Networks and systems

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Week 1 Solutions

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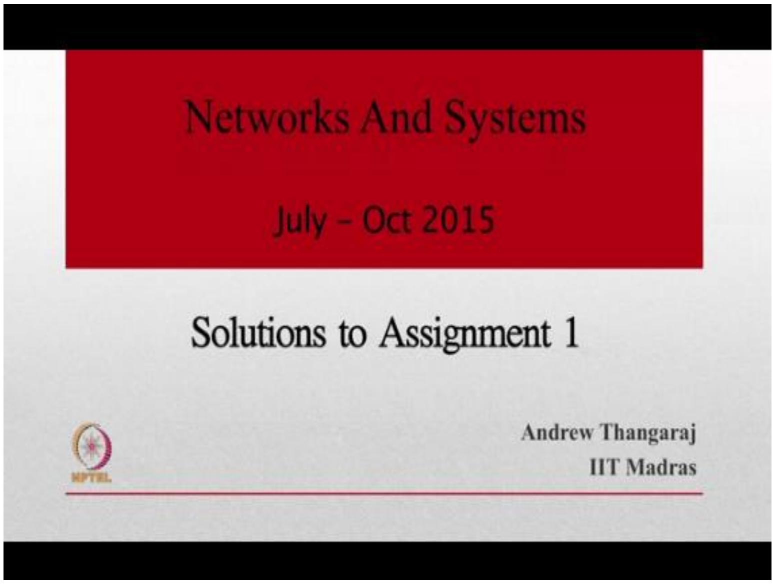

Hello every one, I hope you are having a good time learning about network and systems. Many of you have submitted the first assignment I was very happy to see that and this short video is aimed at describing the solutions to the first assignment. So, I picked a few questions important questions which I think are very representative of the type of questions you might get in this class and I would like to describe the way to go about solving them, the main steps involved in solving them.

The exact solutions will be provided as a separate file in the in the portal as well, but this video hopefully will help you to learn about problem solving in this subject, which is quite important, okay.

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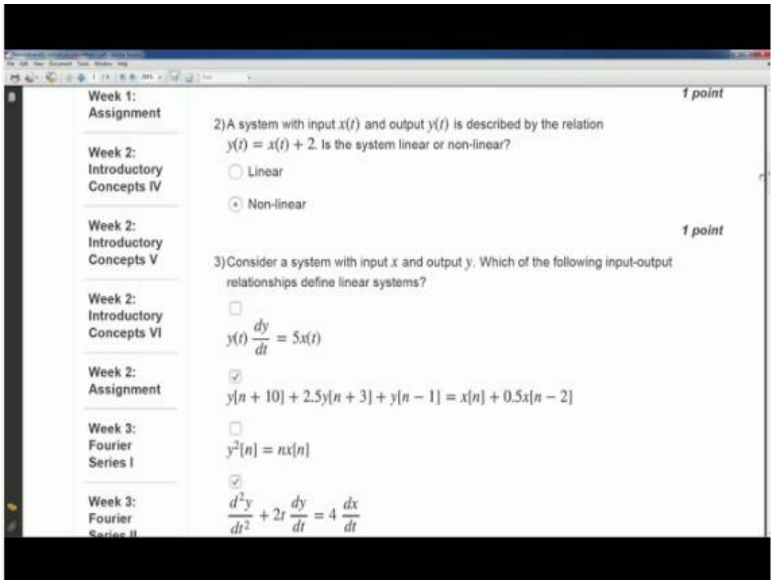

So, let me proceed, so, if you look at the first assignment there are quite a few questions, but there are many questions, which go like this. A system is described first, what do you mean by describing a system, a very typical way of describing system is to provide an input output relationship, a mathematical equation, which can be used to find the output when an input is given.

So, for instance if you look at question number two it reads a system with input x of t and output y of t is described by the relation y of t equals x of t plus 2 so, that is a mathematical description of the input output relationship. If the input signal is x of t, you are going to plug in to this equation and you will get an output signal y of t okay, now that is a complete description of a system.

If you know, for every input how to calculate the output you have already described a system entirely so, that’s a precise and complete description. Once you have such a description, you can answer the questions about properties of the system. So, one question that’s asked in question number 2 is the system linear or nonlinear? Now once again a system is described by the equation y of t equals x of t plus 2 and the question that is asked is the system linear or nonlinear?

So, how do you go about solving a question like this so, let me take that up as an illustration, okay.

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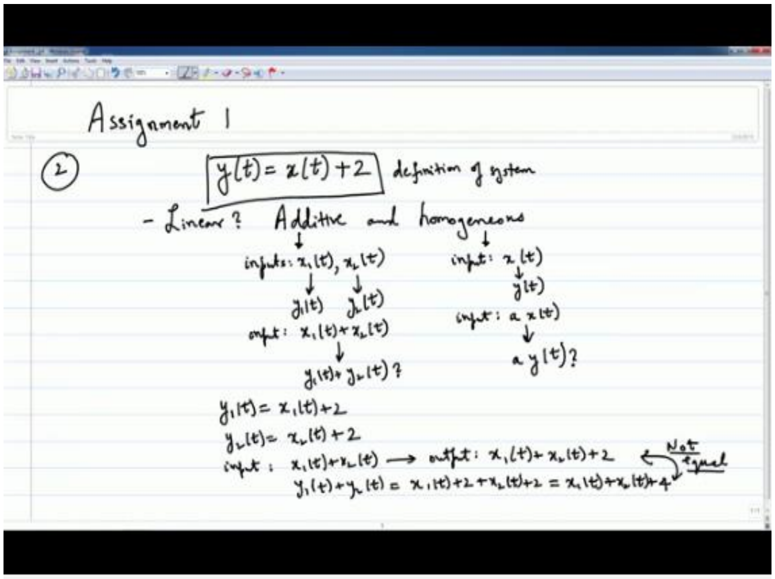

So, lets look at the question number 2 in assignment 1, you have a system which is described by the input output relationship y of t equals x of t plus 2 so, the question that is asked is, is it linear or nonlinear? Okay.

So, how do you go about answering such questions, so, here is a little tip all problems in this class will be quite simple in a certain sense okay so, there will be a certain definition for a property and you will be asked to check whether that property is true, so, this is the very typical way of doing. so, you have been given a system here, y of t equals x of t plus 2 and here is a property, which is linearity of the system.

So, now the way to solve this questions is first you have to read the mathematical definition of linearity of a system and then apply that definition to the description of the system and find out if it is linear or not, that is the way to solve these questions. So, most of you many of you, if you look at this equation you might want to indubitably answer this question now it looks linear or its nonlinear etcetera but that’s not the way to go about actually solving it and system has been described to you a property he has a mathematical definition you have to verify whether the definition is satisfied or not that's the way to solve this problem.

So, what is the mathematical definition of linearity, so, linearity you have to check for 2 properties first is additive and homogeneous? What is additive?. Additive means if you have 2 inputs x1 of t and x2 of t and if x1 of t produces an output y1 of t and x2 of t produces an output y2 of t. The input x1 of t plus x2 of t should produce an output y1 of t plus y2 of t. This is the definition of the additive property.

What is the definition of the homogeneous property? If you have an input x of t which produces an output y of t and then you look at the input a times x of t where x is in general a complex number you should get the output a times y of t. Any system any input output relationship for a system, if it satisfies these two properties additivity and homogeneity then that system is said to be linear okay.

So, now we have a system here y of t equals x of t plus 2 and one needs to check whether both these properties are satisfied by this input output relationship. so, lets look at that if you have, lets look at additivity if x1 of t is the input output y output represented as y1 of t so, now input output relationship, I know that y1 of t equals x1 of t plus 2 right. If x1 of t is the input how to find what the output is, you plug in x1 of t instead of x of t in the input output relationship the answer you get is the output.

Likewise, y2 of t is going to be x2 of t plus 2 okay now after say put question marks here because these things may not be true I have to check if these true or not I have to see if the input is x1 of t plus x2 of t will I get the output as y1 of t plus y2 of t so, now this is the definition of the system right, this input output relationship is the definition of the system.

Now, I have to use the definition to figure out what the output would be that's what I have done here so, y1 of t and y2 of t now if the input is x1 of t plus x2 of t what is the output going to be if x1 of t plus x2 of t is the input, the output is going to be instead of x of t in the definition of the system, I have to plug in the input that I have the input I have x1 of t plus x2 of t so, the output is going to become x1 of t plus x2 of t plus 2 right

According to the definition of the system if x1 of t plus x2 of t is the input, the output you get is x1 of t plus x2 of t plus 2. Now what is y1 of t plus y2 of t okay we know what y1 of t is x1 of t plus 2 y2 of t is x2 of t plus 2 so, y1 of t plus y2 of t is going to become x1 of t plus 2 plus x2 of t plus 2 and that is going to become x1 of t plus x2 of t plus 4 okay, so, now one can compare these two expressions and say that these two are not equal okay

So, if I have 2 inputs x1 of t and x2 of t and if I provide the new input x1 of t plus x2 of t the output I get is x1 of t plus x2 of t plus 2 and that is not equal to the sum of the 2 individual outputs y of t, y1 of t and y2 of t so, if these two are not equal then the system that is defined by y of t equal to x of t plus 2 does not satisfy the additive property and it is therefore not linear okay.

So, this is the way to solve this problem and any other definition of the system that's given to you it could be slightly more complicated than y of t equal to x of t plus 2 you have to follow a similar approach. You have to check additivity and homogeneity and you have to use the input output relationship of the system and check whether it satisfies those properties are not okay. So, let's look at a slightly more complicated type of question, which is say question number 3.

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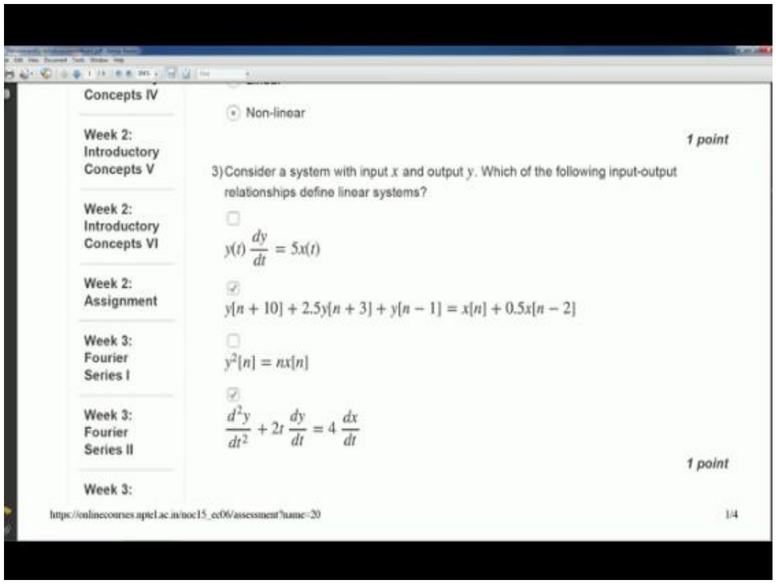

So, here is a there is a similar sounding question but the systems satisfy more complicated input output equations look at the question here. You have a system with input x and output y and which of the following input output relationships are linear that's the question, so, if you look at the first situation it says y of t times dy by dt equals 5 times x of t.

So, now how do you determine whether this is a linear system or not, you have to follow an approach very similar to before, you have to write down x1 of t so, for instance you to check additivity how do you check additivity you have to say x1 of t satisfies certain equations with y1 of t, x2 of t satisfies a certain equation with y2 of t. Now in this case the equation is slightly more complicated it's a differential equation and you may not be able to find y1 of t and y2 of t explicitly in terms of x1 of t and x2 of t.

In the previous question we were able to do it in this case you may not be able to do it. Now if you are not able to do it, it's not a significant difficulty, because you do not need the explicit form for y1 of t and y2 of t, what you really need is whether additivity is satisfied or not. So, lets see how to work with the situation like that.

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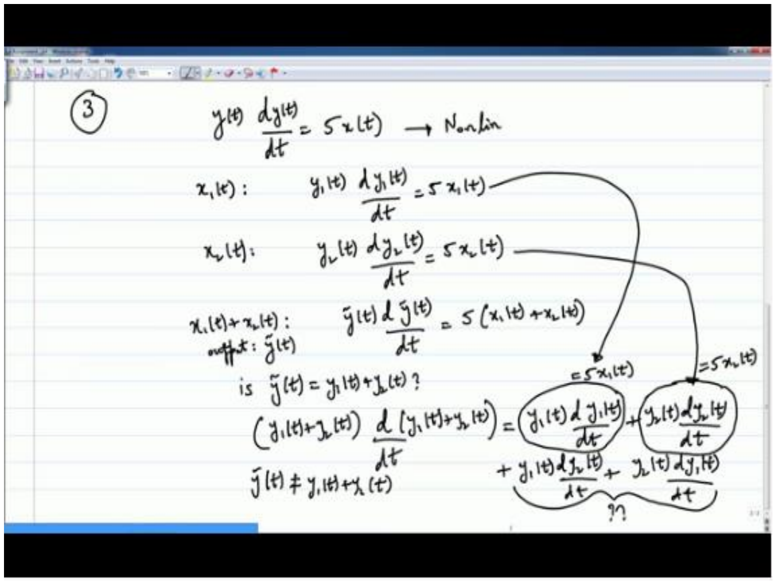

So, to illustrate that let me take the first choice here I show you what would happen, if you look at that so, question number 3 the first choice says y of t dy of t by dt equals let me just look it up its five times x of t. This is the input output relationship okay.

Now if x1 of t is the input then the output y1 of t satisfies y1 of t dy1 of t by dt equals 5 times x1 of t now if the input is x2 of t then the output y2 of t satisfies y2 of t by dt equals 5 times x2 of t. On the other hand, if the input is x1 of t plus x2 of t, then let us say the output corresponding to this we need an notation for that. The output corresponding to this lets say is I will call it y tilt of t okay. Then you have y tilt of t dy tilt of t by dt equals 5 times x1 of t plus x2 of t right.

If a input is x1 of t plus x2 of t and the output is y tilt of t, y tilt of t and x1 of t plus x2 of t satisfies this equations so, if you put an instead of x of t, if you put an x1 of t plus x2 of t and instead of y of t, if you put y tilt of t you should be able to satisfy this equation.

So, now the question is. is y tilt of t equal to y1 of t plus y2 of t that is the question we are asking if we can show in general y1 of y tilt of t is equals to y1 of t plus y2 of t then we know that the system is additive the input output relationship satisfies the additivity property now you have to plug in this, this now you have to check whether this is true or not how do you check whether this is true or not I know y tilt of t satisfies this equation okay.

So, let we try an evaluate the left hand side by substituting y tilt of t equals y1 of t plus y2 of t now if it where true to be equal then the right hand side should work how to 5 times x1 of t plus x2 of t. so, that's the way you check whether additivity is true or not without really solving explicitly y1 of t and y2 of t you can check if an equation involving y1 of t and y2 of t is satisfied or not.

So, lets plug in y tilt of t equals y1 of t plus y2 of t into this equation on the left hand side and use the previous equations and see if we get the same thing as the right hand side or not lets try that y1 of t plus y2 of t okay d by dt of y1 of t plus y2 of t okay. so, we know how to simplify these terms you do the differentiation multiply etcetera you are going to get y1 of t dy1 of t by dt plus y2 of t dy2 of t by dt and then interestingly you will also get cross terms y1 of t dy2 of t by dt plus y2 of t dy1 of t by dt right.

So, one can take this part of the expression now use this equation, I know this is equal to 5 times x1 of t similarly you can take this term right here and use this equation and now that's equal to 5 times x2 of t on the other hand these 2 terms cannot be simplified any further so, those 2 terms till show up okay.

On the other hand what happens if you put an y tilt of t in this equation y tilt of t d y tilt of t by dt equals 5 times x1 of t plus x2 of t. Now if you look at these two equations y1 of t plus y2 of t satisfies this complicated equations with these mixed terms on the other hand y tilt of t d y tilt of t by dt is simply equal to 5 x1 of t plus 5 x2 of t. so, clearly the answer is y tilt of t is not equal to y1 of t plus y2 of t right. If y1 of t plus y2 of t is equal to y tilt of t this right hand side would have simplified to 5 times x1 of t plus 5 times x2 of t these 2 cross terms will not have been that since the cross term showed up clearly y tilt of t is not equal to y1 of t plus y2 of t, which means this equation is not additive and which means it is nonlinear okay.

So, like I said before given an input output relationship you plug in these definitions of the properties look at how to substitute them into the equation test if the additivity property satisfied and you get your answer okay that's why you go about solving these kind of problems and so, let me make a few more comments about comments about linearity etcetera so, lets go back and look at the third question.

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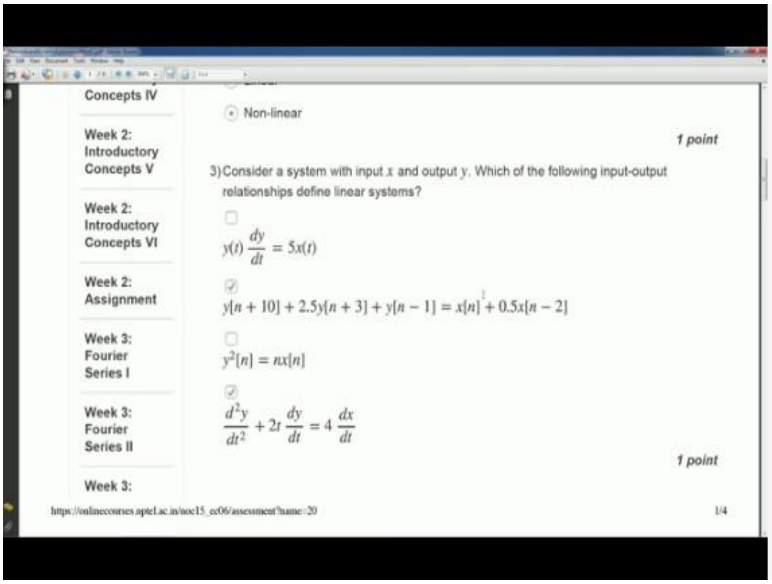

Now if you look at the third question I did some calculations to show that the first choice is nonlinear and if you do if you repeat the same kind of argument for the other choices you will get the second choice is in fact linear okay. The third choice will once again be nonlinear and the fourth choice will be linear okay, so, you can go ahead and think about it and write down the exact steps that I wrote one for the problems and you will be able to show that linearity holds for both the second and the fourth choices.

Remember, you have to check both properties for linearity both additivity and homogeneity I only showed checking up additivity you also have to check the homogeneity property that's very important okay and you can do that, but there are some general rules of thumb to remember if you have an input output relationship and that input output relationship involves a product of terms or a squaring of terms for instance.

Here is an y square n and here is a product y of t times dy of t by dt okay if it involves a product of input output a product of output with some derivative of a product any product of that form for instance y square of n itself is a product of y of n with y of n right if it involves any product taking products of inputs and outputs is not a linear operation so, that is the hint for you that the first and the third are going to be nonlinear.

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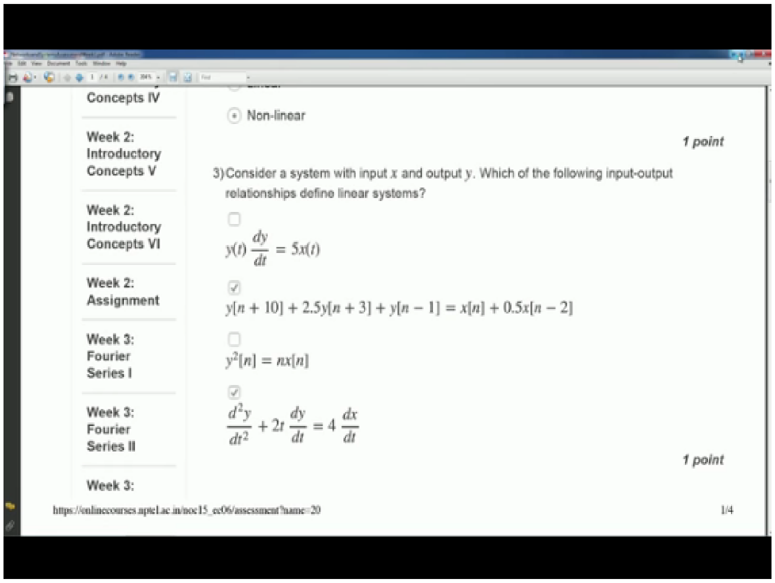

On the other hand if you look at second and fourth you do not have any products of inputs and outputs, you might ask me there is a product of t times dy by dt, but that is not a problem if you have the independent variable multiplying the input and the output that is not a problem. The independent variable is t and that's not really going to affect the input and output linearity or nonlinearity if it multiplies with the signal or its derivative or the outputs or its derivative.

If y on the other hand multiplies with itself then it affects the linearity so, this is the good rule of thumb to remember, but that they itself not proof if you want to show that something is linear or nonlinear you have to write down the equations like it and prove it okay.

Now there is lots of interesting questions in the forum also about linearity and homogeneity and additivity. It turns out there are additive systems, which are not homogeneous and there are homogeneous systems, which are not additive. There are some examples interesting examples which you can use and may be I should highlight one of them to show you how both of these are possible just to give you a little hint us how this works out for instance.

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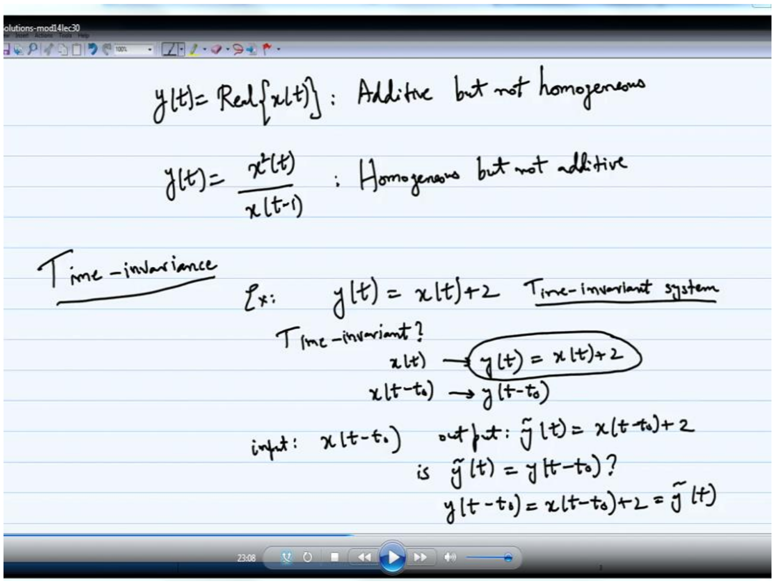

Here is an example if you have a system y of t is real part affects of t it turns out this guy is additive, but not homogenous okay so, think about it if you multiply a times x of t and if a is complex its not the same as multiplying a complex number with the real part and clearly it will not be homogenous.

On the other hand, it will be additive x1 of t plus x2 of t will give you the sum and here is another interesting system, which is x square of t by x of t minus 1 okay. This guy it turns out homogeneous but not additive okay. So, instead of x of t if you put an a times x of t you see the square on top will cancel with at the bottom and you will get back a times the output.

So, it is homogenous, but clearly it is not additive. I gave you the rule of thumb about products and here is a product and a quotient an x square of t divided by x of t minus 1 clearly it will not be linear, but it will not be additive it is easy to show that but it is homogenous.

Okay so, both these conditions are kind of independent of each other and you have to check both any question you have to check both so, that's an interesting little tip bit about linearity and homogeneity etcetera okay.

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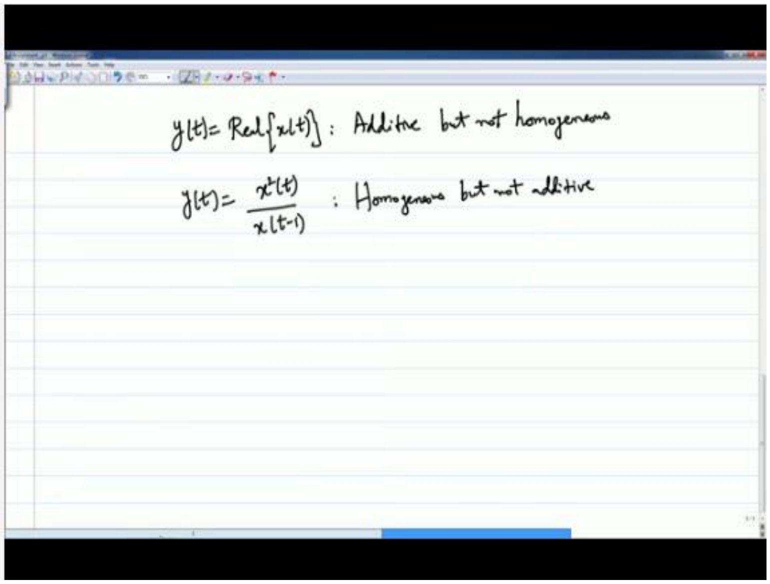

All right so, lets look at a few other questions okay that's third question and the fourth question I believe is very similar but you allow you have asked to check for time invariance and time variance okay.

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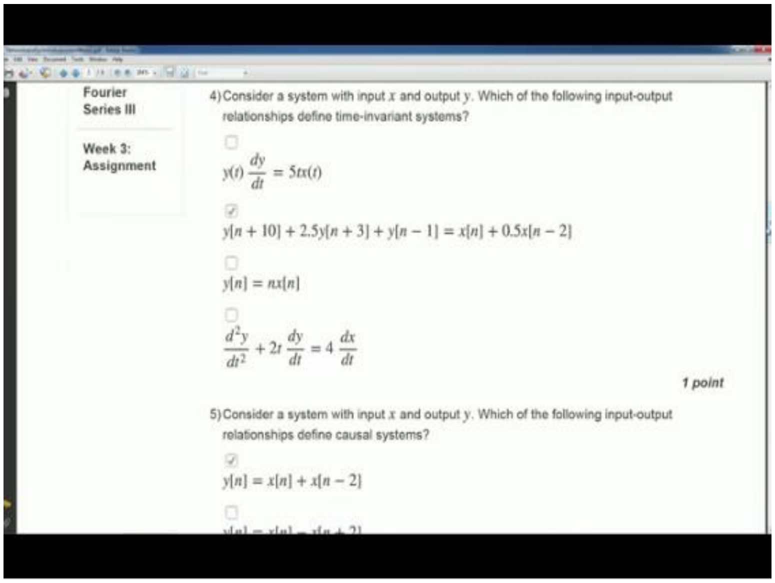

So, now once again let me may be show you one way of checking for main method for checking for time variance and time invariance may be we should do one question of that type okay. So, when we talk about time invariance next and for time invariance for instance question number four in your first assignment is for time invariance it has quite a few complicated examples.

So, I am going to start with a very simple example look at the system, we had in one of the earlier questions y of t equals x of t plus 2 and asked the question we saw that it was a nonlinear and ask the question whether it is time invariant or not is this time invariant or time variant okay, so, how does one solve such questions.

Remember the definition for time invariance is quite simple if x of t produces an output y of t, x of t minus t0 should produce an output which is y of t minus t0 okay. This is the definition for time invariance, and one needs to check them so, if x of t produces an output y of t I know that may output x of my input and output x of t and y of t satisfies the equation okay.

So, y of t is x of t plus 2 I know that okay right now if x of t minus t0 is the input and the output is y tilt of t I know y tilt of t equals x of t minus t0 plus 2 okay is that right so, the question now is, is y tilt of t the same as y of t minus t0 okay. The question is is y tilt of t equal to y if t minus t0 for every t.

That is the question so, how do you answer that you try to find y of t minus t0. How do you find y of t minus t0 i know y of t is x of t t plus 2 this equation is true right. Here instead of t I need to replace it with t minus t0 so, if I do that I see that y of t minus t0 is the same as x of t minus t0 by plus 2 and that as it turns out is the same as y tilt of t. So, it turns out this input output relationship defines a time invariant system. We saw that it was not linear, but it is time invariant okay.

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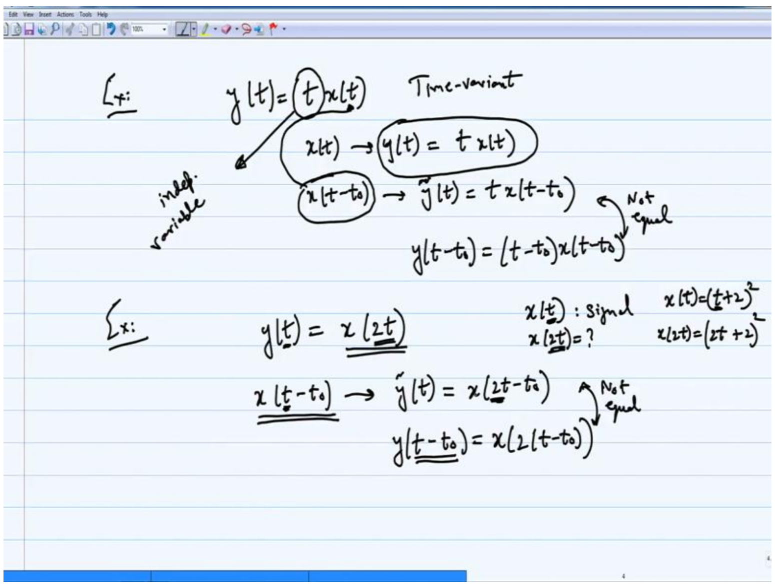

So, let me take another simple example to illustrate how you go about finding time variant or time invariant systems. Here is the other example y of t equals t times x of t. Okay, so, it turns out if the independent variable t appears by itself in the input output equation that’s one of the hints to show that it is a time variance system. It is not a time invariant system.

Okay, the independent variable if it appears by itself in an input output equation that is a hint pointing towards a time variant system. Okay, one can check this very easily if x of t is the input and it produces y of t, we know that y of t satisfies this equation t times x of t.

Now if x of t minus t0 produces an output, y tilt of t, we know that satisfies t times x of t minus t0. Okay remember that that's the equation if the input so, if I say my input is x of t minus t0. How do you find the corresponding output? I have to take that input x of t minus t0 and replace instead of x of t in my input output equation I should put x of t minus t0 I am not allow to touch anything else in the input output equation okay. So, keep that in mind.

If I have an input output equation involving an inputs signal x of t and I want to find the output corresponding to the input x of t minus t0 instead x of t wherever you have x of t you have to put of x of t minus t0. You should not change it to anything else. A lot of people invariably end up saying when if my input is x of t minus t0, the end up saying the output will be t minus t0 times x of t minus t0 that's not what this input output equation says.

This input output equation says y of t equals t times x of t, which means if the input is x of t minus t0 instead of x of t you should put x of t minus t0. Okay it is not the same as changing t by t minus t0 keep that in mind okay. So, y tilt of t is t times x of t minus t0 on the other hand what is y of t minus t0 how do I find what y of t minus t0 is I have this equation for y of t y of t is t times x of t now how do you find y of t minus t0 instead of t, I should put t minus t0 that is the definition thats the meaning of this equation.

If I say y of t is t times x of t instead of t, I have to put t minus t0 to get y of t minus t0. So, there, I will get t minus t0 times x of t minus t0 and you see these two are not equal and this input output equation is time variant okay. So, like, I said a very good hint is to look at this independent variable if the independent variable appears by itself in the input output equation then it is going to be time variant.

This is very useful hint many questions one can use this and quickly solve the problem otherwise you would be going you will be repeating this calculations. Here is the another example, which is quite involved and sometimes people end up mixing it up so, I want to show this example.

Lets look at x of t y of t equals x of 2t. Here, is another interesting input output equation and let us try to find if its time invariant or time variant. Once again what is the way to do it if input is x of t your output is going to be y of t and y of t satisfies the equation?

Now I want to find if input is x of t minus t0, what will be the output, okay so, this is a bit of slightly confusing question okay. So, I have x of 2t here and I have x of t minus t0 okay what will be the output for y tilt of t okay. So, think about it the correct answer is its x of 2t minus t0 okay. Remember why this is true; whatever means by x of 2t, okay everything boils down to what I mean by y of x of 2t okay.

If I give you x of t as the signal, x of t is some signal or a function of one variable as I say. To find x of 2t what should you do what is x of 2t, wherever you have t in the expression of a signal, I have to put 2t that's what this means when I say x of 2t that's what I mean.

Wherever I have t I should substitute 2t. So, for instance x of t could be, I don't know t plus 2 whole square okay how do I find x of 2t, wherever I have t, I should put 2t. I should not touch anything else. I am not allowed to touch anything else right. So, x of t is some function of t, what is x of 2t wherever I have t I should put 2t. Now a question that has been asked here is x of t is x of t minus t0, write the input x of t the function x of t is x of t minus t0. What is 2t instead of t I should put 2t.

So, I get the output y tilt of t being x of 2t minus t0. Think about this once again. This is the meaning of x of t minus t0 being input to the system. The instead of x of t I should think of it x of t minus t0. Now my signal has become x of t minus t0. How do you find x of 2t, wherever I have t I should put in 2t and that's what I get the answer.

On the other hand, y of t minus t0 is what I have an equation for y of t y of t is x of 2t wherever I have t, I have to put no I am sorry y of t is x of 2t okay. Instead of t I should put t minus t0. On the right hand side, instead of t I should put t minus t0 and I will get something like this okay.

So, you see these 2 are not equal and this is the time variant systems. So, once again this is another useful hint to have. If you have constants multiplying the input variable in the independent variable inside an argument, if you have constants inside an argument it's not going to be time invariant okay.

So, the only trick here is the definition of functions independent variable, dependent variable what is the meaning of saying the input to my system is x of t minus t0. Okay so, what I do when I put y of t minus t0, so, how do I evaluate these things? How do you I compare?. There are some definitions here think about what I wrote down here in the example and you will be able to solve these problems okay.

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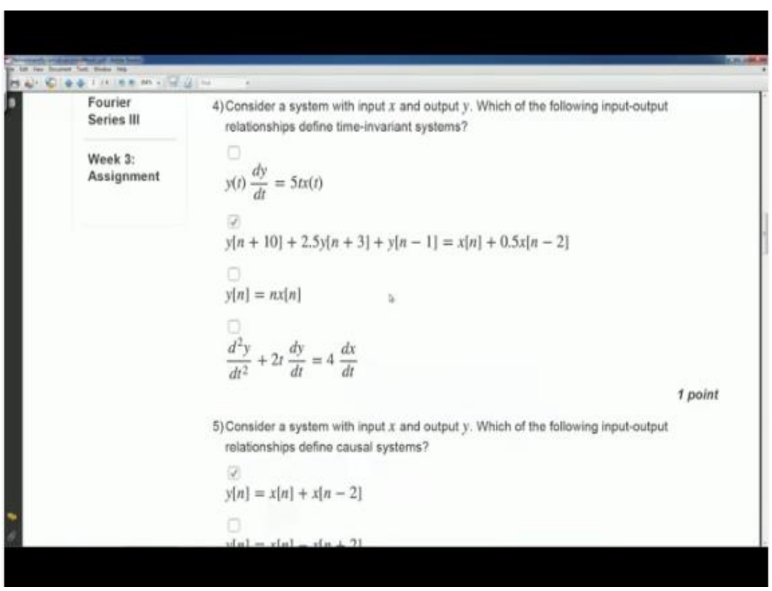

So, that's the discussion on time invariants and if you go back to your fourth question, if you go back to your fourth question you see if you use the rules I told you. The first question has t appearing by itself, third option has n appearing by itself, the second independent variable and the fourth option has t appearing by itself, again an independent variable.

So, the first, third, and fourth are time variant and not time invariant. The second one on the other hand, has no other problems it is a time invariant system. So, one can quickly do this okay. So, I am going to skip the causality question its turns out the answers of first and third, and which of the questions are real, which of the following signals are real.

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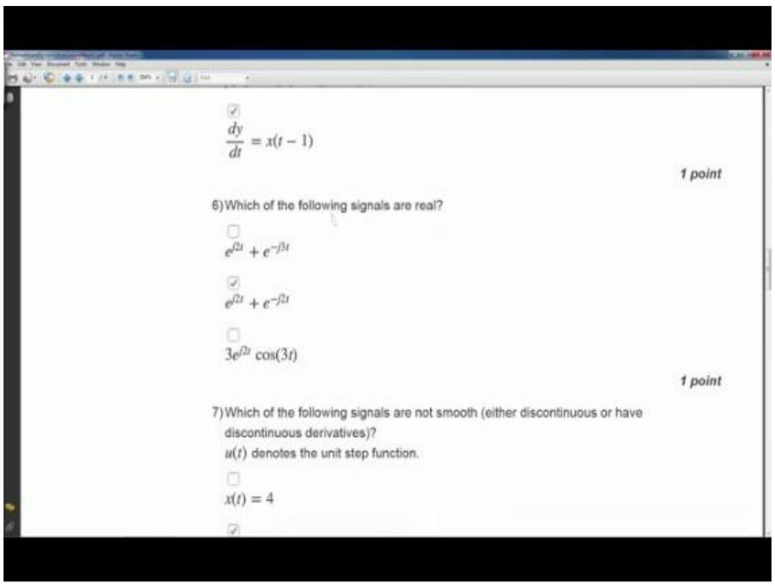

It is again an easy definition of the real number system, a useful hint is number x is real, if x equals x conjugate okay. That's the way to check if some complicated function is real or not. Okay so, if the number is given to you, you can quickly say the imaginary part is 0, but sometimes it might be a complicated way, in which the imaginary parts add together and becomes 0 okay.

So, its may not be easier for you to check, but useful definition for real number is x equals x star. If number x is real than x is equal to x star. Okay the conjugate x is equal to x if only if x is real. So, one can use that and quickly you will see immediately you settled that this guy is a real right.

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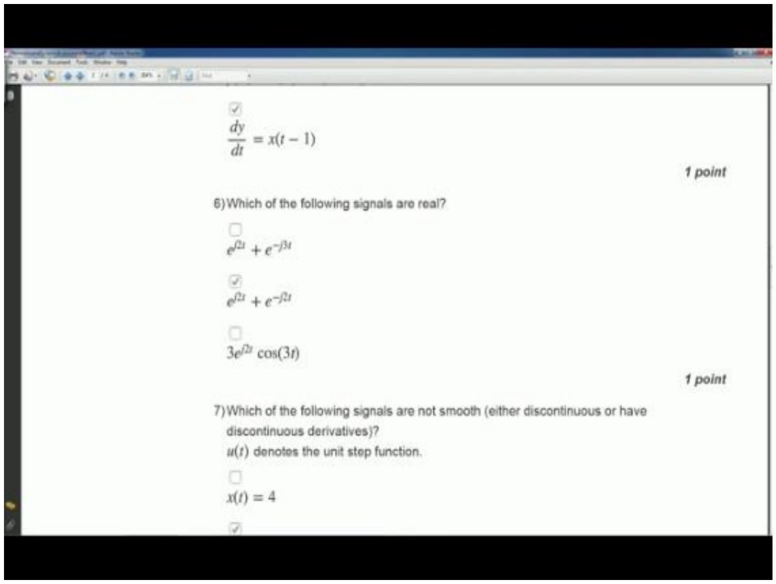

If you take a conjugate, what happens you get e power minus j2t, but then the second term becomes e power plus j2t and is the same as the first one, okay on the other hand other 2 guys will change if you take conjugate okay? The next question is about smoothness of signals okay, which is a continuous signal, which is discontinuous etcetera okay.

So, in this class we are not reviewing calculus too much. When the first weeks lectures we pointed you to a lot of nice web pages on continuity of functions basic calculus, I will urge to you go and look at it. It is not a very hard area, but we are not going to spend too much time on it and in general a very simple Layman's definition is if in the plot of the signal, you have any jumps then its discontinuous and if in the plot of the signal you have any corners then the derivative is discontinuous okay.

So, what do I mean by that let me just quickly show you couple of examples and then we will proceed okay. So, a good simple rule of thumb for if you want to get complicated the definitions okay.

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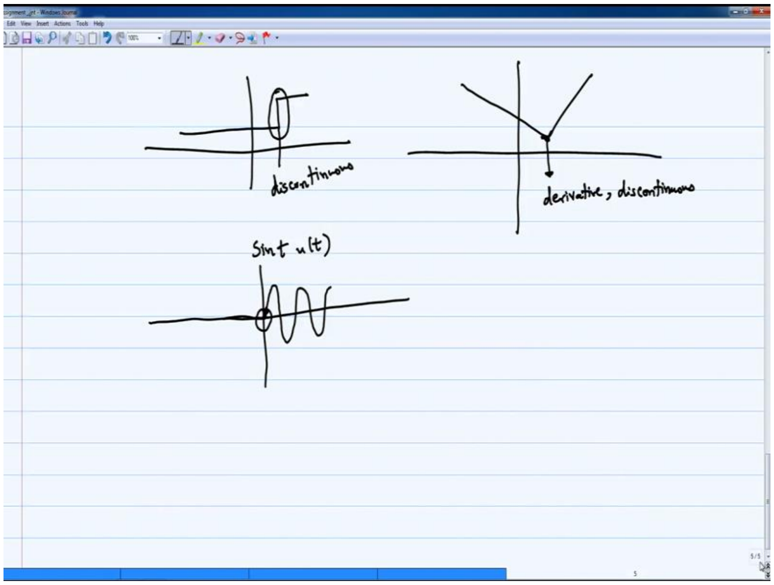

So, here is an example of let say a discontinuous function. If you have a step like this okay this is a point of discontinuity. Okay discontinuous at this point, if you have a corner in your signal, what do I mean by a corner something like this okay.

If you have sharp corner point where the function takes step here it is continuous okay the function continuous to be continuous as there is no jump, but then there is a corner, which means the derivative will be discontinuous here. So, think about why this is true and this is the hint you can use quite often to figure out whether or not the function is continuous or discontinuous and whether the derivative is discontinuous or not.

For instance, one of the questions that I given is say sine t u of t okay if you sketch this guy, you going to get it will be 0 through out on the left and then it will start right here and go like that so, you can notice this is a continuous function this is nothing no problem with continuity.

Then here there is a sharp corner okay takes off like that, so, the derivative is going to be discontinuous at that point okay. So, this is the only hint I want to give you on discontinuous and continuous functions. I will urge you to look at all the notes that’s we pointed you to in the week 0 material on continuity, discontinuous derivatives etcetera. It will help you gain up deeper understanding of were all these things are coming through.

Okay, and the next type of question you have when assignment 1 is evaluating values for the signal I will let you do that.

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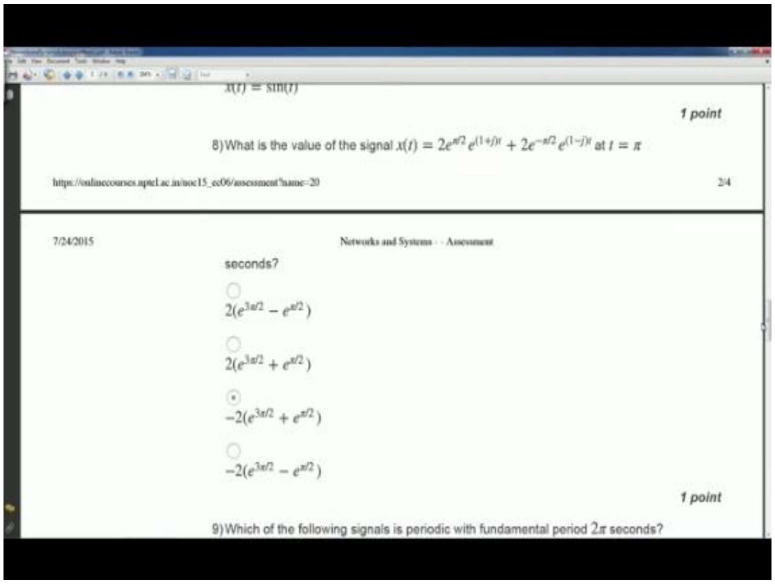

It is not very hard if you look at this question you simply how to put equals pi and remember that e power jpi you remember the unit circle definition I gave you for e power j theta right look on that is a very powerful definition to use. If you simply use that e power j pi you will see is minus 1. What about e power minus j pi okay, whether you do plus pi or minus pi, you hit up the 180 degree angles.

So, you get minus 1. So, you use the power j pi use the power minus j pi is minus 1 you can solve the eighth question. This no more nothing more to it okay.

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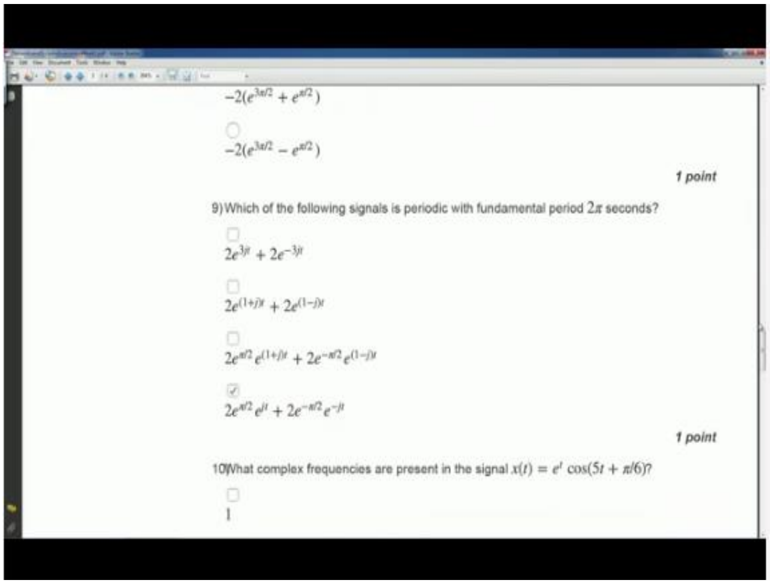

The ninth question is on periodic signals. So, let me spend a few minutes on periodicity and how to go about working with that okay.

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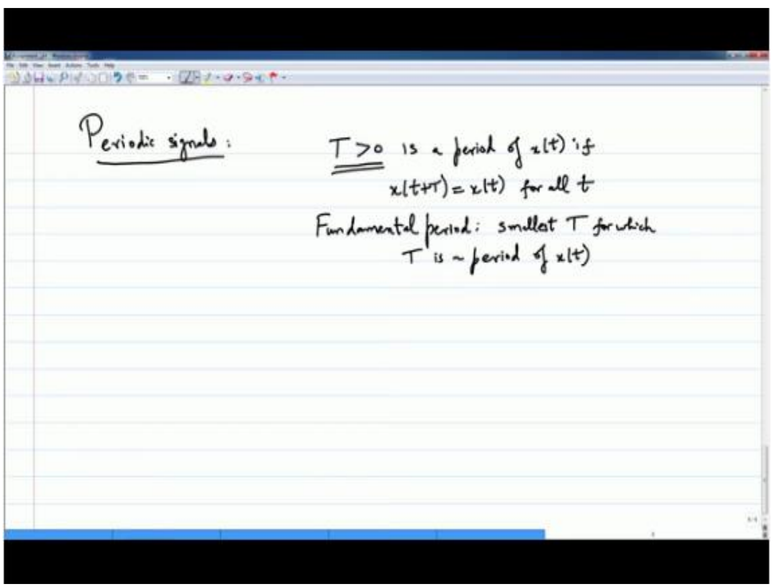

So, periodic signals, so, there is some pitfalls have here, but usually it is not too hard to worry about it okay. So, there are some pitfalls in the definition of periodic signals, but usually it is not too hard to check if a signal is periodic or not.

In many cases it is easy to do that. So, the definition of periodic periodicity is the following okay, t greater than 0 is set to be a period okay, so, let me definite it little bit more carefully. This period of x of t, if x of t plus t equals x of t for all t okay so, that's a way to think of periodic signals.

There is a the time period of capital t over, which the signal has to be looked at and after that for every other t, it simply repeats okay x of t plus t is the same as x of t. So, if you know, x of t from 0 to t you can simply repeat that from t2 to 2t to 3t minus t2 to 0 minus 2t to t and minus t etcetera, etcetera and then get the entire x of t. It goes from minus infinity to infinity, but only a period of t really matters for a periodic signal okay.

So, that's the periodicity and this is greater than 0 is a bit of technicality and it is important okay. So, t greater than 0, it cannot be equal to 0, because equal to 0 every signal will satisfy this so, this is a very technical kind of definition okay. So, there are something lot a fundamental period okay. What is a fundamental period? A smallest t for which t is a period, okay so, I mean I am not making a technical definition, but hopefully you understand what I am saying so, smallest period of periodic signal is said to be the fundamental period of the signal okay.

So, these 2 definitions are the definitions to use you have to check if to check if t is period. You have to simply plug in t plus t instead of t and nx of t and see if it evaluates the same as x of t and for the fundamental period, you have to define, what is the smallest valid period for a signal x of t.

There are some more answers here, but this is the definition and with this definition if you go and look at your problem, if you look at the ninth problem you can quickly see checks the answer, which of the following signals is periodic with fundamental period 2 pi seconds okay.
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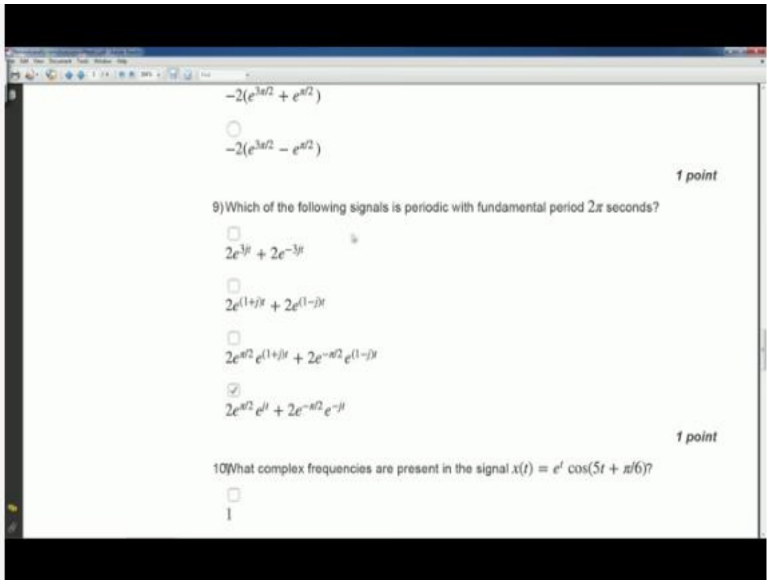

So, the easy questions, easy parts are second and third option okay. So, if instead of t here you put t plus 2 pi. You notice there is e power t term, which will become e power t plus 2 pi okay. Here again, there is e power t term, which will become a power t plus 2 pi okay. Of course, there is e power jt, which will be a periodic part then there is a e power t part, which will make t plus 2 pi not the same as x of t okay.

So, in this equation if you put t plus 2 pi instead of t, you will not get the same form again, so, this is not periodic at all okay. Then in the same thing will hold here, there is e power t guy showing up and does not cancel and that will make a periodic okay. So, fourth option on the other hand is just e power jt and you know that e power j2 pi is equal to 1 okay.

So, instead of t, if you put t plus 2 pi, you are going to get e power jt times e power j2 pi same thing e power minus j2 pi both of them will become one and this will evaluate to exactly the same form I am not writing this down in great detail hopefully its clear to you okay. So, now I feel the first choice, this is a little bit more interesting it turns out 2 pi is valid period for the signal okay.

If you put instead of t, if you put t plus 2 pi, you will get the same form again so, it repeats at 2 pi returns out it is not the fundamental period okay. Instead of t if you put t plus 2 pi over 3 okay, the 3 and 3 will cancel, but still the 2 pi remains and you will get the same form again. Okay so, the fundamental period for this guy is actually 2 pi over 3 is not just 2 pi okay.

So, this is the periodic signal, but the fundamental period is not 2 pi. So, that’s way the first option is not correct okay. So, one can evaluate this and find out whether it is periodic or not okay. The useful question to ponder about is whether D.C. signal is periodic or not okay suppose you look at D.C. signal is just flat. It is periodic or not. It turns out for every value of t, which is non 0. The D.C. signal is periodic with that period t.

Okay that is true x of t plus capital t is going to be equal to x of t, but then what is the fundamental period of the D.C. signal okay. If you think you know the answer do post it on the forum it is an interesting little question to ponder about okay.

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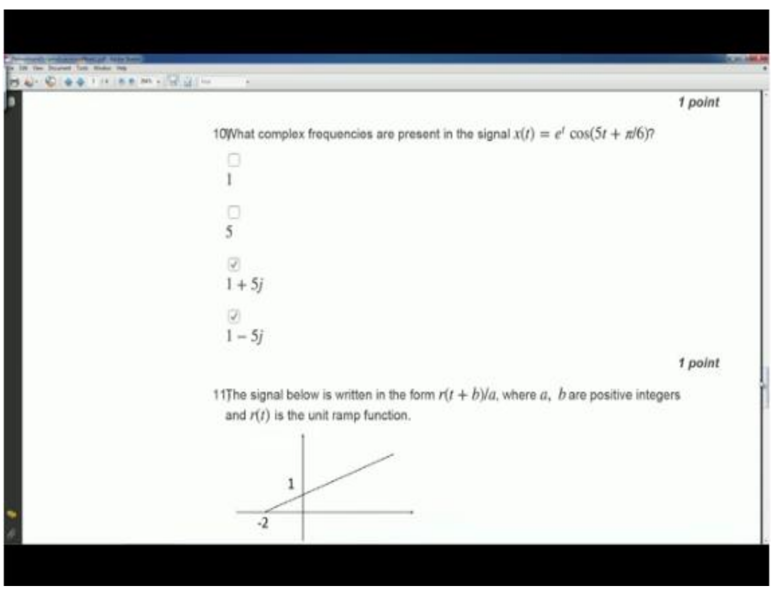

So, the tenth question is on complex frequencies so, there was a question once again asked in the forum about what is the complex frequency. It is actually a relatively simple definition and don’t get confused by the term frequency. It is a frequency is just use for legacy reasons here. Complex frequency is just more or less an notation okay. If you have e power st, where s is complex s s is called the complex frequency of that term.

That is the simple notational definition e power st where s is complex s is called the complex frequency that term. So, in this equation, you have e power t times cos 5t plus pi by 6. So, now we need to replace cos with the exponential form okay. So, let me may be do this alone real quick okay.

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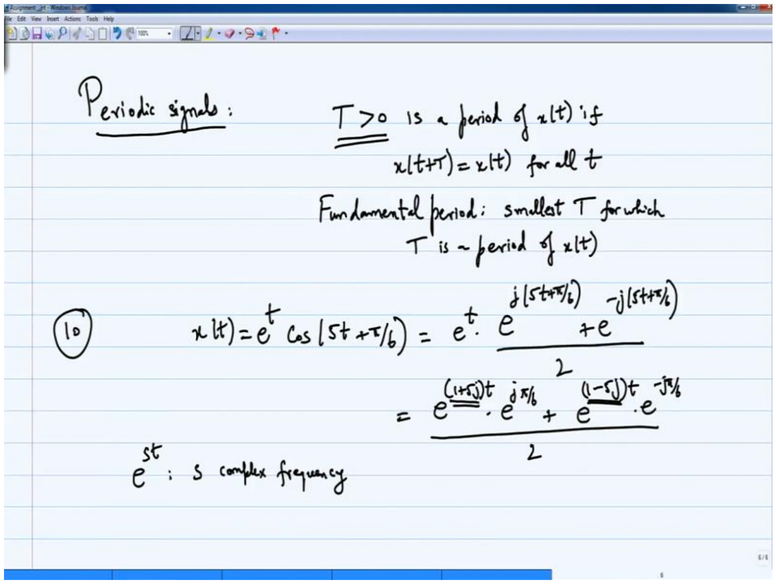

So, if you look at the term, there it says e power t times cos. So, this is question number 10 x of t is e power t times cos 5t plus pi over 6, right so, now one can write this as e power t times e power j 5t plus pi by 6 plus e power minus j 5t plus pi by 6 divided by 2 so, write cosine in terms of e power j theta plus e power minus j theta by 2 okay.

So, once you do this you can multiply it out you see you get e power 1 plus 5 j times t times some complex number e power j pi by 6 plus e power 1 minus 5j times t times another complex number e power minus j pi over 6 the whole thing divided by 2. So, what complex frequencies are present in this, once again what is a definition if e power st is present in your signal, s is the complex frequency.

That's all that is a its like I said it is a notational definition does not have physical meaning be very watchful of that okay. So, what are the e power st terms here where as s is complex. You have 1 plus 5 j and 1 minus 5j and that's it those are the complex frequencies in the signal okay.

Lets move over to the next question that's question number 10 and 11 and 12 are very nice questions these are about the ramps ramp and unit step and how to represent a signal in terms of the ramp okay.

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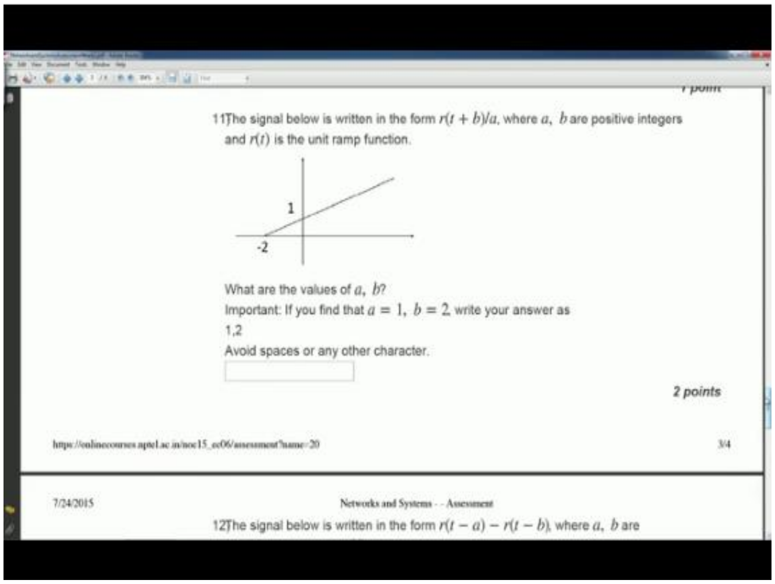

So, shall let me may be spend a couple of minutes on that.

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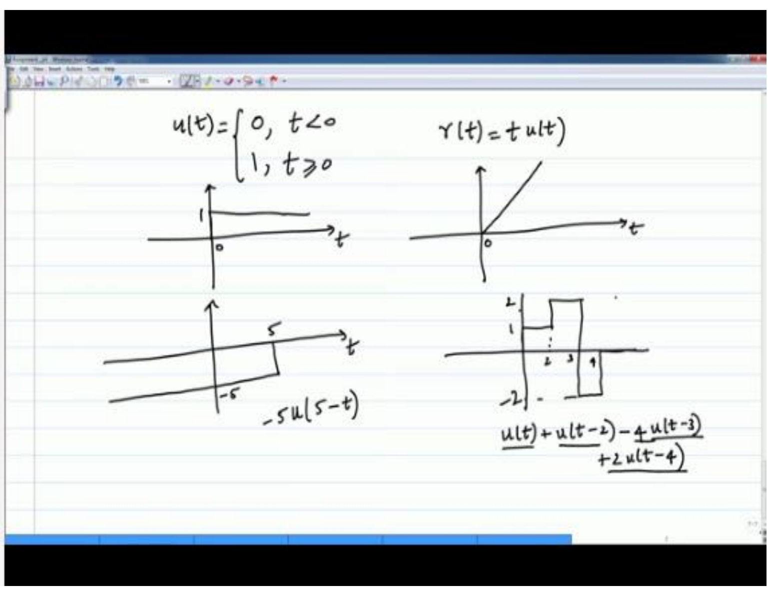

Okay so, now its we study of this basic unit step function u of t, which has several forms one can write it in various forms and I am going to just pick some form, which may not be the best forms may be pick this guy u of t is 0 for t less than 0 and 1 for t greater than 0 and then r of t has this very simple form t times u of t.

So, the sketch is quite interesting to keep in mind 0 it goes up to 1 and stays at 1 forever okay. So, that is u of t versus t and r of t versus t starts at 0 and goes off at 1 at a 45 degree angle towards infinity, so, that is that is r of t okay.

So, now it turns out many signals in practice okay can be written in terms of combination of u of t and r of t. It is quite valuable in many systems your signal is either going to jump from 0 stairs at some value and then change some other value change some other value it is going to be piecewise constant or piecewise linear. It would start somewhere grow up linearly to something and then come down that’s how most signals are.

They are piecewise constant or piecewise linear okay. So, any such signal piecewise constant, piecewise linear signals can be written as combination of u of t and r of t may be in multiple ways. Now why is that very interesting in practice, because once you write it as a combination of u of t and r of t in lti systems.

If you know the unit step response and unit ramp response or speak or given unit step response good enough. You know how the final signal is going to look like. So, given the input signal as a piecewise constant let say. You might be able to roughly sketch how the output would look and you will have an intuition of how your system is working based on some simple ideas.

Okay so, that’s why these things are very important and one very good piece of exercise is to take some piecewise constant signal and write it as a some of shifted and scaled unit steps and unit ramps okay. So, that is a very good exercise and last 2 questions ask you to do that for ramps. One can also do for unit step and all that okay.

So, here is an example for unit step may be slightly nontrivial example okay. Lets say somebody gave you function like this. It stays at minus 5 up to lets say 5 and stays a constant after that okay. so, how would one write this is basically think about it. Its u of 5 minus t is that right its u of minus t and then it's been shifted right shifted right by t so, have u of 5 minus t, then you have to multiply by minus 5 okay.

So, this is the signal, which is again piecewise is constant. It stays constant at minus 5 and then it comes down to 0. So, this would satisfy these equations. So, you have several such things, for instance one may want to write something like this you stay at 1 for while and then at may be at time 2 it moves up to 4 and moves of to 2 and then may be at time 3 it comes down to minus 2 may be minus 2 minus 2 and then may be comes down to 0 at time 4 and stays like that.

Okay so, here is a signal which shows as actually might be an input signal to some system that you build its quite realistic quite often many signals look like this. So, for such a signal how would you write in terms of shifted unit ramps, it is unit signals it is a unit steps its possible how do you do that. So, this one would be u of t okay plus u of t minus 2 okay minus 4 times u of t minus 3 plus 2 times u of t minus 4.

Think about how I did that one can sketch each of these things separately in a separate page okay. Then see how it ends up adding you to give you a adding to give you what do you want okay so, this is an important step. This is not very hard to do, but hopefully you have seen how this is done okay.

So, this is in terms of unit steps and what was asked, what was being asked in your question is a unit ramp right. So, if you go out to the last couple of questions, you are asked about writing a signal like this, which starts at minus 2 and increases up and in a certain slopes such that 0 the value is 1 okay.
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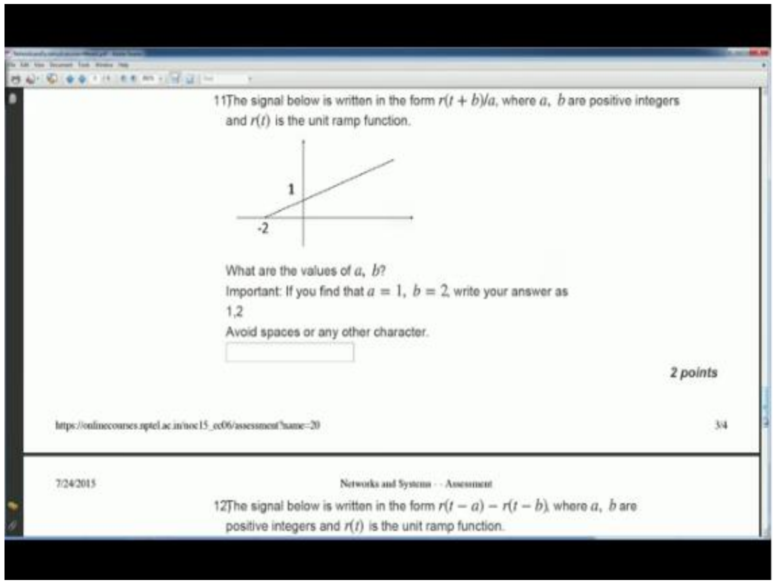

So, that is the signal and you have to write it in terms of the unit ramp rf t plus b divided by a, so, if you look at it very carefully rs is a shifted ramp and it is also scaled. So, it is multiplied by something okay. So, one can think about this very carefully and if you want you can do some careful calculations, but let me just give you the answer here and explained why how I got that answer.

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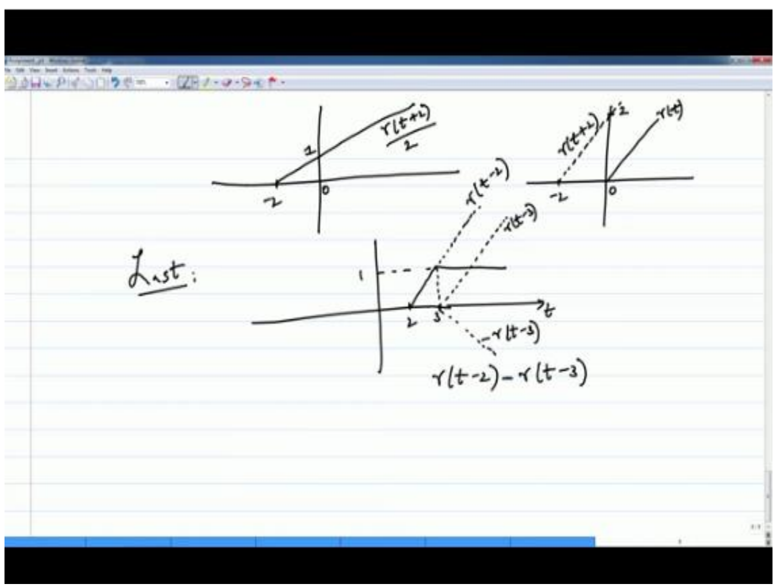

If I start with minus 2 and then you have a signal like this, which at 0 crosses 1 so, now look at what a ramp is the unit ramp just as this okay it starts at 0 and goes at certain angle. This thing is starting at minus 2, so, the first thing is to shift the ramp left to minus 2. How do you shift a signal to the left instead of t, you put t plus the shift okay.

So, if I do that I am going to get signal like that okay. This guy is r of t plus 2 right. This guy is r of t. This guy is r of t plus 2. Okay you shifted it left by 2. Now, if you do a ramp. If you shift the ramp left by 2 what will be this value? This value is going to be 2, but in my signal I want that value to be 1.

Okay so, that tells you what you have to divide by and that tell you it divide by 2 again. It divide by 2 you are going to get back to 1 so, this signal will be r of t plus 2 divided by 2 okay. So, it is an easy calculation to do one is to think about it carefully and you will get the answer okay. So, that is the answer for the ramp question.

The next question is again a little bit more interesting. This is the last question okay. It goes like this you have it starts at 2 increases linearly up to 1 and at 1 I am sorry at 1 it doesn't drop down like that I am sorry it stays flat after 1 at after 3 at 1 right this what happens. After 3 seconds it stays flat at one.

So, now, the signal once again it starts at 0 at 2 and goes off increasing in a slope and that tells you it is going to be r of t minus 2. Right what is r of t minus 2 it starts like this and it keeps on going forever okay. I want to bring it back to bring it back down and make it flat I have to subtract of something.

What you have to subtract of it turns out we will start from here and go in the negative direction and that would be minus r of t minus 3. Okay the ramp shifted right to 3 and then there is a subtraction to subtract out. Once sub add these 2 guys, you are going to get a flat signal, one is increasing a slope point, another decreasing a slope point.

You add these 2 guys they cancelled and give you something straight. This is going of r of t minus 2 minus r of t minus 3 and you have done okay. So, if you do r of t minus 3, you get something like this. This would be r of t minus 3. This is r of t minus 2 to subtract these 2 beyond this point, it is going to stay flat okay. So, these 2 are parallel lines going like this. If you keep subtracting you are going to get the same value of 1 and that is going to stay a constant okay.

So, it is useful like I said to write piecewise linear or piecewise constant functions in terms of r and u and then if you know how your systems reacts to r and u. You know how your system reacts to any input like this. So, this is a valuable thing in practice okay.

Thank you very much that was a description of the assignment 1 and questions on assignment 1 will also put up the solutions on the portal. Do have look at it and keeps solving problems you will really learn all about this subject quite well.