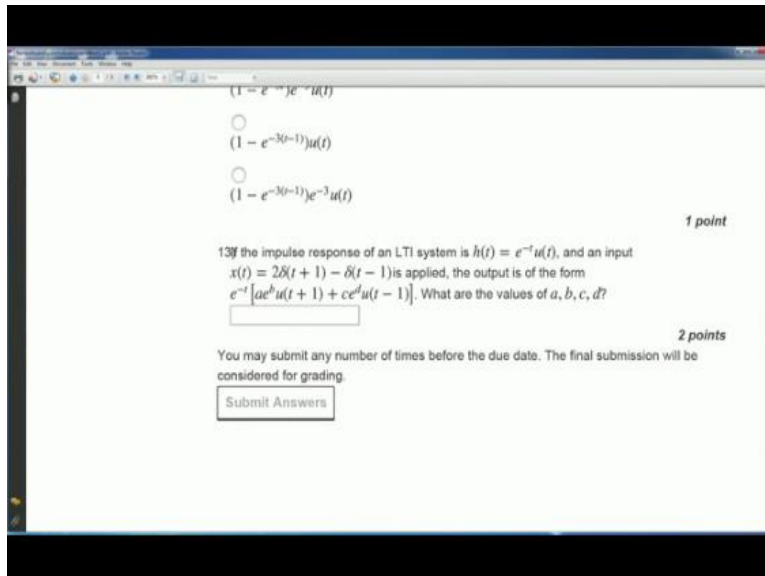
So, this is an interesting question you might get a fairly surprising answer, but this will test your grasp of how this how convolution is. If you can do this convolution quite well, I think you doing okay so this is good practice I would encourage you to try this convolution and practice see if you understood the operations quite well okay. So, let's go back to the assignment questions, so, this question just as basic convolution.

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And once again you come to the 12th question, which is again a classic question on LTI signals LTI systems. You have an impulse response x of t and now you have slightly more complex input signal is delta of t plus 3 times u of t which you know this is an LTI system you can use so many different properties and convolution etcetera, etcetera and you will get to the answer quite easily its the standard LTI system question.

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And the 13th is also a pretty standard LTI system question you have the impulse response you have the input I mentioned the property. If how to find the output is a convolution and here you have this delta functions and unit step showing up and these kinds of questions build up your comfort in dealing with delta functions in calculations how do you do how do you describe how do you do convolution with delta functions.

How do you convolution of delta of t plus 1 okay this is a property of the delta function, which is described in the lectures you can go ahead and use that and solve the problem okay. So, this was a short video on hints for solving assignment 2 hopefully this hints will help you to go ahead and look at the assignment 2 questions once again in a different light and score high marks in the assignment 2. Thank you very much.