Networks and Systems
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Lecture-25
Signal Power and Related Ideas

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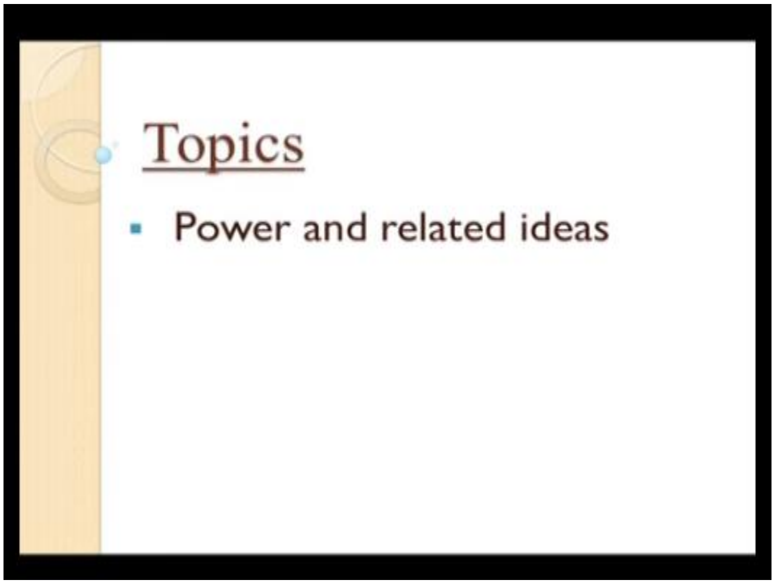

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Continuing our discussion of Fourier series today, we will take up consideration of power and related aspects associated with periodic signal.

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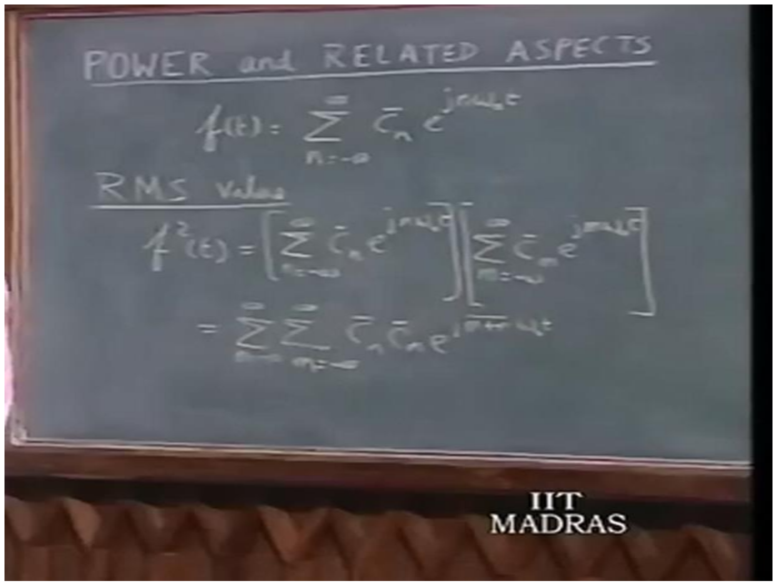

We recall that the Fourier series expansion of periodic f of t can be put in this form in its exponential representation. To start with let us, try to calculate the RMS value of this periodic signal in terms of the Fourier coefficients cn in its exponential representation. So, you like to calculate the RMS value of f of t you can it in time domain as the average of the mean square of a ft, but we like to carry out this analysis in terms of the complex coefficient cn. Let us see, how we do it.

Suppose I like to calculate f square of t then this can be written as cn into e o the power of j n omega 0 t n of course, ranging from minus infinity to plus infinity multiply by the same series in order to facilitate our taking stock of the various terms. I would like the index here to be m instead of n m from 1 to minus infinity to plus infinity. This of course, can be written as, summed of m from minus infinity to plus infinity and summation on n from minus infinity to plus infinity.

So, each term here gets multiplied by each 1 of the term of the second summation. So, I have cn multiplied by cm e to the power 2 j m plus n omega 0 t. Now, in order to find out the mean value of this f square t we should find out the average of this whole series. So, the average of the summation is nearly the summation of the averages. So, we will like to find out the average of each 1 of these terms, if you do that then we can sum up this averages and say that is in the average of f square t. Because the RMS value of f of t is the square root of the mean of the square.

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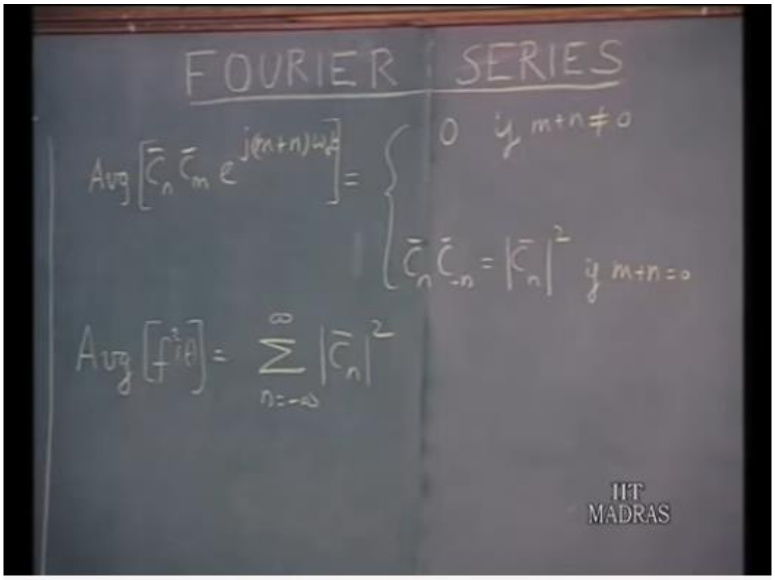

Let us see the average value of cn cm e to the power of j m plus n omega 0 t what it is going to be. When we are talking about the average, we always understand that is the average over a full cycle the complete period of the fundamental. So, this is the value depending term cn

and cm are constant.

So, what do we have if m plus n is an integer then e to the power j k omega 0 t, if you integrate over a complete period it is going to vanish because it is after all cos m plus n omega 0 t j sin omega 0 t any sin term cosine term integrated over a complete period or the integral number of period is going to 0. Therefore the average of this will be 0,if m plus n is not equal to 0.

So, if m plus n is some non zero constant the integer that is going to be 0. On the other hand suppose, we have a case if m equal to minus n or m plus n 0. If m plus n is 0 then this whole term becomes 1 and the average of cn cm is cn cm itself. Therefore the average of that would be cn multiplied by cm and what is m m is equals minus n, if m plus n 0 m equals to minus n.

Therefore, cn multiplied by c minus n and since, we know that cm and c minus n are complex conjugate of each other here multiplying a complex number by its conjugate the result is simply the magnitude cn whole square. So, we have that the average of each 1 of the terms here will be either 0 or an expression like cn square.

Therefore, we can now come to conclusion that the average of f square t the average of this term is the sum of the averages of these. Now let us see, what happens if i freeze a particular value of m and allow the m to take increments from minus infinity to plus infinity. So, m goes on changing from minus infinity to plus infinity 1 m takes minus n then you have non zero average for all other values of m the contribution is 0.

So, when we take a particular value of m in the first summation m equal to the minus n is the only term which gives the contribution non zero contribution and that we do for all values of m. So, the result is this will be m from minus infinity to plus infinity for each value of m the contribution can come only when m equals to minus n and that, condition the contribution is cn squared.

Therefore, this will be cn magnitude square and what is the average of f square t it is the RMS value of the periodic function square, square of the RMS value mean square value.
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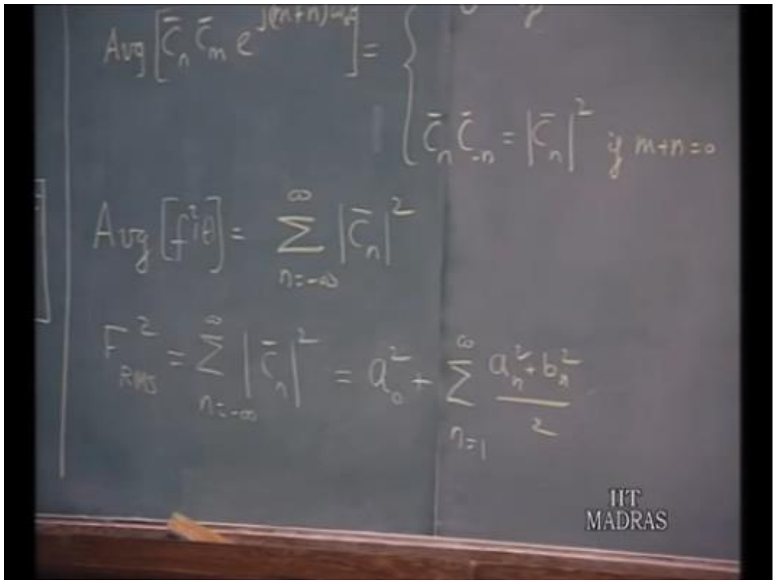

Therefore if I write as Frms of the effective value of this 1 that is equal to cn magnitude square n ranging from minus infinity to plus infinity you can see this also, can be written as c 0 squared or I put it a 0 square because I like to put in the terms of the coefficient of the trigonometric series.

So, when n equal to 0 cn equal to c 0 that is a 0 square and cn square and c minus n square together are both equal to each other. Therefore, this will be 2 terms cn square in terms of turned out to be an squared plus bn squared over 2 n ranging from 1 to infinity because an squared plus bn squared is 4 cn square because square root of an square plus bn square is 2 cn as you recall.

Therefore, an square plus bn square is 4 cn square up on cn squared and m equals k and m equals to minus k together contribute 1 cn term will come for positive n another for negative n together they constitute an squared plus bn squared over 2.

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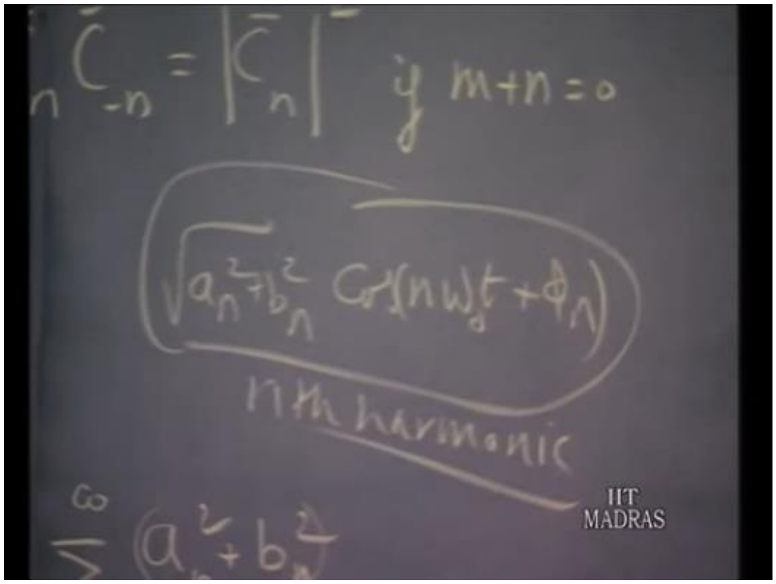

So, what we find that the RMS value of a composite way periodic wave this square of that equals, the square of the square of the dc term and an square plus bn square root of an square plus bn square cos n omega 0 t plus ph i n as you call is the n'th harmonic.

So, what we have here is the peak value square by 2 which means; the RMS value is the n'th harmonic squared root of an squared plus bn squared over 2 is the RMS vaule the n'th harmonic therefore, the RMS value the n'th harmonic squared an squared is the this continue squared.

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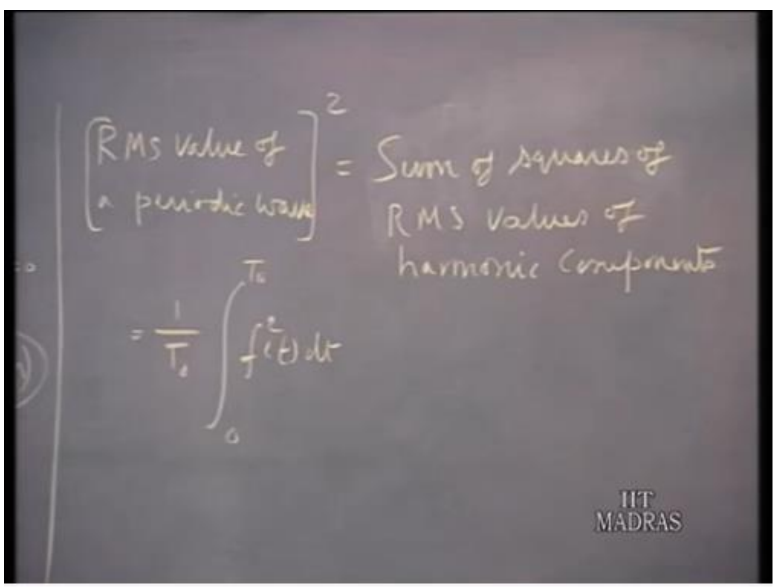

So, the final conclusion is the RMS value of a periodic wave is square of that, equals the summation of or I can say the sum of the RMS value squared of the harmonic components sum of squares of RMS values of harmonic components including the dc.

So, that is easy way remembering how we can calculate the RMS value of periodic wave once we have got the Fourier series analysis for that, this of course, this is also equal to 1 over T 0 this is also equal in time domain 1 over T 0 f square t dt. So, you calculate the RMS value either in time domain or in terms of the harmonic components, which are convenient to you at a particular contest a particular contest.

Now, let us continuous this and see as far as f of t is concerned, it could be voltage signal, it could be current signal. So, you can calculate the RMS value of voltage signal or current signal using this kind of formulation. Now let us see, how we calculate power when we are given 2 periodic signals the 1 is being voltage the other is the current.

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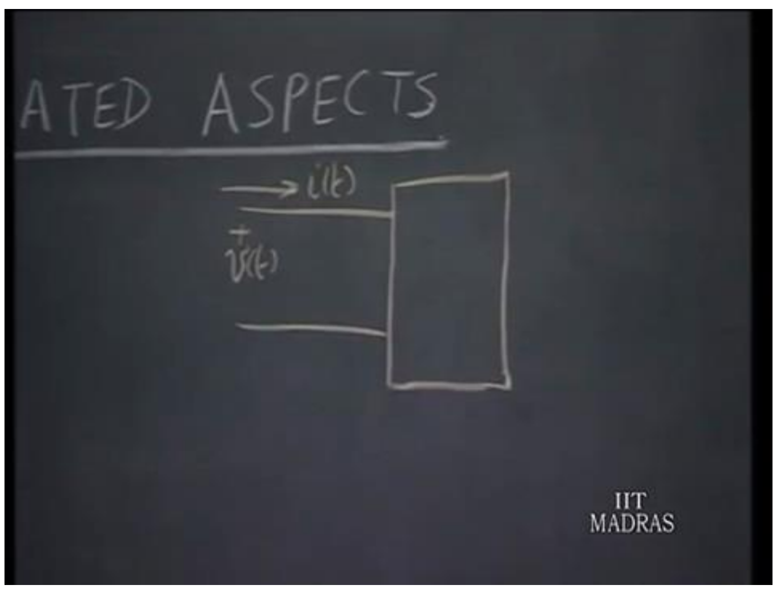

Let us assume that we have a network or a network element which has the periodic voltage upward cross it and periodic current I (t) going to the terminals.

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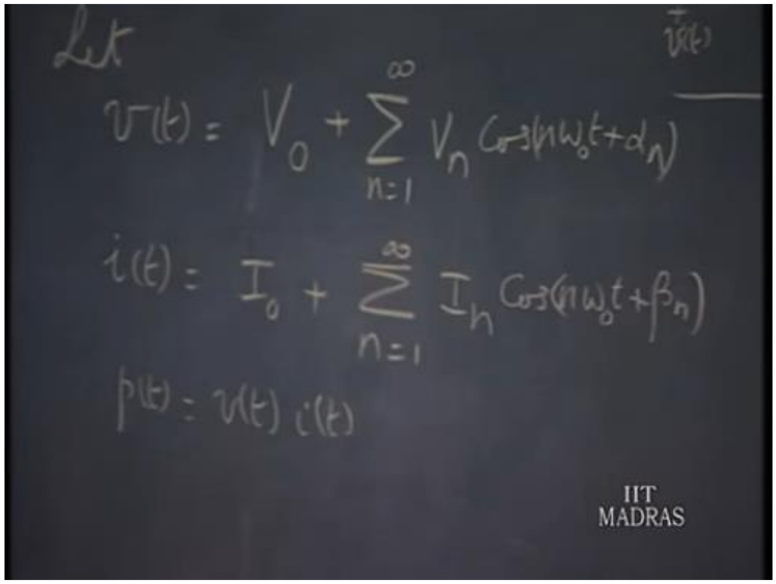

Now, let the voltage v of t be v 0 plus vn cos n omega 0 plus alpha n n ranging from 1 to infinity. So, that is the Fourier series expansion for the voltage wave, which is periodic the fundamental frequency omega 0 this is the amplitude n'th harmonic and this is the dc terms and likewise the current wave form be, i 0 plus n from 1 to infinity of In cosine n omega 0 t plus beta n. Now, the instantaneous part p of t is of course, the product of v of t and i of t.

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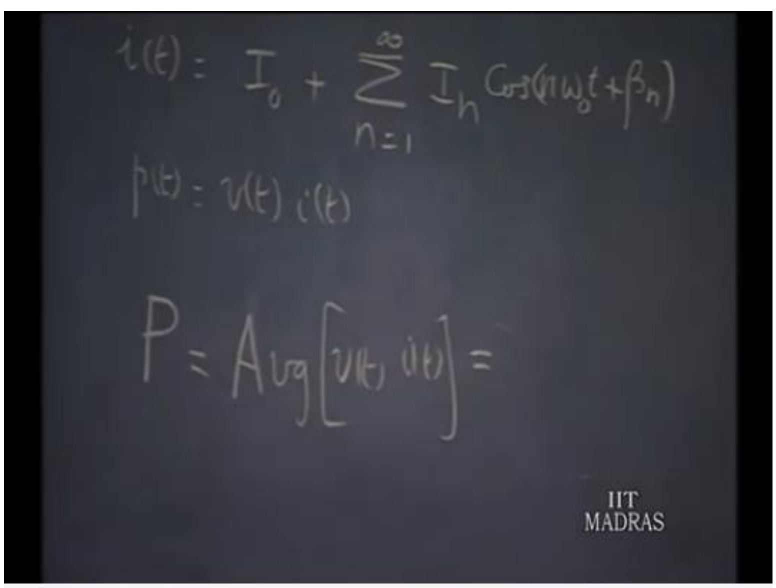

And when you talk about power associated with any periodic phenomenon, we always imply the average power over a fundamental period or integrate multiple of such periods therefore, when we say power associated with this voltage and current that is the power import into this network N. Simply, call this P it is the average of the product of v of t times i of t always the average over a complete period.

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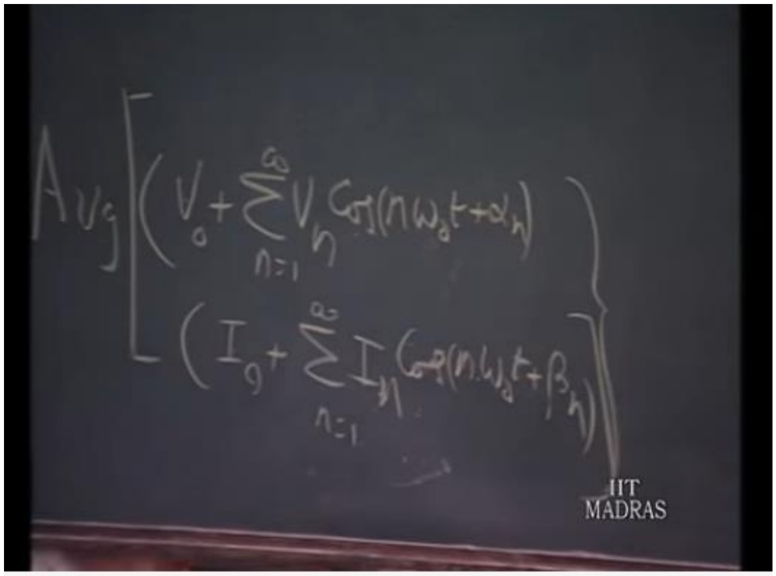

So, we take these 2 series and multiple them out. So, you have the average of v 0 plus vn cos n omega 0 t plus alpha n multiplied by i 0 plus im cos n omega 0 t plus beta n. Now, when you look at this you have whole a lot of terms here, but luckily for us we are interested only the average of the these terms the sum of these terms.

So, you notice that v 0 gets multiplied with i 0 get multiplied with In cos omega 0 plus beta 1 i 2 cos 2 omega 0 plus beta 2 and. So, on, but when we are talking about average, it is only this v 0 and i 0 which results in nonzero average v 0 multiplied by i 1 cos n omega 0 t plus beta 1 is 0 v 0 multiplied by i 2 cos 2 omega 0 plus beta 2 is 0 and so, on and so, forth.

So, as far the v 0 is concerned the contribution to this average to come from, the term v 0 i 0. Similarly, we will take the fundamental here v1 cos omega 0 t plus alpha 1 that multiplied by i 0 will be the average the 0 average v1 cos omega 0 t plus alpha n multiplied by I1 cos omega 0 t plus beta 1.

We have an average after all, it is the voltage and current of the same frequency that, the average will be of course, the product of v1 and I1 divided by 2 the product of the 2 RMS values times cosine of the phase angle between alpha and beta alpha 1 minus beta 1 all other terms the fundamental here multiplied by second harmonic, third harmonic, fourth harmonic we have 0 average.

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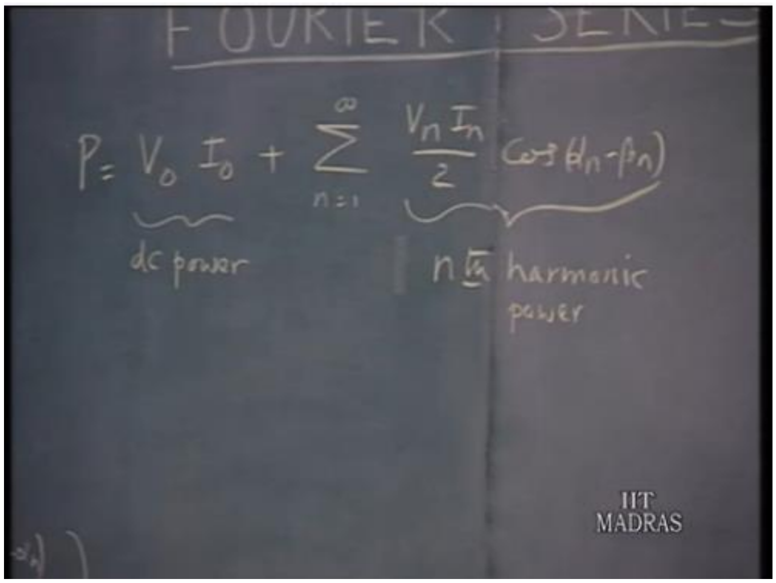

So, consequently P will turn out to be v 0 i 0 that is the product of the dc terms the n'th harmonic term will be vn In divided by 2 cos alpha n minus beta n this is the power factor associated with the n'th harmonic component n ranging from 1 to infinity. So, this is the dc power, this is the power associated with n'th harmonic.

So, when you have non periodic when you have periodic wave forms which are non sinusoidal and if we make the Fourier series expansion of vt and it as far the power is concerned you can calculate the power for each frequency separately. The product of v 0 and i 0 is the dc power the n'th harmonic power is calculated taking the n'th harmonic voltage component, n'th harmonic current component multiplying their RMS values times cosine of the phase angle difference between them.

As a rule power is a non-linear quantity associated with voltage and current non-linear function of voltage and current. However, in the particular case where the components are different frequencies you can calculate superpose the power for each component separately as we have seen here.

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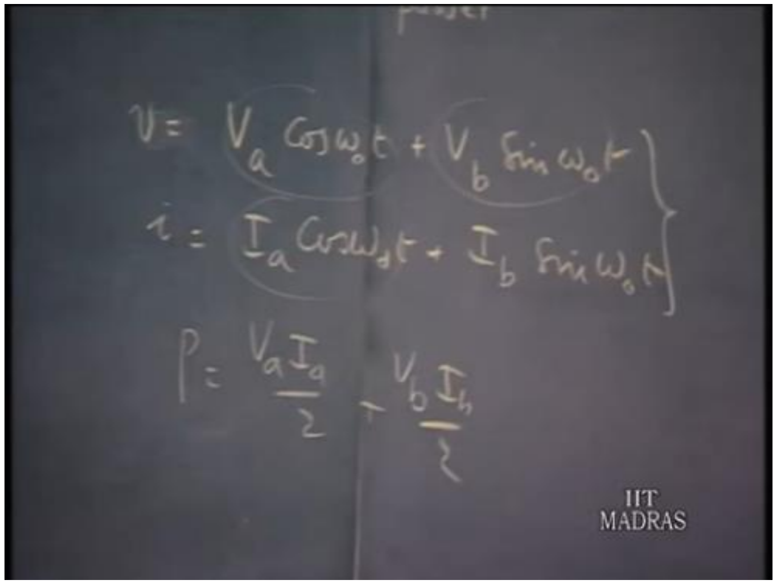

Normally we have a voltage v as the sum of v1 plus v2 and the currents i1 plus i2 you cannot say v1 plus i1 plus v2 i2 is the power because you cannot it does not work out that way. But, as long as you are calculating the power associated with a pair of current and voltage components of different frequencies than this; summation will be valid and this is; what we are having here you superpose power.

Therefore, if you calculate the power associated with each set of pair of frequency components current and voltage you can do that also, if you have the same frequency. Suppose, you have v let us say va cos omega 0 t plus vb sin omega 0 t and current let us say, is Ia cos omega 0 t and Ib sin omega 0 t then associated with this.

You can say power is va Ia divided by 2 plus vb Ib divided by 2 here also you will have some super position principle will be working out will work out that is; because there is ranged phase difference between this component and voltage and the other component and voltage here also there is 90 degree difference between this current and this current.

So, the associated the components associated with cos omega 0 t terms can be multiplied together and the component associated with sin omega 0 t will be multiplied together and they can be superposed. So, power can be superposed under these conditions. If we have components at this same frequency, but with 90 degree phase difference you can superpose the 2 components of power or as in this case.

If we have got different frequency components you can calculate power separately for each 1 of these component, that is why we can always terms these quantities as a fundamental dc power. Fundamental power, second harmonic power, third harmonic power and so on and each harmonic power can be calculated independently of the values of voltage and currents at the other harmonics.

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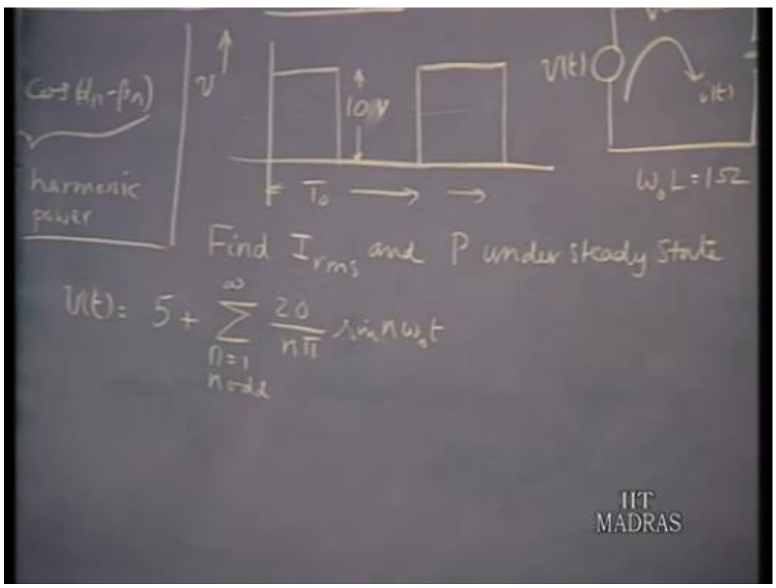

Let us, take an example suppose, I have a 10 volt signal the voltage an half set square wave with the period T 0 and this voltage signal is given to RN circuit. R is equal to 1 Ohm and L is such that; omega 0 L is 1 Ohm we have naturally a current i of t. The question that will be asked is: find Irms and the power P under steady state.

So, this is the question that has been asked, what we proposed to do is we split of the given input voltage v of t into its various harmonic components under the influence of each of the harmonic component, we will calculate the current under steady state. So, we get expressions for v of t and i of t in the form of Fourier series and once we have them it is easy to calculate Irms and the overall power the manner we have illustrated earlier.

So, v of t contains the dc components the average value of this periodic wave is 5 volts therefore, that is the dc value and once we remove the dc value we have the familiar square wave that, have been talking about. So, often in the past. So, it will have Fourier series expansion of 4 times this amplitude divided by n pi and you have only odd harmonic terms are presented and sin terms only will be presented because once, we remove the dc term this will turn out to be an odd function of time.

Anyway the final Fourier series would be sin n omega 0 t and n from 1 to infinity and n is odd because this as once, we remove the dc terms this will have odd half wave. So, we have v of t and now we do the steady state analysis circuits and to do that; we need to find out the impedance of the circuit for different frequencies.

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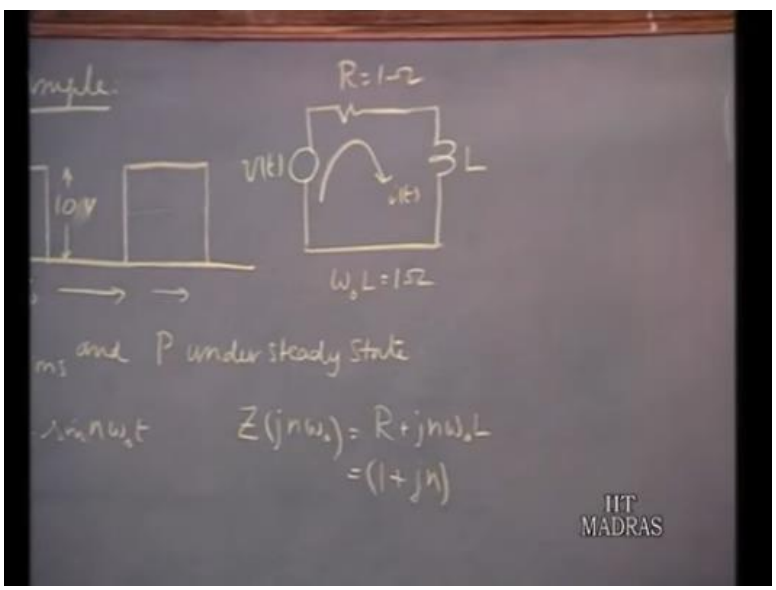

So, z jn omega 0 so, far the n'th harmonic frequency the impedance of the circuit is R plus jn omega 0 L and since R is equal to 1 Ohm and omega 0 L is 1 Ohm this is jn so, many Ohms.

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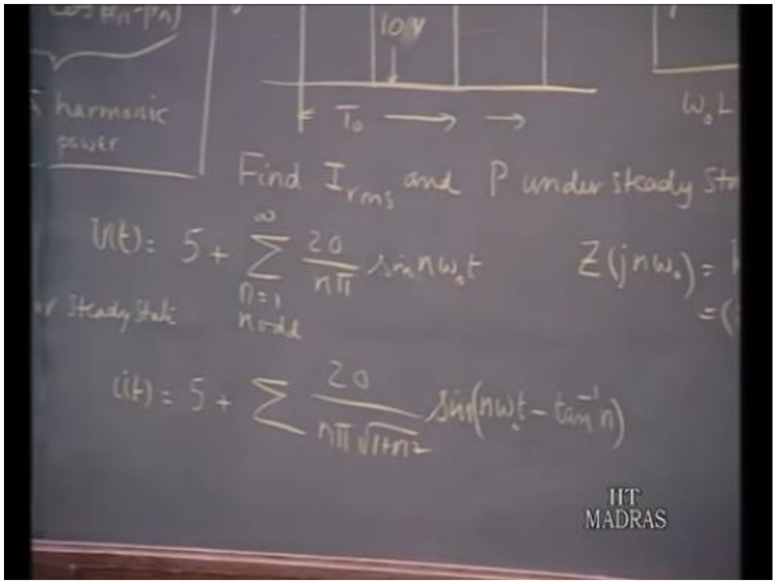

So, we have the impedance for different frequencies we have the voltage under steady conditions, under steady state i of t will be for each component separately we find the steady state current. If the dc voltage is 5 volts in a circuit consisting of R and L the current will be B divided by R. So, 5 up on 1 that is 5 and the n’th harmonic component the voltage is 20 by n pi.

So, the peak value of the current will be divided by this voltage divided by the value of the impedance which is square root of 1 plus n squared sin n omega 0 t. Since, the impedance such an angle tan inverse of n therefore, there will be a phase difference between the current between the voltage and the current this will be tan inverse of arc tangent of n.

So, we have the complete description of the voltage and current in terms of the Fourier components. So, we can calculate the whatever, results that are required Irms and P under steady state conditions.

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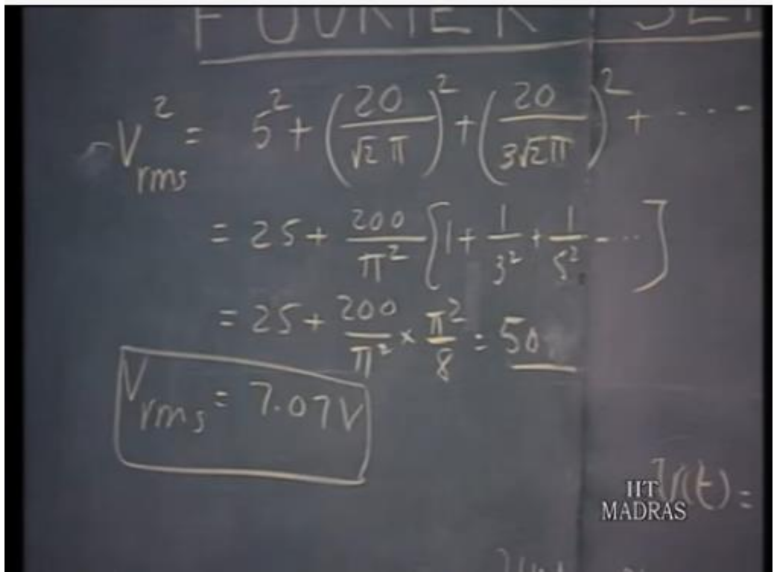

First of all let us, also calculate the Vrms just for sake of interest. So, Vrms squared will be the square of the dc term 5 squared plus the fundamental amplitude square divided by 2 because, we have the RMS value of the fundamental RMS value of the fundamental is 20 up on root 2 pi squared.

The third harmonic component 20 upon 3 root pi whole square and etcetera and if you calculate this this will turn out to be 25 200 up on pi squared times 1 plus 1 by 3 squared plus 1 by pi squared etc etc and this series is going to have summation equal to pi square up on 8. Therefore this will turn out 25 plus 200 up on pi squared into pi squared of up on 8. So, this is total 50 watts.

So, Vrms equals 7.07 that is: 50 volts the results which we have obtained straight way from here working out in time domain quite simple once, the peak values turn hold you integrate you get this result straight away. But just wanted to demonstrate the use of the formula for RMS value in terms of v t that is; how we got this we could have got this same results working out time domain straight away, but it is not so, you could not have got this time domain.

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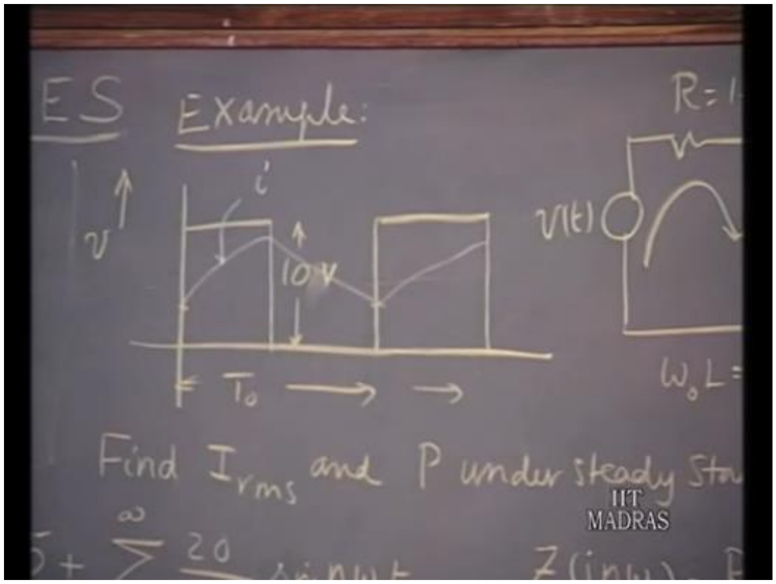

So, easily for i of t Irms because the once we have this type of voltage the current under steady state would have a characteristic like this: under steady state that would be the value of the current will vary. So, you must, if you want to find the rms value the current you would have to find out the expression for this i of t in this manner.

So, that the starting point is equal to the same at the end of the period to fit the initial conditions suitably and then do the integration. Now we have got the term expression for i of t in the form of Fourier series.

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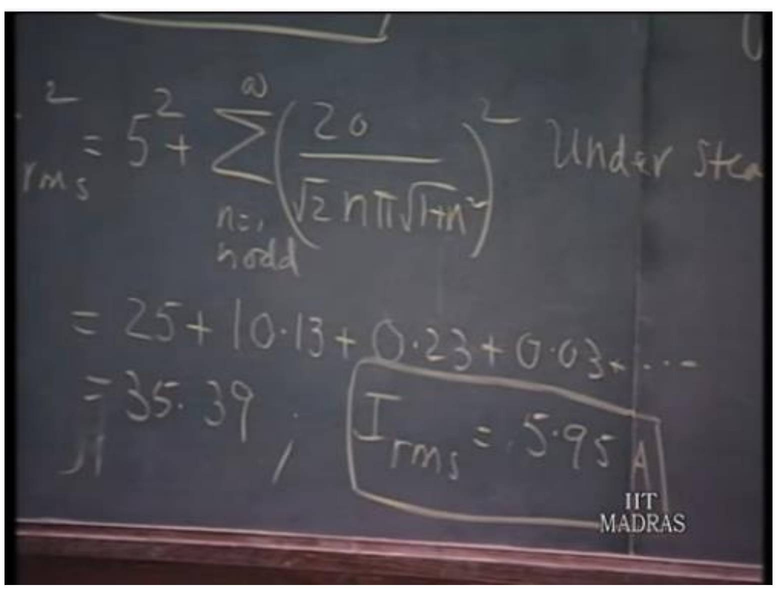

We can calculate Irms in terms its various harmonic components. So, 5 squared the dc term plus summation 20 up on root 2 n pi root over 1 minus n squared whole squared because this is the peak amplitude the RMS value will be obtained by dividing by root 2 and then n from 1 to infinity n is odd.

Calculate the first a few terms you will get 25 from the fundamental from the dc from the fundamental you will get 10.13 for the third harmonic you get 0.23 for the third harmonic you get 0.03. For the fifth harmonic, you get 0.03 and therefore, afterwards it is negligible. So, the this will be equal to 35.39 and Irms will be 5.95 Amperes.

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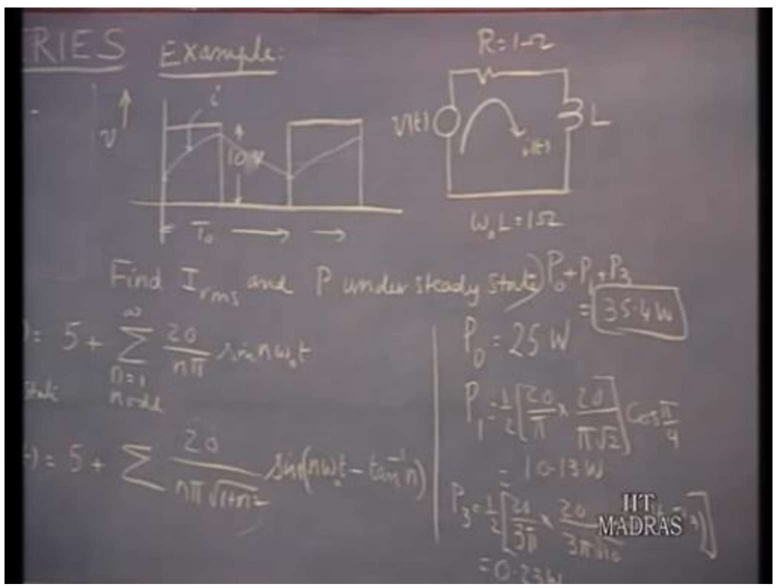

What about power we have v of t and i of t here. So, power we can calculate taking the particular term here and they associated current terminal, current term here and finding of the phase angle. So, let us see the dc power P 0 product of these 2 the dc voltage and the dc component of the current 25 watts.

Fundamental power, fundamental power we have the amplitude the fundamental voltage is 20 up on pi the amplitude of the fundamental component of the current is 20 up on pi n equals 1 square root of 2 and of course, n equals to 1 these are the product of the amplitudes.

So, multiplied by half because we wanted to take the product of the RMS value and cosine of the phase angel difference is tan inverse of 1 that is equal to pi by 4. So, cos pi by 4. And this turns out to be 10.13 watts similarly P3 will be given by half of 20 up on 3 pi multiplied by 20 up on 3 pi and when you substitute n equal to 3 this root 10 cosine of whatever, angle you get tan inverse of 3 and this will turn out to be 0.23 watts.

The total power, if you assume all other harmonic component powers are negligible P 0 plus P1 plus P3 will be 35.4 watts. So, that will be the total power that, will be delivered by the source into the circle. If you look at closely you will find that once you have got RMS value you could have calculated power without going to this analysis after all you know the RMS value of the current in this circuit.

So, if you know the RMS value this kind of the circuit the RMS value squared multiplied by, the resistance will be the total power in the circuit. So, indeed it turns out IRMS squared 35.39 and the power that, we got also the same thing 35.4 watts. But it only illustrates the procedure the principle that is: involved that is you can calculate the power for different harmonic components separately and add them up to get the total power in this circuit.