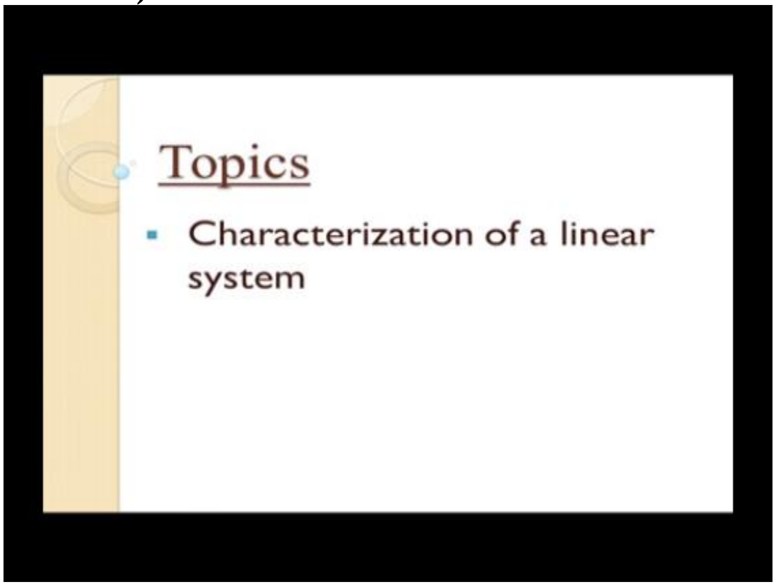
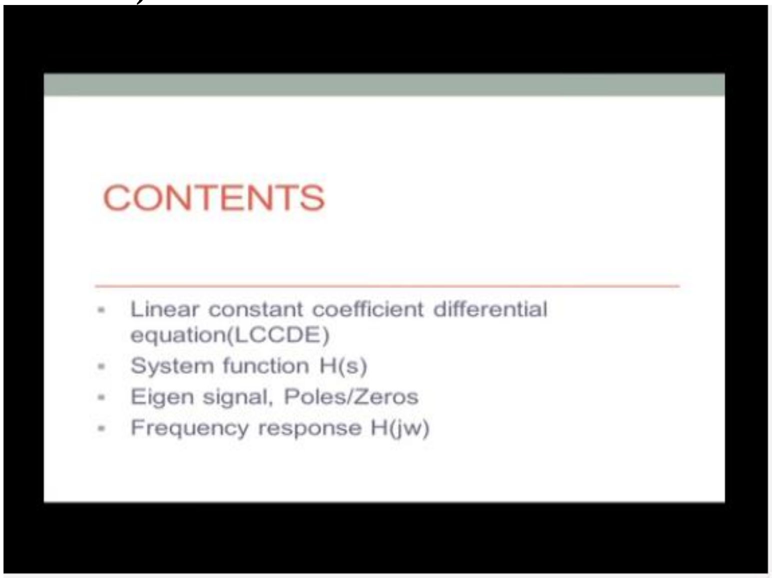
Networks and Systems
Prof V. G. K. Murti
Department of Electronics Engineering
Indian Institute of Technology – Madras

Lecture- 13
Characterization of a Linear System

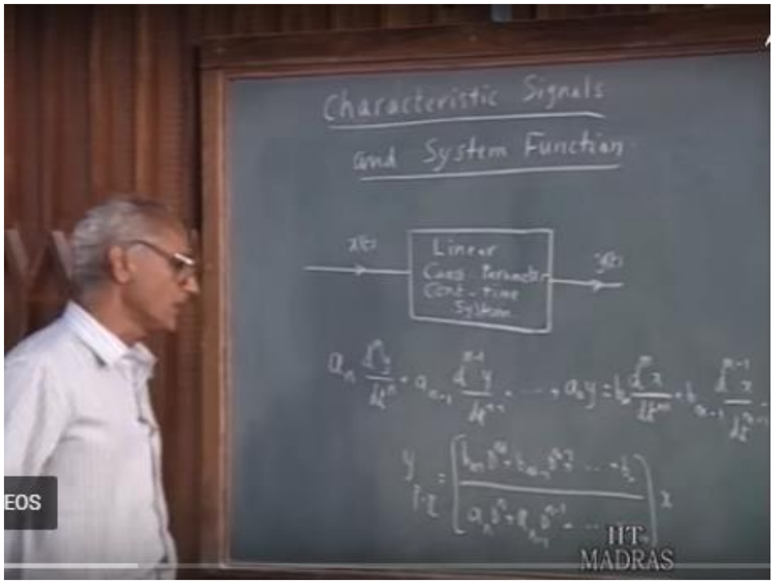
(Refer Slide Time 00:16)



(Refer Slide Time 00:20)



(Refer Slide Time 00:36)



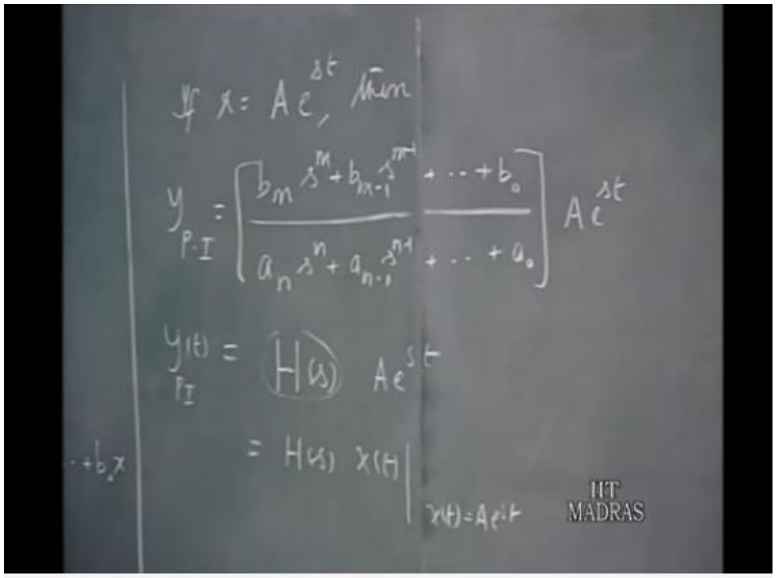
Let us now, introduce ourselves to the concept of characteristic signal of a linear system and the concept of system function. We will consider a linear time invariant constant parameter time invariant and constant parameter mean 1 and the same continuous time system. So, linear, constant parameter, continuous time system we have an input quantity and an output quantity y of t. An electrical network consisting of r l c elements being a specific example of this.

In general, the output y t and the input x t are connected by a linear differential equation with constant coefficients. Let us assume that the differential equation connecting these 2 is of this form. A n d n y d t n plus a n minus 1 d n minus 1 d t n minus 1 and so on and so forth plus a 0 y equals d m x d t m plus b minus 1. You can put b m if you like b m minus 1 x d t m minus 1 so on and so forth plus b 0 x.

So, this is a nth order differential equation connecting the output quantity y with the input quantity x. Now, the particular integral solution of this is given by the operator b m d m plus b m minus 1 d m minus 1 down the line to b 0 divided by a n d n plus a n minus 1 d n minus 1. So, on plus a 0 this is a operator a function of d operating on x. So, depending up on the input quantity you can calculate the particular integral solution for this.

This is what we do in the case of solution of the differential equation. And the electrical network which we considered in the last lecture is a specific example you remember that we ended up with the second order differential equation. For which we can calculate the particular integral solution in the same manner as here.

(Refer Slide Time: 03:29)



Now, it turns out that if the input function is of a particular type exponential function a e to the power of s t. Then the particular integral solution is this operator operating on a e to the power of s t. And from the theory of differential equations whenever you have an exponential function here, this operator function can be thought of as an algebraic function where d is replaced by particular value of s.

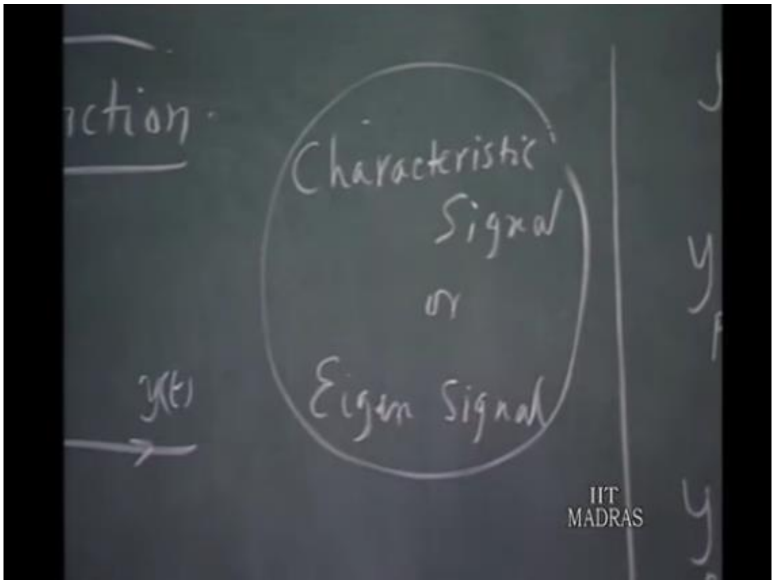
So, it turns out that the particular integral solution will be b m s to the power of m plus b m minus 1 s to the power of m minus 1 down the line to b 0 divided by a n s to the power of n a n minus 1 s to the power of n minus 1 up to a 0 times x of t which is a e to the power of s t.

So, the solution for this differential equation in the particular integral solution all you have to do is substitute s for d. So, if x is a e to the power of s t y p i is some quantity times a e to the power of s t. We often call this h of s a e to the power of s t. So, the particular integral solution the function of time is h s times a e to the power of s t or h s times x t when, x t equals a e to the power of s t.

So, this is a very interesting property that, the time function describing the force part of the solution and the excitation have got the same function of time. Except that it is multiplied by h of s which is independent of time. So, we can think of this as proportionality factor this is a function of only s, but not a function of time. So, the input time function and the output time function as for the particular integral solutions are concerned they are exactly the same. Except that it is scaled down by a factor or scaled up by a factor h of s.

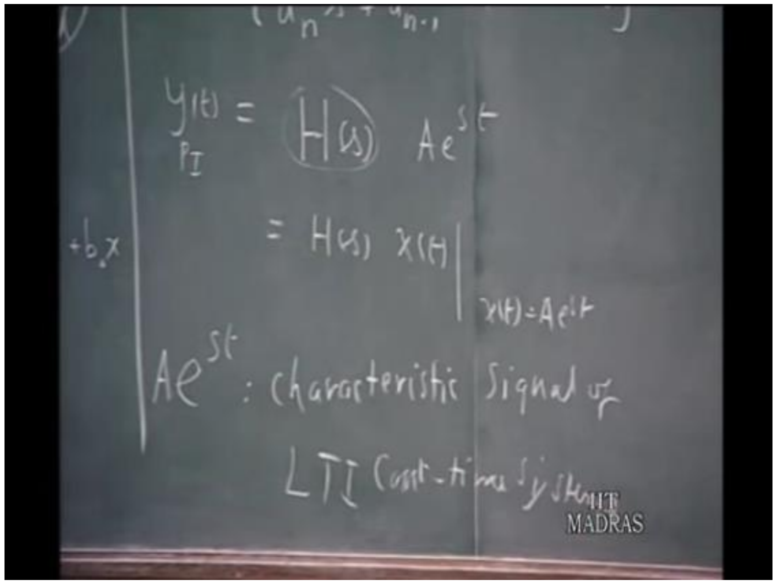
So, where the input excitation function and the output function have the same form of time function it is said to be a characteristic signal of a system. The characteristic signals are sometimes called Eigen system Eigen function. Characteristic function or characteristic signal are also called Eigen signal.

(Refer Slide Time: 05:55)



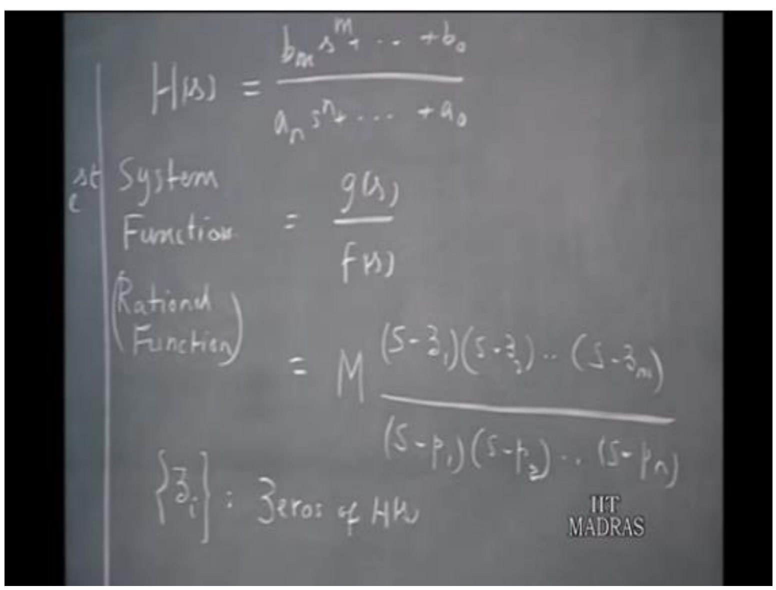
Again, the meaning of an Eigen signal or a characteristic signal is it is that particular signal which if it is given as input. The output will also will have the same time variation except for a scale factor and that scale factor is h of s in this case.

(Refer Slide Time: 06:26)



So, e to the power of s t is a characteristic signal e to the power of s t with a constant multiplier any constant is a characteristic signal of linear time invariant continuous time systems. This is a very important property. Any linear continuous time invariant system will have this as characteristic signal and therefore, if an input is in that form the output is obtained by merely multiplying this by h of s where, h of s is a kind of proportionality factor.

(Refer Slide Time: 07:11)



And this h of s is called the system function and in describing the characteristic signal s can be complex s need not be real. It can be need not be imaginary it can be complex in general.

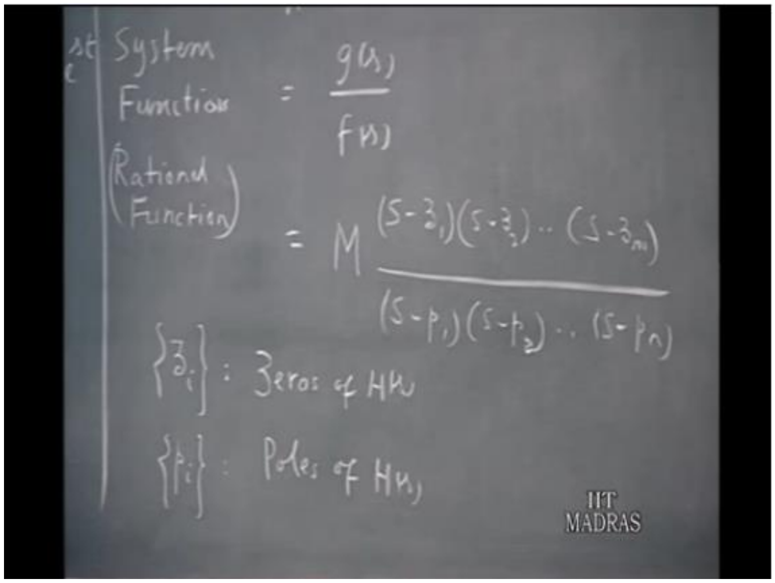
Therefore, the complex exponential signal of this type a e to the power of s t mathematically a can be complex as well. Therefore, a e to the power of s t is a characteristic signal and the complex frequency associated with this is s. And the system function which represents the proportionality constant between the output and the input is called the system function.

It is the function of the complex frequency of the characteristic signal e to the power of s t. This is called the system function and this will be of the form b m s to the power of m plus b 0 divided by a n s to the power of n down the line to a 0. So, if you know the differential equation then the system function can easily be obtained. After all you have got b m s to the power of m b m minus 1 the same coefficients applied appear in the 2 polynomials constituting the rational function h of s.

So, the system function is a rational function. For lumped parameter systems which we are dealing with rational function is the ratio of 2 polynomials and for linear lumped parameter systems the system function turns out to be the rational function. It is the ration of 2 polynomials if i call this g of s over f of s g and s are 2 polynomials and these 2 polynomials are easily set up if you look at the differential equations being coefficients in the differential equation constitute the constants in these 2 polynomials.

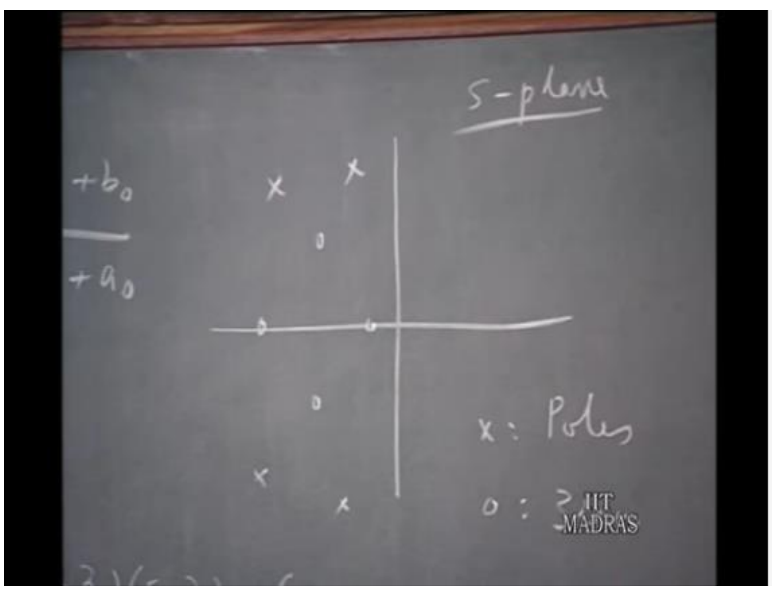
So, and further it is since it is a rational function of a complex frequency variable s 2 polynomials. We also would find it useful to represent this in this manner some constant m times s minus z 1 s minus z 2 so on. Since, this is a n th order polynomial you have s minus z m divided by s minus p 1 s minus p 2 and so on, s minus p n. So, you can factorize the numerator and denominator in this form and the set of values are called the zeroes of h of s these are the values of s which make h of s 0 go to 0.

(Refer Slide Time: 10:07)



If s is equal to z 1 or z 2 or whatever it is the h of s becomes 0 that is why, they are called the zeroes of h of s. The set of values of p which make the denominator vanish are called poles of h of s. These are terms which some of you may be familiar with from your study of complex variable theory. So, in any case at the frequencies of s at s values of s which correspond to a pole of h of s h of s becomes infinitely large. It blows up that means, the denominator is made equal to 0 on the other hand s equal to z i any 1 i equals 1 to m. Then the system function becomes 0.

(Refer Slide Time: 10:45)

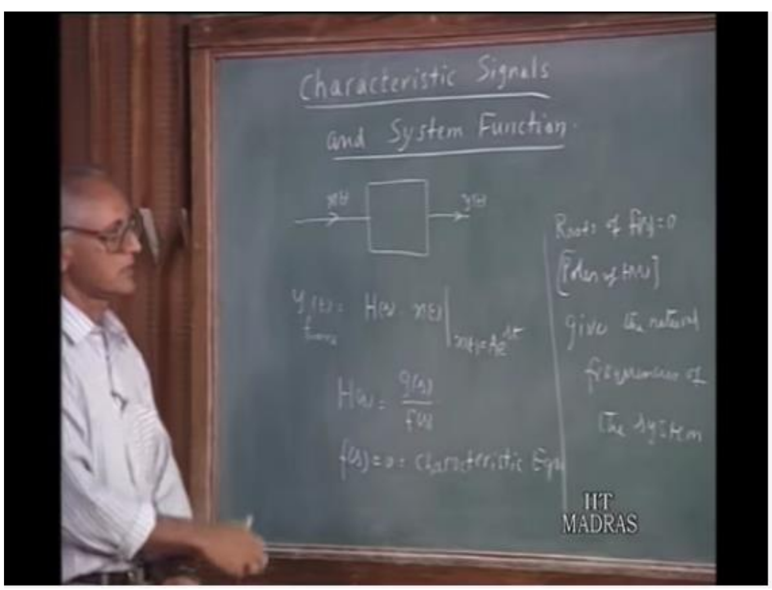


These are usually represented in the complex plane by the 0 locations are given like this and the pole locations are given by crosses. So, these represent poles and the zeroes are represented by small. Since, these are values are in general complex then they can be represented in the complex plane this is called the s plane or complex plane and the values of the zeroes and poles are represented in this fashion.

You recall that, whenever these coefficients are all real whenever there's a complex pole it is accompanies by its conjugate whenever there's a complex 0 it is accompanied by its conjugate So, they appear in pairs. So, as far as real zeroes and real poles are concerned they can appear of unit order singly without second pole or 0 appearing as a conjugate because, 0 or pole by itself is real there's no necessity for a conjugate pole or 0 to appear since they are real.

Now, system function plays a very important role in linear system studies as we will see later and particularly when you take up Laplace's transform techniques. This system function is very important tool in our analysis of linear systems. But, this has nothing to do with Laplace transforms really. It is a, we can regard this as the ratio of the force response to the excitation when the excitation is of the form a e to the power of s t that is all we need to know about it.

(Refer Slide Time: 12:22)



So, what we have seen now is that if w are talking about a linear time invariant continuous time system represented by this black box now put in an input x t and y t Then the force response of the system instead of particular integral system i am calling y of s that is force response is h of s times x t if x t is a characteristic signal.

This is the summary of what we have discussed just now this proportionality is valid only when s t is of the form a e to the power of s t not for other signals. And secondly, we saw that h of s is the ratio of 2 polynomials in s g of s over f of s. And if you look at the way in which we calculated g of s and f of s we took the operator function giving the particular integral solution and form the f of s and g of s.

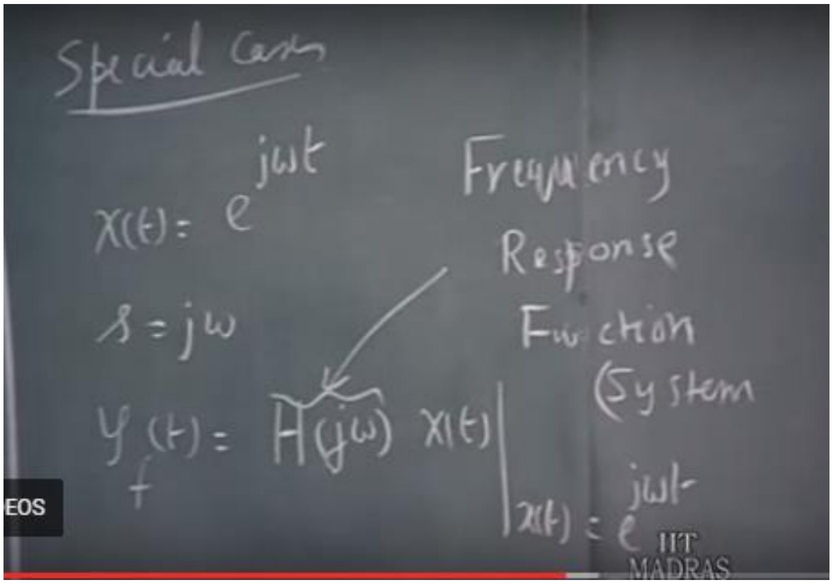
F of s equals a n s to power of n a n minus 1 s to the power of n minus 1 and so on and so forth. So, f of s equal to 0 is a characteristic equation that is the characteristic equation from the differential equation that is what is called the auxiliary equation. So, the zeroes of roots of this equation which are the same as poles of h of s, we said poles of h of s are the zeroes of f of s or the roots of the equation f of s equals 0.

So, the roots of f of s equals 0 that equation which can be called also as zeroes of f of s. The roots of the equation f s equals 0 can be termed as zeroes of f of s, values of s which makes f of s 0, which are also the poles of h of s. These are gives the natural frequencies of the system. What is meant by natural frequencies? These are the frequencies present in the complementary solutions or the homogeneous differential equation solutions.

So, this h of s builds in itself a lot of properties. Not only it gives the force response for this type of excitation, but it also gives you the form of the complementary solution straight away. So, whatever information is available in the differential equation is built in this system function.

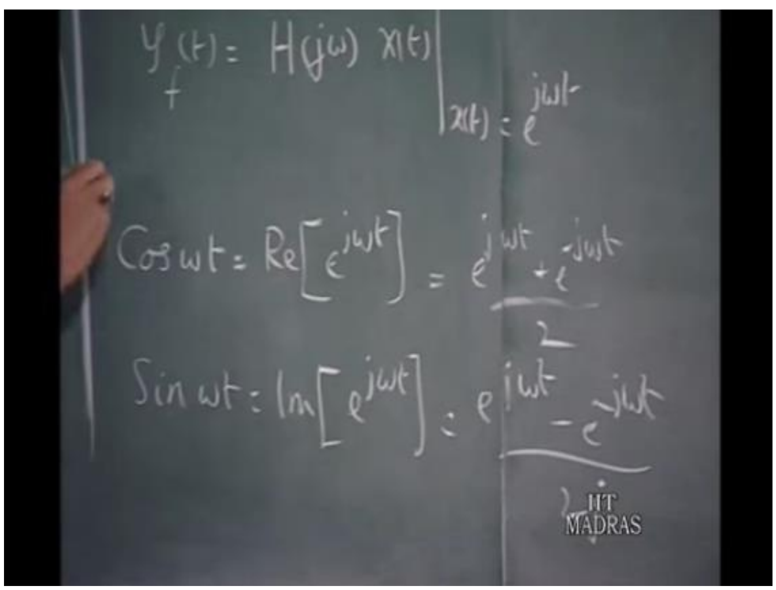
Once, you have the system you do not have to look at the differential equation again because, all the information that is available in the differential equation is given here. Because from this you can set up the differential equation if you wish, but you do not even have to do it because the complementary solution if you want all you have to do is solve for f of s equal to 0 that equation. It gives you all the terms that are present in the natural part of the solution or the complementary solution.

(Refer Slide Time: 15:56)



The special cases it often turns out that we would like to have x t of the form e to the power of j omega t. This is the special case of x t being e to the power of s t if x t is e to the power of j omega t that means, you are taking s equals j omega t. In which case, the forced response will be h instead of s j omega times x t where x t is e to the power of j omega t.

(Refer Slide Time: 16:46)



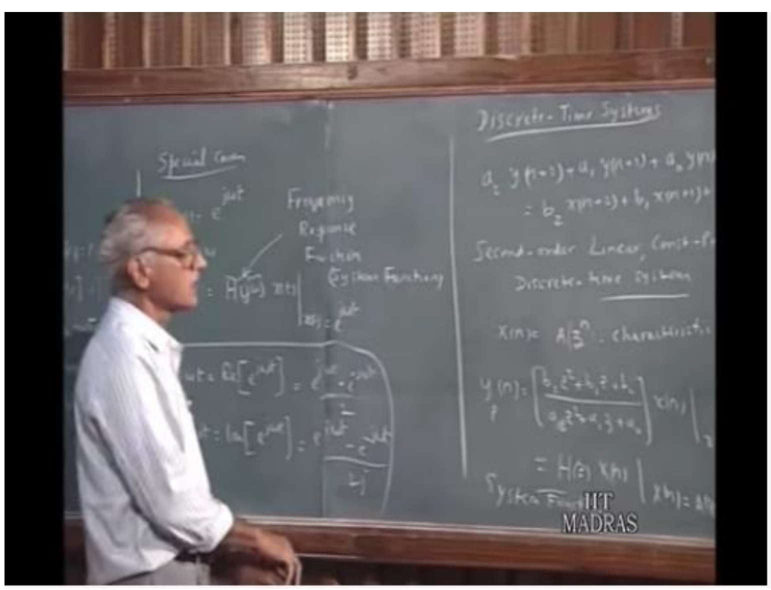
Now, this is important because whenever we are talking about sinusoidal quantities as the excitations or inputs sine omega t cos omega t they can be related to e to the power of j omega t. For example, cos omega t can be thought of as the real part of e to the power of j omega t, alternately, you can think of this as the sum of 2 exponential functions as i already mentioned earlier. Similarly, sine omega t can be regarded as the imaginary part of e to the power of j omega t or the combination of 2 characteristic signals.

So, e to the power of j omega t is closely related to the sinusoidal functions which are very important in system studies. And therefore, the response of a system to a signal of e to the power of j omega t can be regarded as a special case of the response to an exponential signal e to the power of s t.

After all, e to the power of s t and e to the power of j omega t are closely related. So, this is also important and when we study later on in Fourier transform methods and Fourier series methods h of j omega becomes very important. And this h of j omega it is in itself sometimes called system function, but more commonly we can call it the frequency response function frequency response. Also sometimes, when there is no scope for confusion this is also called system function.

So, this is very important that is h of j omega is also regarded as a system function or a frequency response function. This is the special case of h of s when h is equal to j omega and this becomes very important when we analyze any system on the basis of sinusoidal inputs. Because e to the power of j omega is as good as cos omega t sine omega t as far as analyses are concerned.

(Refer Slide Time: 19:01)



We then before we move on to other topics let us see what kind of system function you have when we have discrete time systems. As an example let us take a second order discrete time system. Now, this is a second order discrete time linear constant parameter discrete time system.

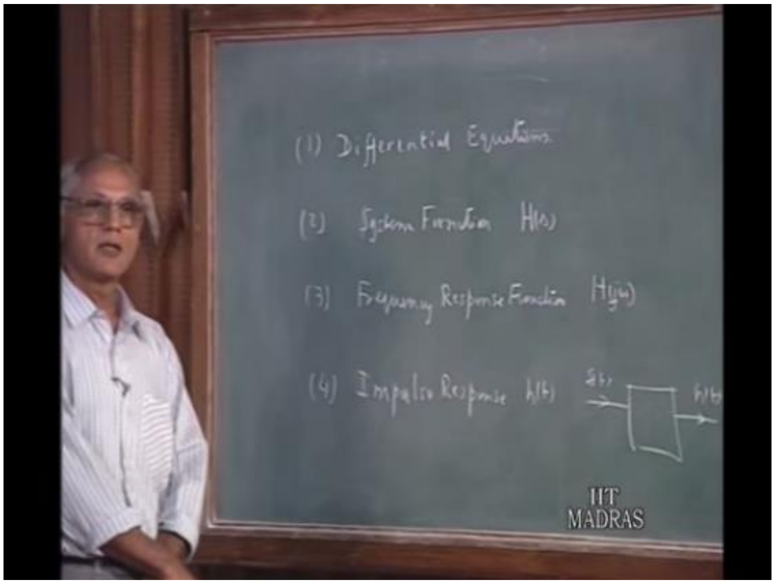
What we have studied earlier is a continuous time system this is a discrete time system. It can be shown that x n equals some constant times z n is a characteristic signal. So, just like e to the power of s t being a characteristic signal for continuous time domain in the case of discrete time system a times z to the power of n is a characteristic signal. A is a multiplying constant we disregard that z to the power of n is a characteristic signal.

And the force response y n the particular integral solution for y n can be written as b 2 z square plus b 1 z plus b 0 divided by a 2 z square plus a 1 z plus a 0 multiplied by x of n when x of n is a characteristic signal. So, if x of n is a characteristic signal the force response of the second order discrete time system will be b 2 z square plus b 1 z plus b 0 by a 2 z square plus z 1 z plus a 0 just in the same way, as we had in the continuous case.

From these coefficients you form these 2 polynomials and this move we will write this h of z where h of z is once again x n is a characteristic signal is a z power n and h of z is called a system function. So, this is a discrete time system function which is a function of z h of z just like h of s, you have got h of z here and the working is quite analogous to the continuous time case. The characteristic signal is z to the power instead of e to the power of s t and the system function is h of z instead of h of s, but z is the variable that you get in this.

We will deal with this later at the end of the course when you deal with discrete time systems. So, this is a parallel development as applicable to discrete time systems it completely follows the same lines that, we have for the continuous time systems. So, so far we have talked about 3 different ways of characterizing a linear system.

(Refer Slide Time: 23:02)

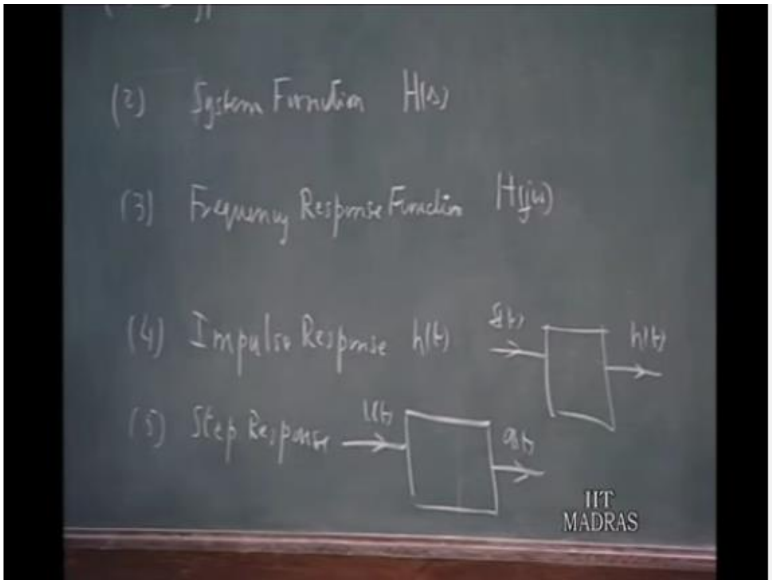


What are the different ways? Let us see what they are. So, far we have discussed different ways of dealing with linear systems, let us see what they are recapitulate what they are 1 is the differential equation solution of differential equations will give us the constant solution. We saw the difficulties associated with this. Second method is system functions h of s.

You can use the system function it gives the same information as the differential equation h of s then we also saw an equivalent is a frequency response function h of j omega. It turns out that when we later study Laplace transformation methods the system function some into its own we will try to exploit the properties of the system function to deal with the transient.

And when we study later the Fourier transform methods we will use the frequency response function h of j omega. There is also a fourth way of describing a system response this is called the impulse response. So, if you give an impulse to a system the input is the impulse the output we will call it impulse response unit impulse this h of t.

(Refer Slide Time 24:14)



This is an independent way of characterizing a system all are equivalent. That's if you know the system function h of s we will be able to analyze the find out the response for any given excitation. Similarly, if you know the frequency response function h of j omega we should be able to find out the response to any excitation. And likewise, an impulse response h of t is also an independent way of characterizing a linear system.

If you know the response to an impulse you should be able to find out the response to any given input x of t.

That is also an independent way of doing this there is yet another way, what is called the step response. So, here you are giving u of t unit step as the input and what you get is the output a t. That's the step response these are all different ways of characterizing a linear system and all are equivalent. Each will give complete information about the system which you need to solve for the response under any given excitation.

We have studied the differential equation to some extent and we will leave it at that we have noted the methodologies that is involved and we also noted some of the difficulties. The difficulties will be overcome when you use the system function approach using the Laplace transformation methods which we will take up at some point later point in this course.

And we will also study how to exploit the information that is available h of j omega the frequency response function when we study Fourier series and Fourier integral methods. But, we do not have to go to this complex frequency or the frequency omega. We can carry out the work in time domain using the impulse response or the step response.

They are independent ways of characterizing the input output relations of a linear system and this is something which we will look up we will examine in some detail in the next lecture. But, we will use this information again further when we go to Laplace transformation methods we will see how, the h of s system functions and the impulse response h of t.

How they are closely related to each other that will be taken up at a later point of time. But in the next lecture, we would have a closer look at the impulse response method of characterizing a linear system.