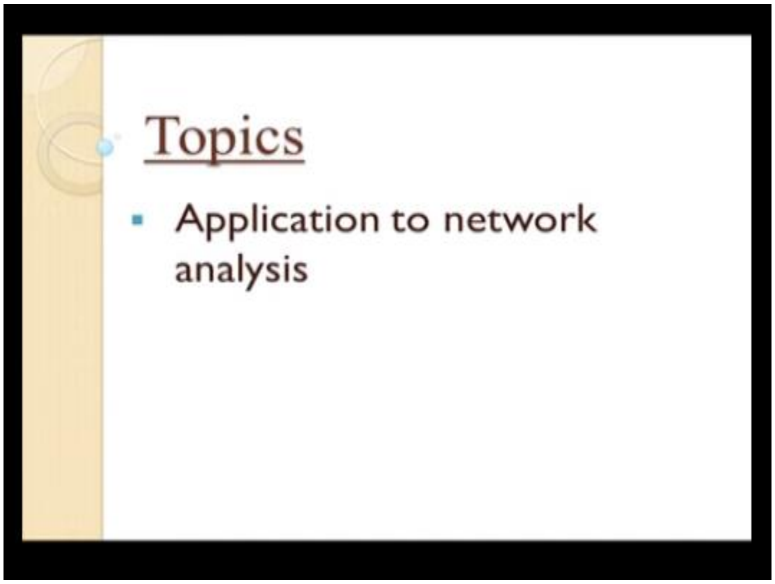
Networks and systems

Prof. V.G.K. Murti
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture-21

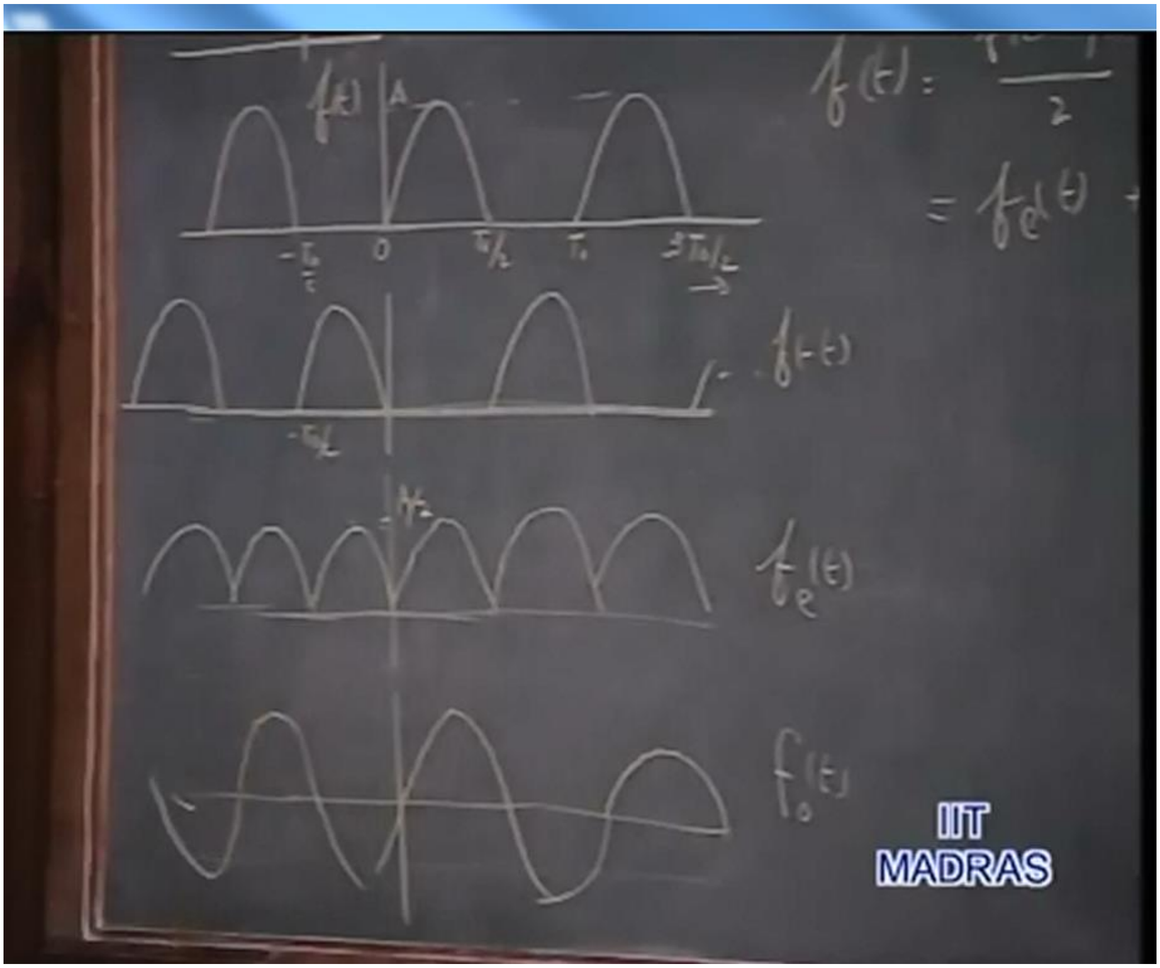
Application to Network Analysis

(Refer Slide Time: 00:14)



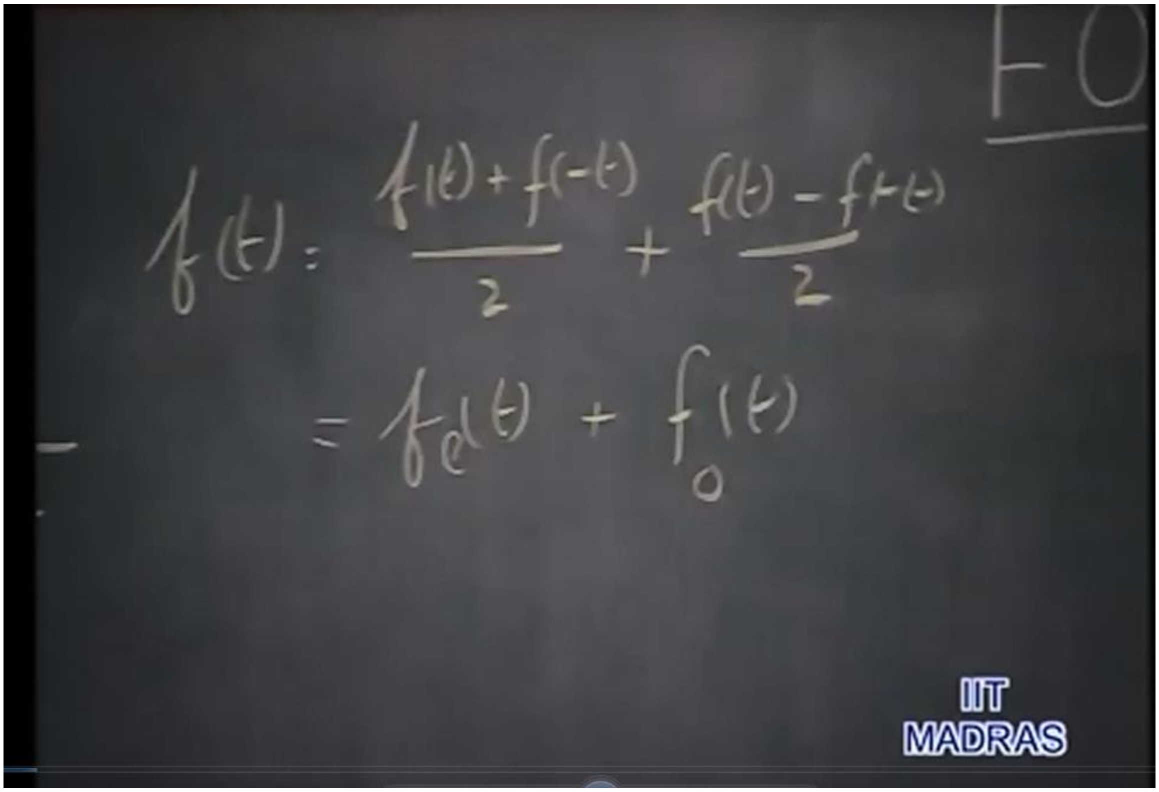
In the last lecture, we have familiarized ourselves, with some symmetry conditions relating to the Fourier coefficients. We would like to continue this discussion, with an example where we make use of the symmetry conditions. And use them to calculate the various Fourier coefficients, quite effectively.

(Refer Slide Time: 00.42)



Consider this example, where we are dealing with a f of t, which is a half wave rectified sine wave of amplitude a. And a fundamental period T not. Now, when you look at this, this waveform does not have either an even symmetry or an odd symmetry. You recall that we had observed earlier, that any given function can be split up, as it is even part and odd part respectively.

(Refer Slide Time: 01:16)



In other words, if you have an f of t, if you break this up as f of t plus f of minus t divided by 2 plus f of t minus of f of minus t upon 2. This becomes the even part and this becomes the odd part.

Because, when you change t for minus t, the value of this part does not remains unchanged. But, if you substitute minus t for t, the value of this function gets reversed. Therefore, any f of t can be split up into it is even part and odd part respectively, in this fashion. And when you do that sometimes, you will find some additional symmetries, which are not present in the original waveform. Let us see how this goes.

Suppose, this is f of t, then let us construct what f of minus t would be. The sequence of values, which this function takes for positive t, it will assume in the reverse direction. Therefore, you have corresponding to this. And another loop corresponding to this, on the negative side. And corresponding to this, you have a loop like this and it goes on like this. This is f of minus t.

To get the even part of the function, then we have to add up these two waves. And then, divide by 2. So, if you do that, then what you have is whenever this is blank, you have this half cycle of sine wave. Whenever, this is blank this is a half cycle sine wave you have got. Therefore, you have like this. Where the amplitude now, the peak amplitude is A, but you are dividing by 2, this becomes A upon 2. And this is the even part of this.

On the other hand, if you subtract f of minus t from f of t and divide by 2. You get the odd part, f of t minus f of minus t upon 2. So, when you subtract the second waveform from the first, this gets reversed, the sign gets reversed. Therefore, this becomes a negative half cycle. And this also becomes a negative half cycle. And therefore, these negative half cycles, fit in smugly into these blank intervals. And the result is, that you have a waveform like this, which is a pure sine wave.

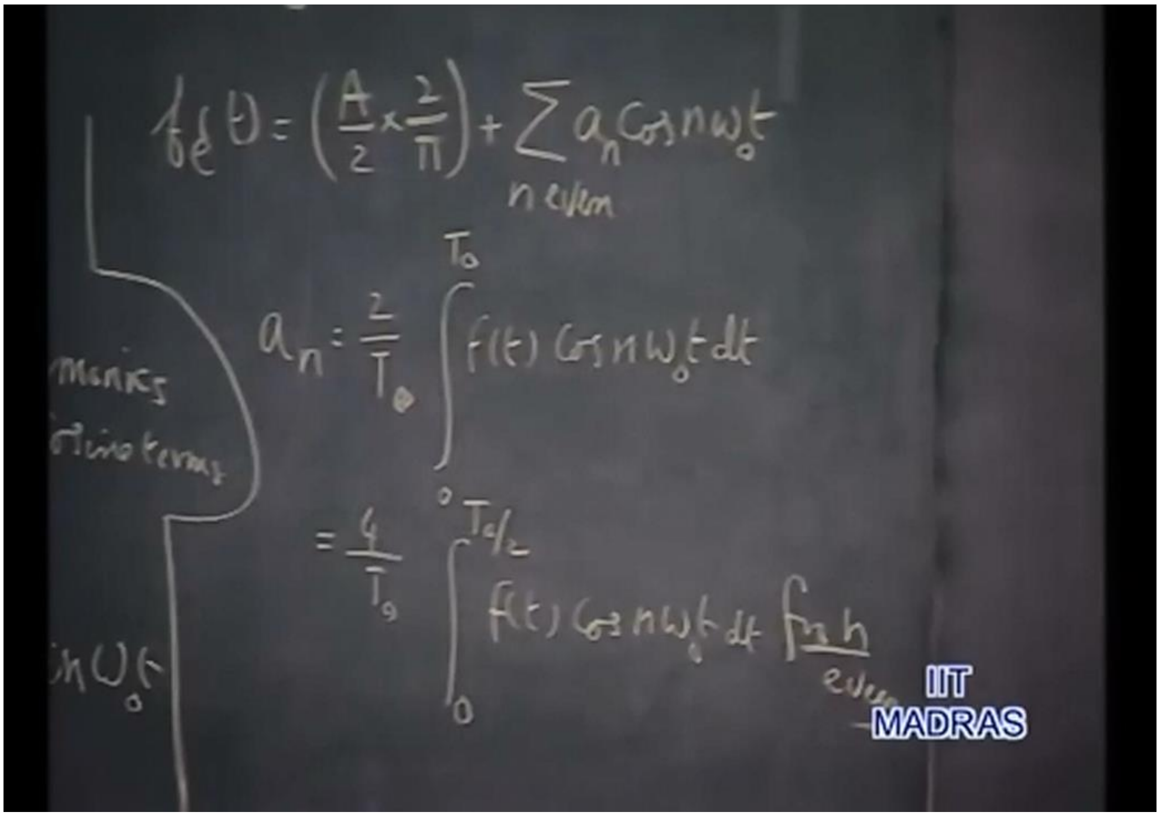
Because, each half cycle sine wave is reproduced in the proper direction here. And this is f of, the odd part of t f not. So now, if look at this constituent parts f e t and f not of your original f of t, you immediately observe that there are symmetries here. For example, f not of t is certainly a pure sine wave. Therefore, there is no Fourier series expansion necessary for that. That itself constitutes the entire Fourier series, A not by 2 sine omega.

So, this will be A not upon 2 sine omega not t. But, as far as the even part is concerned you observe that, this has got what we described as a kind of half wave symmetry. The function repeats itself, every half cycle.

And you also recall that, we mentioned in the last class, just last lecture. That whenever you are having a waveform like this, we would like to still continue this as the basic period, not as this. Because, we are going to relate this basic period, to the parent waveform from which it is generated. After all we want to talk about fundamental frequency in relation to this.

Therefore, we will continue to have the same fundamental frequency, when describes in this waveform as well. Consequently we regard this as the basic period, in which case the function f of t happens to be f of t plus t not upon 2. Therefore, this will have only even harmonics present. And since the function is even, only sine terms, only cosine terms will be present. So, if you make the Fourier series expansion of this. And add to this the Fourier series expansion of f not of t, which is this. Then, you get the Fourier series for the entire function.

(Refer Slide Time: 06:26)

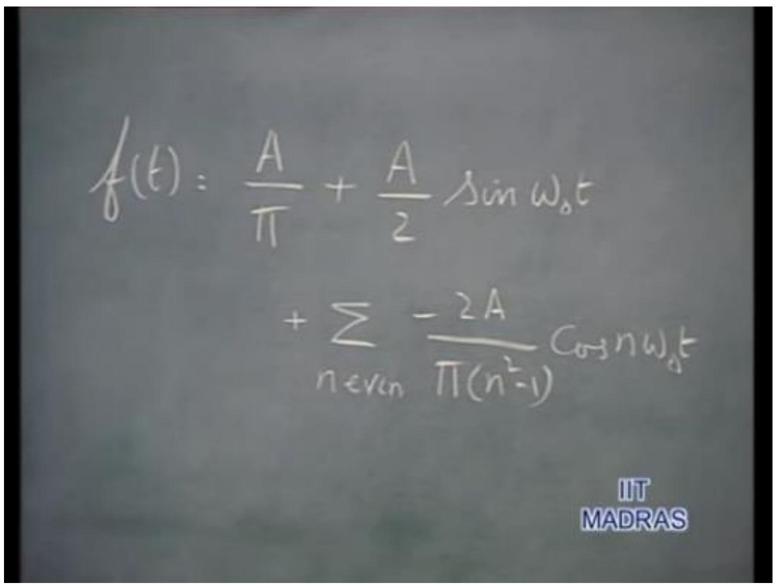


So, let us do this. F e of t suppose has a d c term. The average value of this, as you know any sinusoid, we are talking about the absolute average value 2 upon pi, that is the d theta. Plus you have A n cos n omega not t; where you need to have n even only. Because, odd values of n would be absent, because it contains only even harmonics. To calculate A n, you take twice the average of the function 0 to T not f of t cos n omega not t d t.

And it can be shown that, the contribution coming from this integral. For 0 to T not by 2 will be the same, from t not by 2 to T not provided n is even. Exactly the same arguments which we used, in discussing the half wave symmetry case, where we have odd harmonics present.

Exactly the same way, we can show that this is equal to 4 upon T not 0 to T not upon 2. That means, you are taking the average of this function over half cycle. F of t cos n omega not t d t for n even only. You see this particular problem, may not be valid for n odd. I will not go to the resulting integration. It can be shown that, this will lead to minus 2 A upon pi n square minus 1, that is the value. So, finally, the Fourier series expansion for this can be written. I will write it here.

(Refer Slide Time: 09:15)

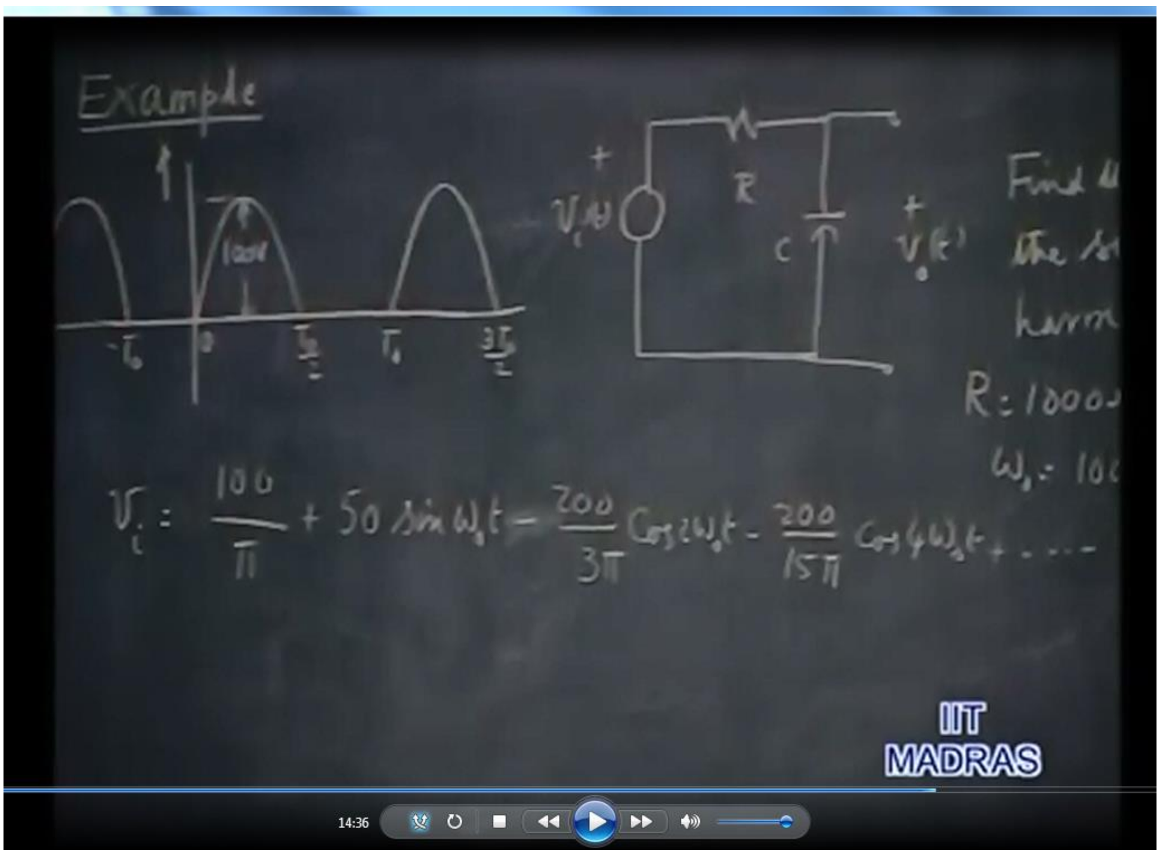


So, the Fourier series for this waveform would be, f of t equals the d c term A upon pi. Plus the fundamental term which comes from the odd part, A upon 2 sine omega not t. Plus the remaining terms in the Fourier series expansion of the even part of the function, which will be minus 2 A over pi times n square minus 1 cos n omega not t. For n even starting from n equals to onwards.

So, you observe that, even though this function as such does not appear to have, any of the symmetries that we have talked about. By splitting this up into even and odd parts, you are able to find some symmetry, at least in one of those parts. In the other of course, falls out. It just breaks down into a single term.

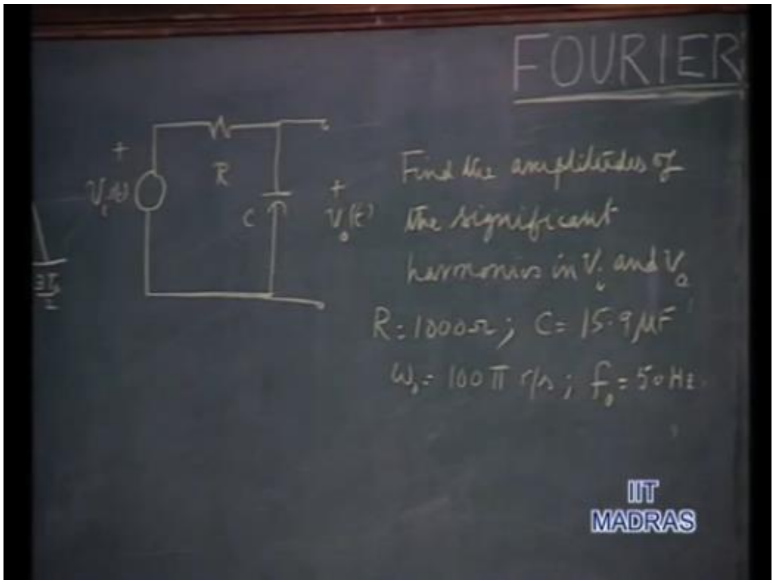
So, it would be sometimes worthwhile for us. Before, we proceed to get the Fourier series expansion of any waveform. To see if we can produce some symmetries, by resolving this function into it is various constituents. One of them being the even part and odd part respectively. You can think of other ways of resolving this, but we need to confine our discussion only to this. Now, let us use this, work that we have done here. To work out another example, where such a waveform is applied to an electrical circuit.

(Refer Slide Time: 11:05)



In this example, we will consider that a waveform of a voltage, of this shape, is applied to an R C circuit. And the voltage across the capacitor is taken to be the output voltage. So, we will imagine that the peak value of this half wave rectified, sine wave is 100 volts. Now, this voltage is applied across the R C circuit. And we are interested in finding out, the significant harmonics both the input and the output voltage.

(Refer Slide Time: 11:57)

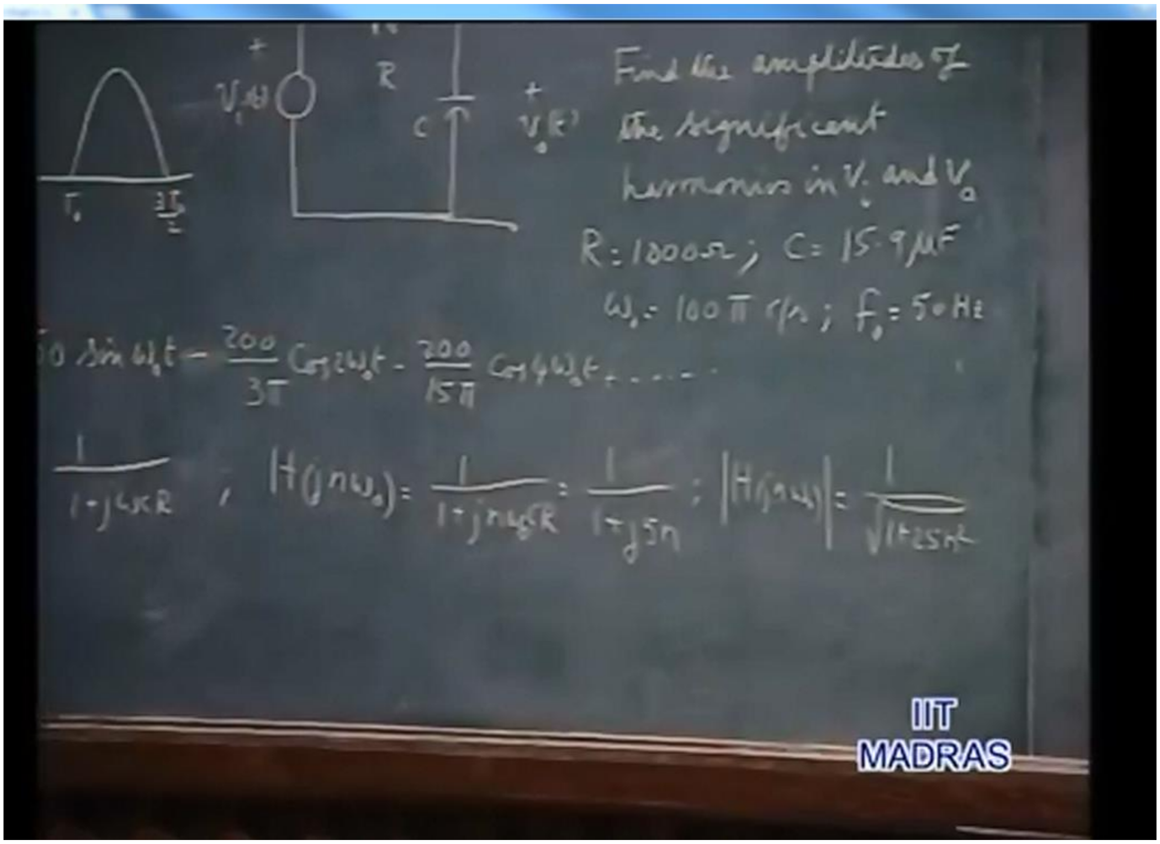


So, the problem is find the amplitudes of the significant harmonics, in v i and the output voltage v not. The data that is given R, let us say is 1000 ohms, C 15.9 micro ferrets. And omega not corresponding to this period T not is 100 pi radiant per second, which means the fundamental frequency is 50 Hertz.

Now, since this input wave form is not sinusoidal, we cannot apply straight away the phasor methods, for calculating the output voltage. On the other hand, the Fourier series tells us that such an input voltage, can be decomposed into a number of sinusoids. And so for each one of this sinusoidal components, we can find out the corresponding output, using the phasor methods.

And superpose all the solutions to obtain the output. Or in the problem like this, we are only interested in knowing the magnitudes of the harmonic components, in the output voltage v not. So, for each harmonic, we can apply the phasor notation and the phasor algebra.

(Refer Slide Time: 13:31)



Now, we know that v i, can be expressed by means of Fourier series, using the result that we have obtained. In the previous example, when we have considered just this kind of wave, half wave rectified sine wave. The answer there was, if you substitute the numerical values, it will turn out to be hundred upon pi. That is the d c component, plus 50 sine omega not t, that is the fundamental. And in addition we have a number of even harmonics.

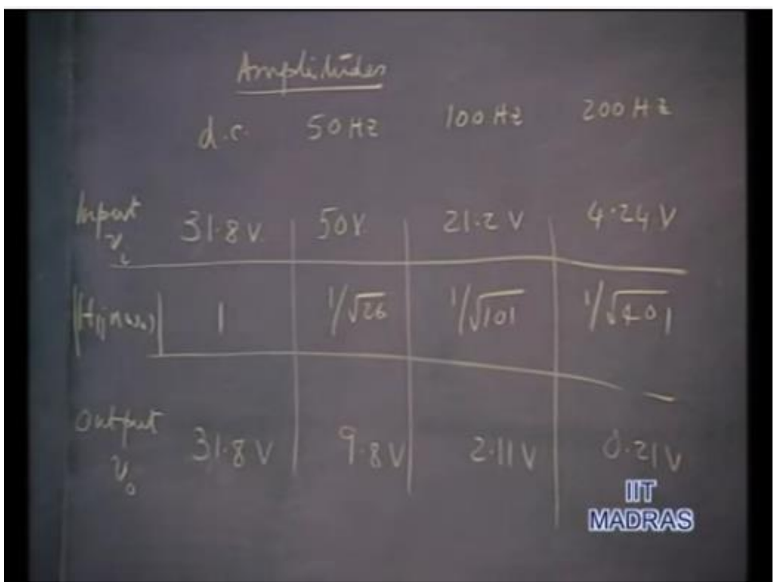
The first two even harmonics are, the second harmonic and the fourth harmonic. This will be 3 pi and this is 200 by 15 pi cos 4 omega not t. So, for each one of these, plus other terms which are insignificant, which we will ignore. For each one of these, we would like to find out the corresponding output quantity. And to do that, we must find out, the output voltage to the input voltage ratio, as a function of frequency.

So, the system function in this case H j omega by potential divider action, is 1 over j omega C divided by R plus 1 over j omega C. That will be 1 over 1 plus j omega C R. This is the general system function, as a function of frequency omega. But, we are interested in evaluating this for particular values of omega, which are omega not, 2 omega not, 4 omega not and so on. Consequently, we will find out H j n omega not for a general n.

This will be 1 over 1 plus j n omega not C R, which when you substitute the numerical values for R C and omega, will turn out to be 1 over 1 plus j 5 n. In particular, we are interested only in the amplitudes of the harmonics. So, we are not really interested in the angle associated with H of j omega not. So, we would like to know, only the magnitude in our problem this will be, therefore 1 over square root of 1 plus 25 H square.

So, we know the amplitudes of each one of these harmonic terms. We know the magnitude of the system function. Now, therefore, we can find out the amplitudes of the output voltage.

(Refer Slide Time: 16:11)



We can do it, we can organize it in this fashion. The amplitudes of d c, d c there is only one quantity. We do not have to talk about amplitude. But, we can talk about the amplitude of the fundamental component, the second harmonic and the fourth harmonic.

These are the significant harmonics, that are present here. So, the input voltage has a d c component, which is 100 upon pi. That is 31.8 volts. The 50 cycles component is 50 volts. The 100 cycles component is 200upon 3 pi, that is 21.2 volts. And the 200 Hertz component is 200 upon 15 pi, that turns out to be 4.24 volts. So, these are the amplitudes of the different harmonic components, as far as the input is concerned.

And the value of the system function. When you apply a d c input here the same d c comes out across here. Because, there is no current passing the circuit and therefore, that is equal to 1. The input output ratio is 1. In the case of the fundamental n equals 1, therefore this is 1 over square root of 26. In case of second harmonic n equals 2. Therefore, this will be 1 over square root of 101.

And in the case of the fourth harmonic n equals 4. So, 16 times 25,400 is 1 over 400 and 1 square root of 400. So, when you multiply these amplitudes with the corresponding magnitude and system function. As far as the output is concerned, the various components will turn out to be this multiplied by this, 31.8 volts. The 50 cycles component, turns out to be 9.8 volts, 100 cycles 2.11 volt and this is 0.21 volt. So, this R C circuit here essentially acts as the filter.

You have an input voltage which is non-sinusoidal, which is a rectified sine wave, half wave rectified sine wave. And we would like to have a filter like this, to swamp out the ripples. So, all the ac components, should be reduced to the extent possible. And we would like to have the output to be as pure a d c as possible. Now, how good is this filter. Let us see in this case, in the input you have 31.8 volts d c.

But, the harmonic components are quite substantial, 50 volts fundamental 21.2. Second harmonic 4.24, fourth harmonic, but as for the output is concerned, the harmonic amplitudes are brought down. Considerably you compare it with the d c. Therefore, this is a good filter, as far as suppression of the various harmonic components are concerned.