Networks and Systems
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Lecture- 20
Symmetry Condition Examples

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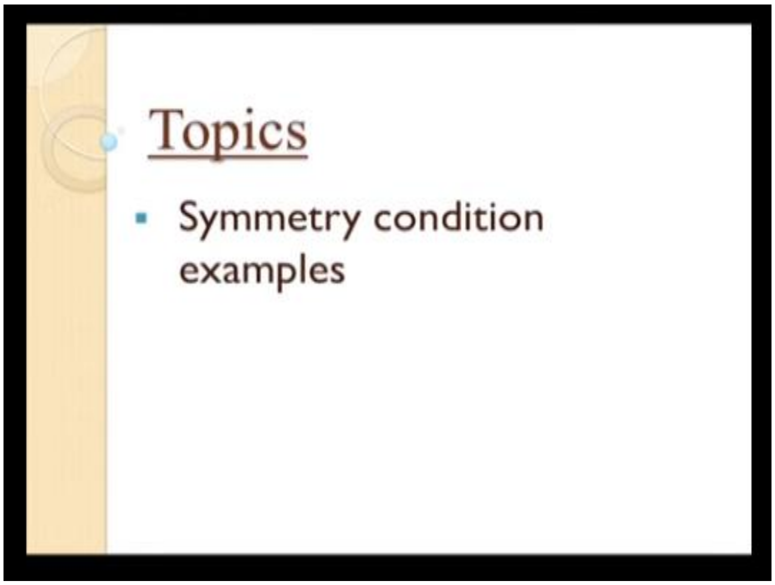

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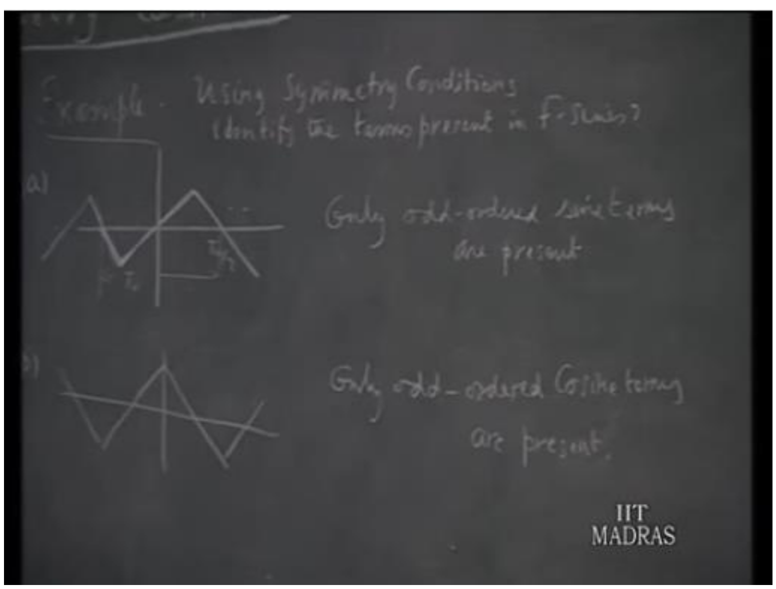

Put down some waveforms, let us find out first of all what kind of harmonic components are present making use of the various symmetric conditions that we have discussed.

Suppose, we have this kind of waveform using the symmetry conditions identify the terms present in the Fourier series or you can say what are the terms that are absent in the Fourier series.

So, if this function is there first of all you can straight away see this is an odd function because f of t equals minus f of minus t. Therefore only sine terms are present and further whatever waveform you are having here is repeating in the next half cycle with a reverse side.

Therefore, this has the half wave symmetry of the type which we have talked about so the result is that this can have only sine terms but odd harmonics. So, odd ordered sine terms are present you cannot expect cosine terms you cannot expect second harmonic fourth harmonic and so on certainly not this.

Suppose, i shift this waveform you have something like this triangular wave is symmetrically situated the origin, then what do we have this is an even function of time. Therefore cosine terms are present but that half way symmetry that we talked about still persists whatever occurring in this half cycle will repeat itself therefore we can say now it will have odd ordered cosine terms are only present.

We must carefully distinguish between the oddness and the evenness of the function and the oddness and the evenness of the order of the harmonic right. When you have cosine terms you say it is an even function sine terms will give only an odd function that relates to the function itself f of t and f of whether it is f of minus t or minus of f of minus t what we are talking about is the order of the harmonic the index of the harmonic.

Second harmonic, fourth harmonic or even ordered harmonics and fundamental third harmonic etc are called odd ordered harmonics.
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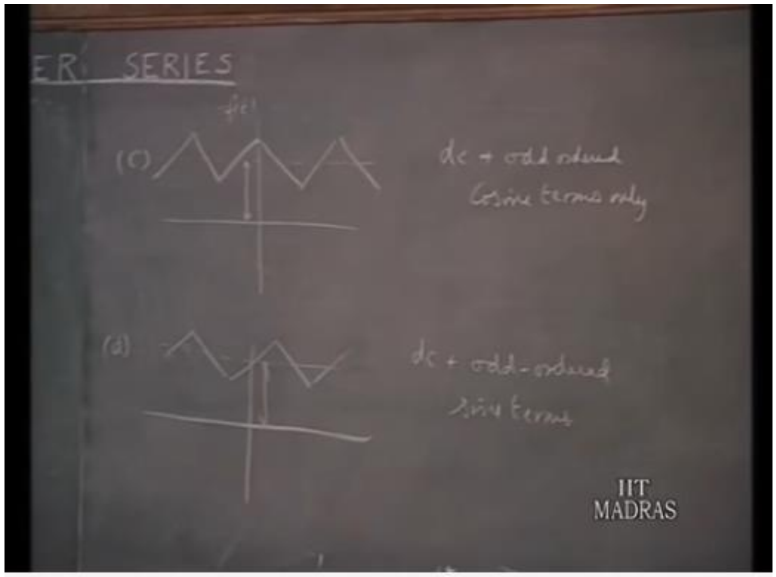

Suppose, I have a function like this which is the same function shifted upwards by a certain amount. So, in other words if i draw a line here parallel to the x axis and subtract this value from this f of t this will be the result. So what do we conclude from this it will have all the harmonics that you are having here plus a d c term right. So, you have because if you subtract this value from then resulting waveform will be exactly this therefore we have d c plus odd ordered cosine terms only.

Similarly, if this waveform is shifted up; so if this is the line then you have if you subtract that amount this value from the waveform you have a waveform which is given in a therefore you have d c plus odd ordered sine terms.

One thing you would notice from this example is that a same function if we shift the origin in one case it may be an even function other case it may be an odd function. Therefore, the placing of the horizon plays an important role but that does not affect the amplitude of the harmonic. As we will see a little later and once again it is also possible for us to note that when you remove the d c term from a given signal sometimes you find out certain characteristic which may not be present in the original signal as these examples show.

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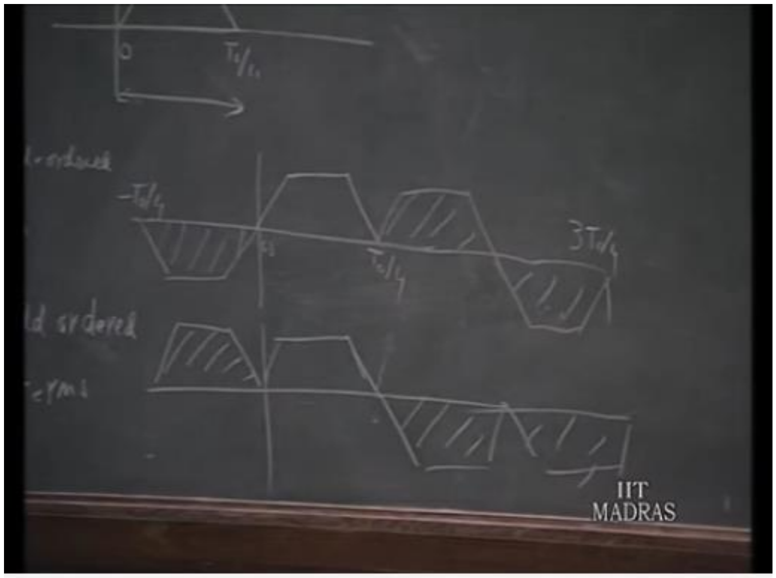

Now, let us work out another example so a waveform is given only for this interval of time now certain initial conditions are given and we are asked to find out identify the complete waveform right. So, let us say the conditions are f of t has only odd ordered sine terms, so if the waveform for the entire cycle is not given only partial specifications are given but this condition is given.

So, this is the data that is given to us, so if f of t has only sine terms which means the function is odd therefore if 0 is t by t not upon 4 is prescribed to us. We immediately conclude that this must be its image on the negative time axis. So, this portion we can fill up because only sine terms are present and then we are also given that it has odd ordered harmonics only which again tells us that whatever sequence of values it takes in 1 half cycle is getting repeated with a negative sign in the subsequent half cycle.

So, starting from this in this half cycle this is the sequence of values therefore the negative of that will be this and therefore this also is filled now that corresponds to 3 t not by 4 so we have completed the waveform a full cycle right.

So, we can make use of the conditions of symmetry and then extend this to satisfy this the given data and the condition that are given. So, suppose it is said that f of t has odd ordered sine terms, cosine terms then how does it go? This is the waveform that is given to us and since it has only cosine terms it is an even function. Therefore, this would be the situation from minus t not upon 4 to t not upon 4 and since it has only odd ordered harmonics this entire waveform gets repeated with a negative sign.

So, this is the extension that we are having and that would be the situation for a case where f of t has only odd ordered cosine terms.

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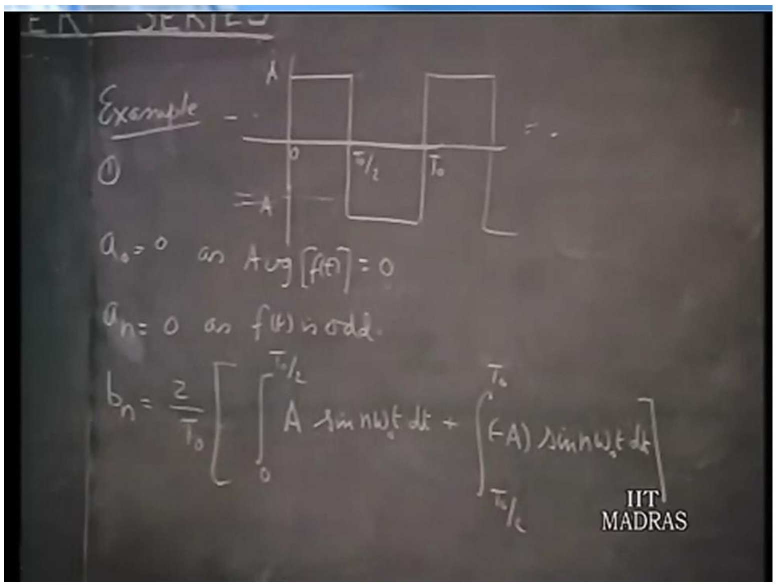

Let us have another example we will take the square wave which is a very important waveform of course its repeated itself in the negative time also like this and we are asked to find out its Fourier series expansion. We can see this take away that the positive half cycle and negative half cycle are equal areas that calculates each other out for the average of this function f of t over a complete period is 0 Therefore a not is a 0. We also can see without further do that b n is 0 because the function is i am sorry the function is odd therefore a n 0.

So, the only coefficient that can represent are the b coefficient so b n would be 2 upon t not 0 to t not upon 2. I will break up this integral into two parts from 0 to t not upon 2, the value of the function is A sine n omega not t dt plus t not upon 2 to t not in this interval of time. In this interval of time the value of the function is minus A sine n omega not t dt.

You can evaluate this second term into two integral separately and then arrive at the result alternatively. We can also make use of one property which you are discussed already when you are talking about half wave symmetry conditions we said which is exactly this situation whatever is happening in positive half wave cycle occur and in the negative second half cycle.

Therefore, at the time if you recall exactly similar situation this integral is equal to this integral for odd n. Therefore, we can as usual write this as for odd n this will be 4 by t not like this alternatively.

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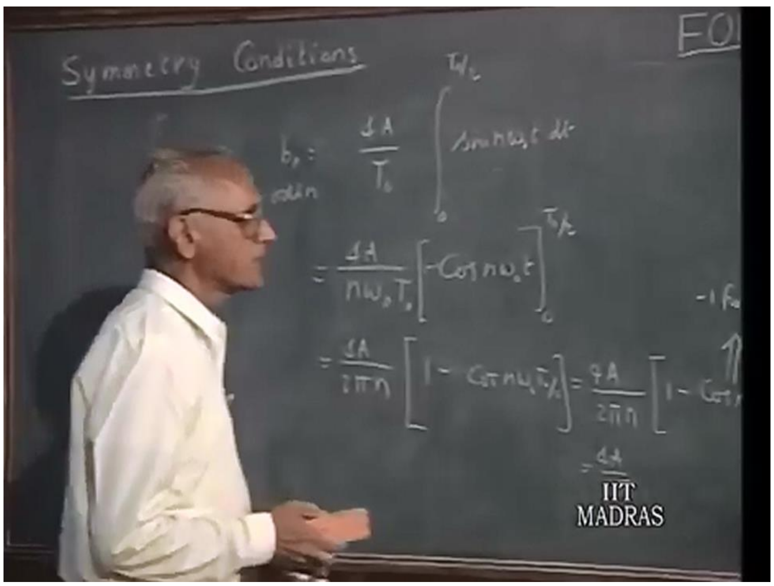

We can calculate this also independently, so we can write this 4 by t not 0 to t not upon 2 A sin n omega not t d t for odd n only you must be very careful in writing this. You cannot have these and then evaluate for even n will give a result but that is wrong for even n this time will cancel these and it becomes 0. So, when you write this expression you must you must make sure that you know that is valid only for odd n only its 0 even n.

So, you work this out then it will be b n will be for odd n 4A by t not 0 to t not upon 2 sin n omega not t d t. So, 4A by n omega not t cos n omega not t 0 to t not upon 2 right.

Therefore, this will be 4A by omega not t 2 pi t not omega t not is 2 pi n and here we have 1 minus cos n omega not t not by 2. Therefore this is 4A by 2 pi n times cos n omega not t not by 2 cos n pi and we are talking about odd n so cos n pi will be minus 1 and therefore this will be 4 a by n pi so only b n terms are present.

So, finally we conclude that the Fourier series expansion for this would be.

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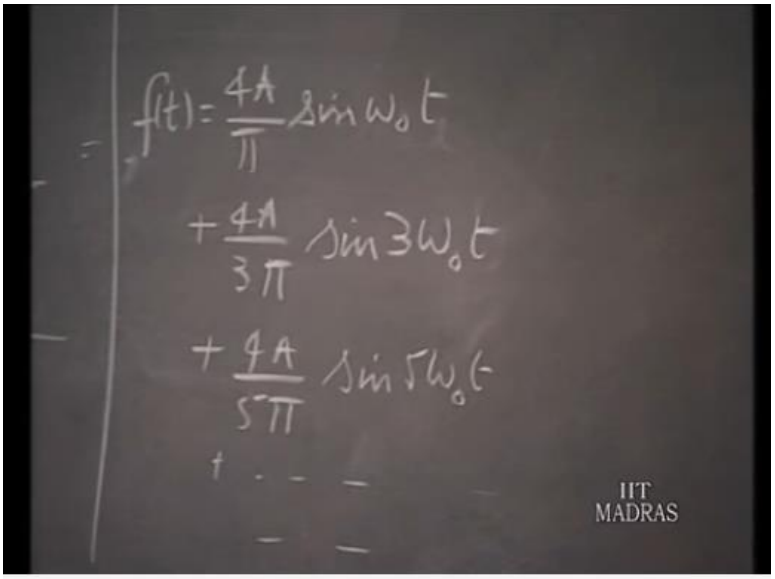

I will write here f of t 4 a by pi sine omega not t plus 4 a by 3 pi sine 3 omega not t plus 4 A by 5 pi sine 5 omega not and so on and so forth. So, this is a important result that it would pay us to keep this in mind for a square wave like this we can amplitude A.

The fundamental component has an amplitude 4A by pi so that means the fundamental will be something like this. The third harmonic will be having one third that amplitude so it will be fifth harmonic in one fifth. Actually in the figure which we have seen in the earlier lecture corresponds to this so the amplitudes go down inversely as the order of the harmonic.

We will continue this discussion or to sum up first of all what we have learnt in this lecture is that the symmetry conditions on the given functions of time enable us to calculate the Fourier coefficients with less effort that it would otherwise be necessary in particular. We saw if the function is even then there can be only cosine terms present.

If the function is odd that is if f of t is minus of f of minus t only sine terms can be present and the third type of symmetry that we talked about is when whatever waveform you have for 1 half cycle is repeated for the subsequent half cycle with it is negative sign then we call that half wave symmetry. It is quite common in electrical machines for example and in that situation you have only the odd ordered harmonics you have the fundamental the next harmonic, the third harmonic and the fifth harmonic and so on.

There is another kind of symmetry which we mentioned in passing that is when the function repeats itself identically. For every half cycle in that case only the even ordered harmonics are present that means the second harmonic fourth harmonic and so on.

This as i said is we call this function to have a period twice which we normally expect or twice of what we expect because this is usually derived from another signal whose fundamental frequency is already defined using these symmetry conditions. We worked out a few examples which show how the Fourier will be evaluated making use of the various symmetries that are present in the next lecture.

We will talk about another method of setting up the Fourier series not in terms of trigonometric functions but in terms of exponential functions which will give a more compact notation and certain advantage of the competition this we will take up in the next lecture thank you.