Networks and Systems
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Lecture-26
Convergence of Fourier Series

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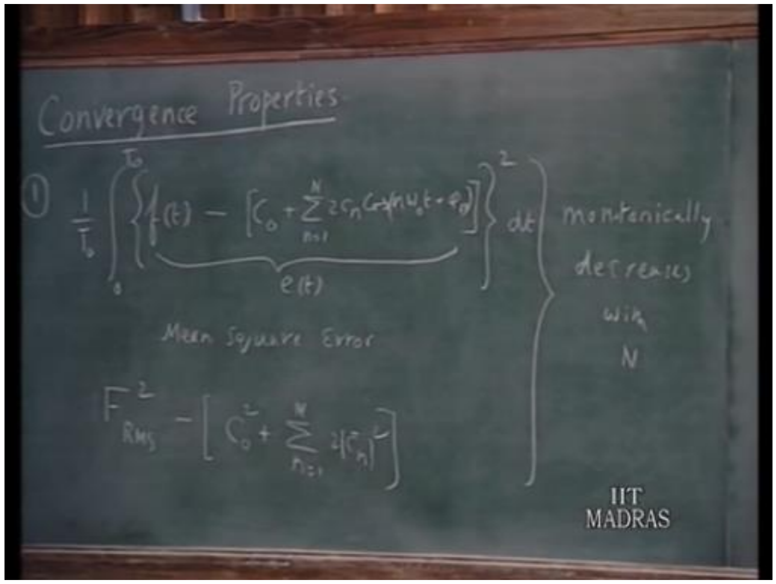

The last example, showed that the size of the various harmonic components and the powers associated with the harmonics go down as the other harmonic increases you should like to see, how fast these various coefficients go down and that is; related to what is meant by, convergence of the Fourier series you should look at the convergence like to look at the convergent properties of the Fourier series.

So, first let us consider this particular point suppose, f of t is approximated by the Fourier series up to order capital N that means; we have c 0 plus 2 cn cos n omega 0 t phi n if you take n from 1 to infinity that constitute the entire Fourier series now suppose, I calculate the series at capital N.

So, this is the approximated Fourier series we take only finite number of terms and the difference between the actual f of t and the truncated Fourier series will call that the error e of t. So, this error now depends up on the value N that we take, now if you take square of the error integrate this from 0 to T 0 and take its average then this is referred to as the mean square error, mean square error.

So, the square of the mean the mean of the square of the error is called the mean square error. So, we see that this decreases the capital N the value of N that you take close to related to this and actually this expression equivalent to another term like this. Suppose, you take the RMS value of the given wave form and it is square FRMS square assuming that f of t is a voltage signal this represents square of the RMS value the voltage signal and this voltage is applied to 1 Ohm resister, this is the power dissipated in the 1 Ohm resister.

Therefore, this is the voltage signal FRMS square can also be part of the power available from the signal. However, if you take only the sum of the RMS values of the various harmonic components up to n; that means, you take c 0 square plus n from 1 to capital N 2 cn magnitude square 2 cn magnitude square is the sum of the square of the RMS values of the harmonic components 1 to N because after all 2 cn equals square root of an square plus bn square.

Therefore, 4 cn square is an square plus bn square and RMS value is an square plus bn square up on 2 therefore, 2 cn magnitude squared is the square of the RMS value of the n'th harmonic component. Therefore, the difference this is the power available from the signal this is the sum of the power available from the various harmonic components of order N.

So, this quantity can be shown to the equal to the other quantity which we have written earlier both these will monotonically decrease with N. So, the large of the value of N you take the less will be discrepancy will be the error and the decrease is monotonic and as you take large and larger values of n these terms go down.

So, the next question we like to ask is how fast do the cn term decrease with N. So, that is the next property of interest that we like to know.

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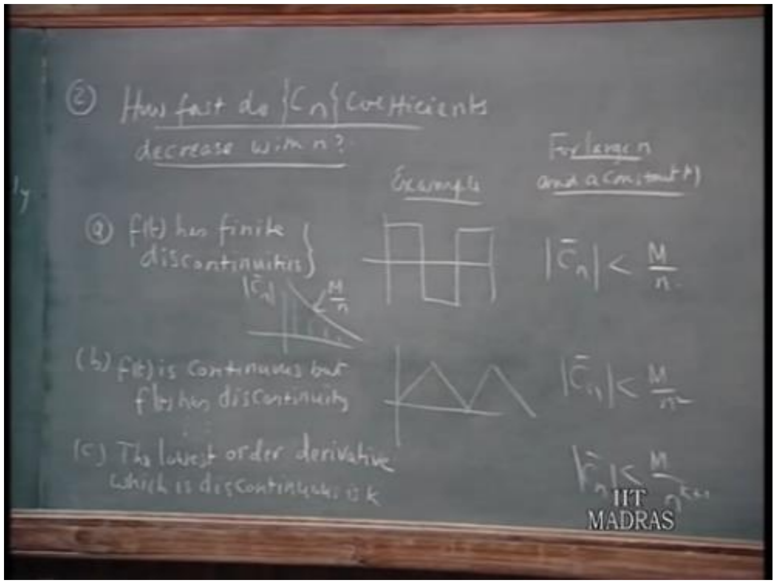

So, the question that we ask is: how fast do the cn terms the cn coefficients decrease with n we like to give the answer in the following manner suppose, we take f of t has finite discontinuities example. Suppose, wave form like this square wave familiar, square wave there is a jump here there is a finite discontinuities.

For such wave forms it can be shown that for large n and a constant M the Fourier coefficients for large n are constrained by a factor like this. So, as you take n to be the large more and more then the coefficients go down as M over n ,that means: the spectrum for this suppose it is like this the values of various coefficients are constrained by a line like this M by n.

And the other hand suppose, f of t is continuous, but the derivative f of t has discontinuity; that means, the function itself is continuous, but then there is a discontinuity its derivative and example function like this will be like this.

Suppose, I have sample like this, this is the periodic function it is continuous there is no jump anywhere, but you take its derivative, the derivative here will be like this the derivative here will be negative; that means, the derivative function of this will be something like this.

So, there is the derivative of the discontinuity, but the function itself is smooth in this event the various Fourier coefficients go down as m by n square; that means, they decreases as 1 over n square; that means, the decrease is faster in this case because this is smoother function than this you can continue like this.

Suppose, you say the lowest order of the derivative which is discontinuous is k; that means, first k minus 1 derivatives are continuous the first time the derivatives going to be discontinuous is order k then it can be shown that cn goes down as m over n times k plus n to the power of k plus 1 this is much smoother function than the earlier 1.

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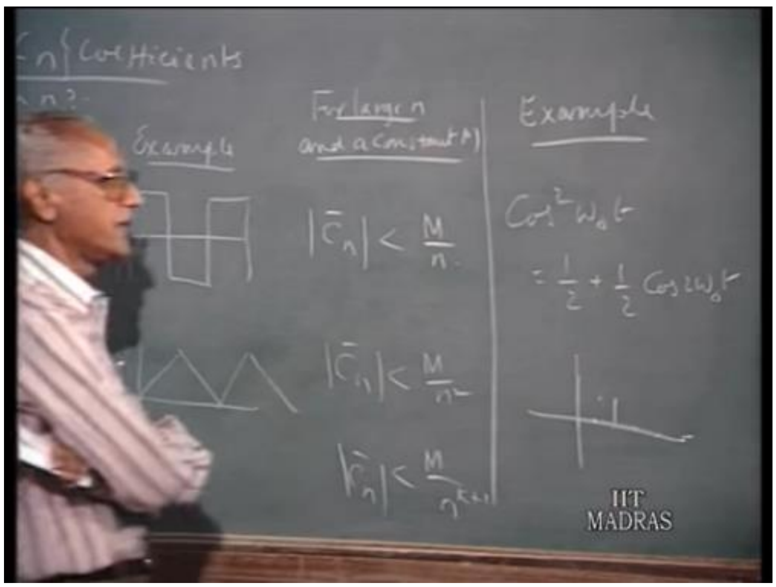

Therefore, the decrease of cn coefficient will much faster as an example suppose, you take cos square omega 0 t this is the very smooth function all its derivatives are continuous and the Fourier series expansion really half plus 1 half of cos 2 omega 0 t. So, you have the dc term the second harmonic term all the other terms have 0.

So, you have really the dc term and the second harmonic term here. Further terms are all 0 because this particular function of time is very smooth all its derivatives are continuous therefore, the Fourier series go down very fast and in fact, they become 0 after n equals 2. So, this is the example of the very smooth function; that means, in other words if the function is smoother than the faster will be the rate of decrease of the cn coefficients that is the summary of what we have discussed under this header.

The third question we would like to ask is: what is the behavior of the function at the point of discontinuity. Let us say, this f of t has the discontinuity at the point x.

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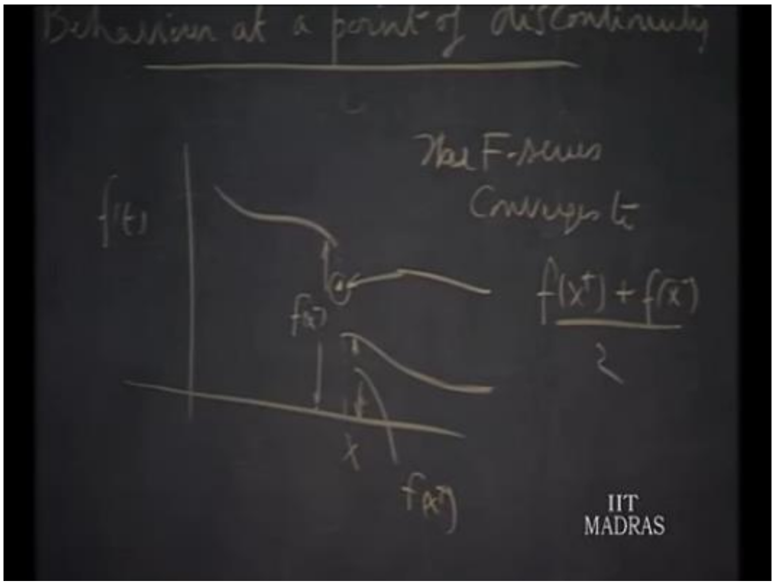

Now, if you take the Fourier series and evaluate the value of f of t at f of x in turns out that the series the Fourier series converges to f of x plus, plus f of x minus divided by 2 the limit of f of t as you approach x from this direction this value is f of x minus the value here is f of x plus.

So, depending up on how you approach x from the right or left you get x minus or x plus, but the series will converge to mid point between this 2 this is the point to which the series will converge irrespective of how you define f of x. So, you may define f of x some value not necessarily this, but as far the series is concerned it converges to this.

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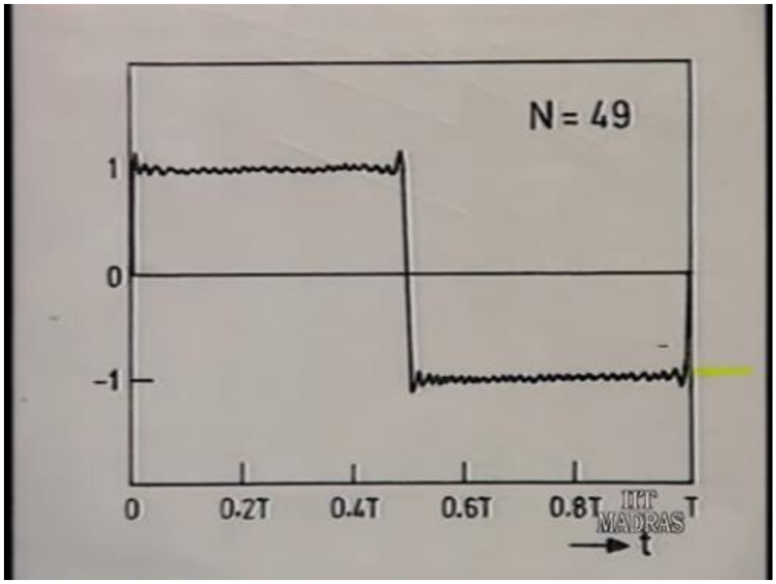

Now let us look at this figure, where we put the Fourier series for a square wave and we have taken up to the 49th term. Now, the series converges to 0 at this point which is the proper thing to do which is the average of the left limit and right limit, but you should also observe that, there is small amount of overshoot just before the jump and just after the jump and this overshoot is before to literature.

Gibb's phenomena and the amount of overshoot is 9 percent of the total jump and this is something which will persist of no matter to what order harmonic you go to.