Networks and systems
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Lecture-12
Special Initial Conditions

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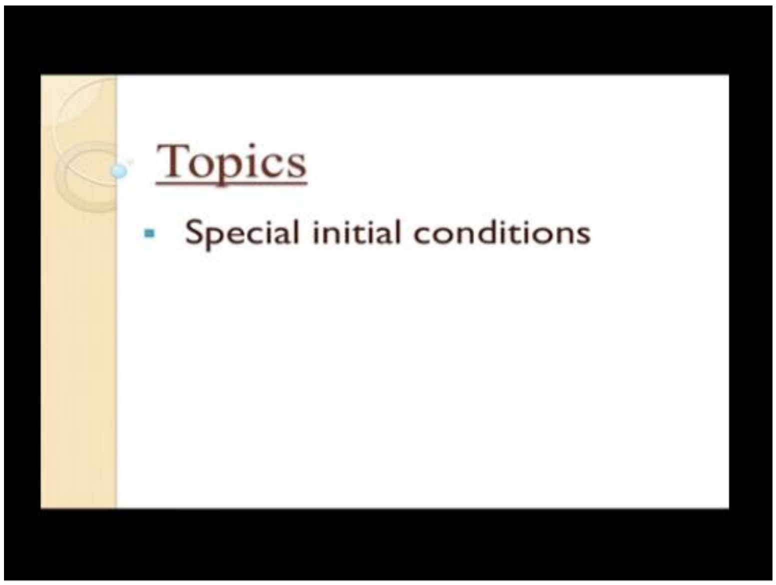

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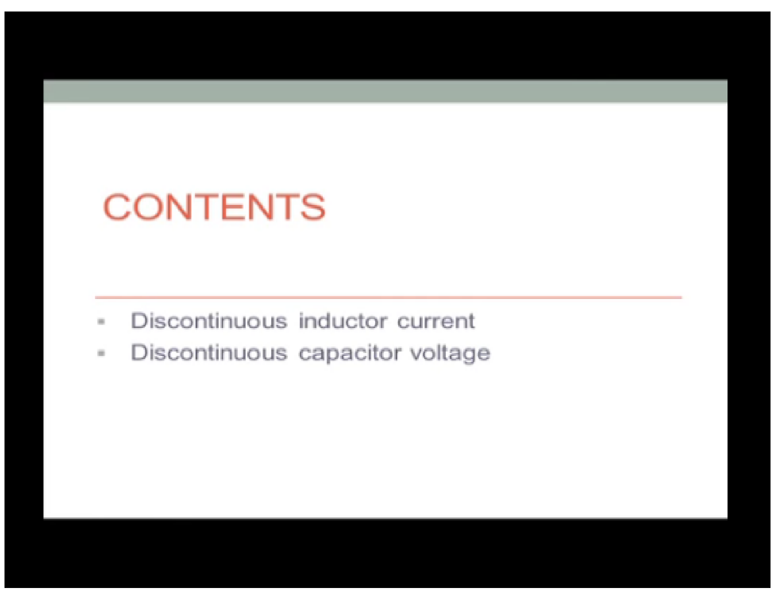

We had a look in the last class at the classical differential equation method of solution of a transient problem. Let us briefly recapitulate the features of solution by this particular method. The solution consists of 2 parts 1 is the complementary function part of the solution; this is obtained after solving the characteristic equation of the system. And the roots of the characteristic equation gives us the natural frequencies or the force free part of the solution of the system.

And these natural frequencies are characteristic of the particular system irrespective of the type of excitation and the complementary part of the solution consists is also referred to as the free response of the system. This consists of various natural frequencies with coefficients whose values are a priori not known to start with. Then there is the particular integral solution which consists of frequencies which are present in the excitation or reforcing function.

This is also referred to as a force response of the system. The force response of the system depends up on not only the parameters of the system, but also the excitation function or the forcing function. Now, together the particular integral solution and the complementary solution together determine the total solution of the system. But, to evaluate this solution specifically we need to evaluate the arbitrary constants that are present in the complementary part of the solution.

To do this we need to know some initial conditions regarding the variables which we have to solve for. And normally this transient solution is calculated based after switching operation say t equals 0. Information about the reactive elements in a network capacitors and inductors are known to us before the switching operation. Let us say at t equals 0 minus using this information, we assume the continuity of the capacitor voltages and the continuity of inductor currents in the time from t equals 0 minus to t equals 0 plus.

Assume the same values to hold and therefore, the initial values or the inductor currents at t equals 0 plus and the capacitor voltages at t equals 0 plus are known to us. Using this information, we will have to find out the initial values of the response quantity and its various derivatives depending up on the order of differential equation that is involved. And this particular process involves some manipulation as we have seen in the particular example.

Now, it often turns out sometimes turns out that the assumption that the capacitor voltage is continuous and the inductor current is continuous breaks down because, of the nature of the particular circuit. Today we will take up 2 examples where we cannot assume continuity of an inductor current or the capacitor voltage. So; that means, the inductor current may jump from t equals 0 minus to t equals 0 plus and so, can a capacitor voltage. We will look at 2 specific examples of such situations and then move on.

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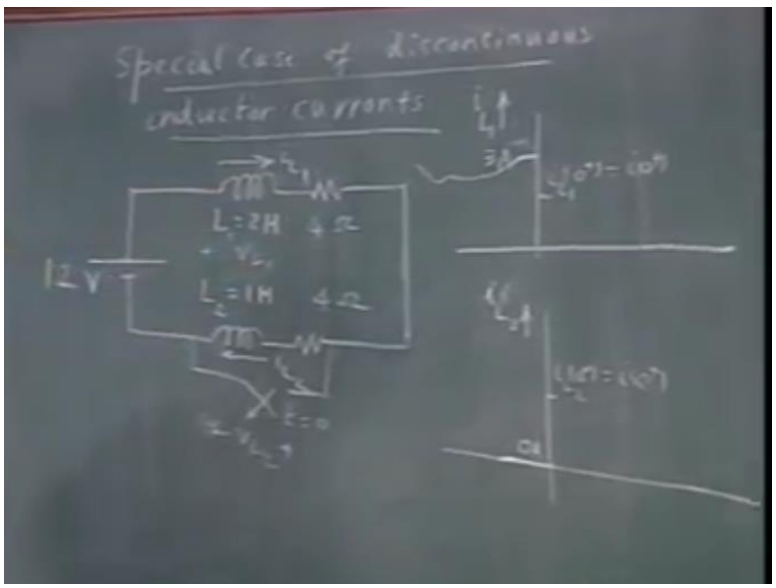

So, we will talk about special cases of discontinuous inductor currents. Let us take a circuit in which we have a 12 volts d c source is connected to a series combination of 2 inductors and 2 resistors. Let us call this l 1 call this l 2 1 henry and 4 ohms and let us have a switch, which is kept close for a long time and this switch is opened at t equals 0. Now, if i call this i l 1 the current in this inductor as i l 1 and the current in this inductor as i l 2.

Now the switch is opened at t equals 0 after it has been closed for a long time. So, let us see the nature of i l 1 and i l 2. This is the current in the first inductor this is the current in the second inductor i l 2. Now, when the switch is closed twelve volts drive a current to a 4-ohm resistor. Therefore, there is 3 ampere current because these are short circuit and therefore, at t equals 0 minus this current is 3 amperes.

So, whatever maybe it is reached that 3 amperes here. As far as, i l 2 is concerned because this is shorted the current passes through shorted switch avoiding this l 2. Therefore, it is 0 current is 0 here. Now, when the switch is opened out by Kirchhoff's current law the same current must pass through them. So, there must be a common current which is certainly cannot be equal to 3 amperes and 0 amperes at the same time.

Continuity of current in this inductor will demand that the i l 1 continues to be 3 amperes. The continuity of the current in the inductor will demand that i l 2 continues to be 0 amperes, but both these conditions cannot be matched because at t equals 0 plus the same current must pass through both. So, we have the situation that i l 1 0 minus is 3 amperes i l 2 0 minus is 0 amperes and we further require that i l 1 0 plus must be equal to i l 2 0 plus whatever that is.

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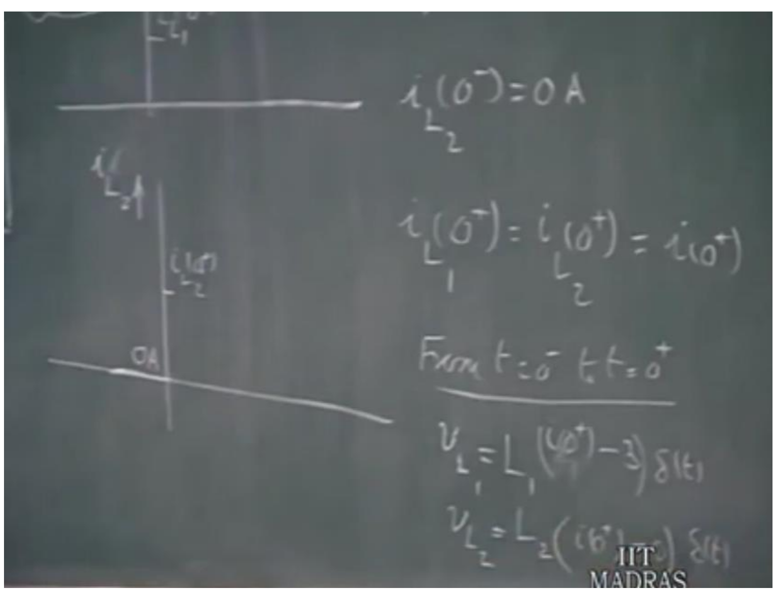

It certainly cannot be, if it is equal to 3 then the continuity of current in this is violated. If it is equal to 0 amperes then the continuity of the current in the first inductor is violated. Therefore, we must now find out i l 0 plus i l 2 0 plus. How do we do that? Now; obviously, there must be a jump in current because; obviously, this cannot continue at 3 amperes at 0 amperes.

So, there must be some kind of intermediate value therefore, if the current has jumped from 3 amperes to some intermediate value let us say i l 1 0 plus and this current had jumped from 0 to i l 2 0 plus both being equal to each other. And let us call this simply i 0 plus. Then there must the voltage across this inductor v l 1 and the voltage across the inductor v l 2 let us say v l 2 like this.

They must have some kind of some jumps in current therefore, there must a impulse present in this voltage as well as this voltage. So, the impulse in v l 1 from t equals 0 minus to t equals 0 plus. In this range, in this small range elementary range in the jump from 0 minus to 0 plus the voltage across inductance l 1 must be described as, l 1 times current had jumped from 3 amperes to i l 1 0 plus.

So, i l 1 0 plus which is simply i 0 plus right i 0 plus minus original current plus 3 amperes. So, that is the strength of the impulse. So, there must be an impulse voltage which is equal to l 1 times i 0 plus minus 3 times delta t because, suddenly this current had jumped from this to this. So, it is negative impulse as a matter of fact if i 0 plus is smaller than this is a negative impulse. And v l 2 is l 2 times i 0 plus minus the original current which is 0 times delta t.

So, the current in this inductor has jumped from 0 to i l 2 plus which is i 0 plus, which we will call i 0 plus. So, other voltages in this circuit are finite this is finite; the currents are finite. Therefore the resistance drops are also finite therefore; this impulse voltage must be matched by this impulse voltage. So, that Kirchhoff’s voltage law is satisfied.

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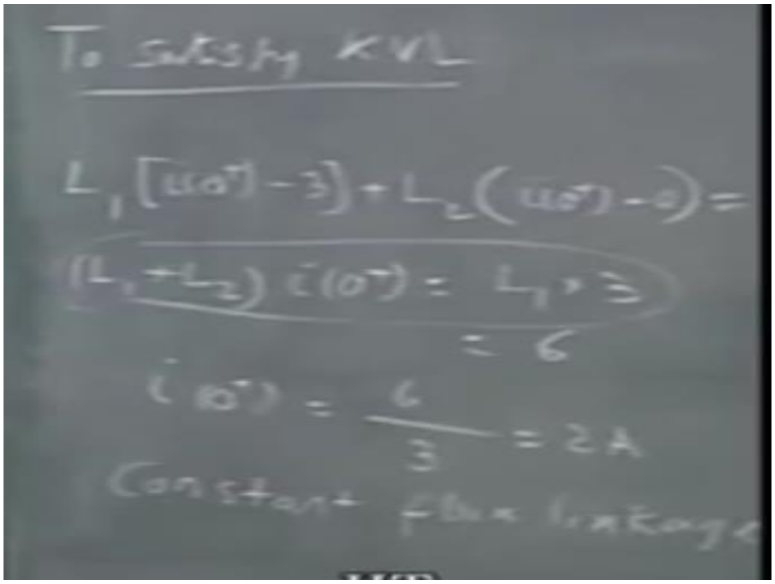

So, to satisfy Kirchhoff's voltage law we require therefore, that this these 2 impulses must be matched. So, l 1 i 0 plus minus 3 plus l 2 times i 0 plus minus 0. This must be equal to 0 because the strength of the impulse plus the strength of this impulse v l 1 plus v l 2 must be equal to 0. The 2 impulses must add up to 0.

So, solving this we get that l 1 plus l 2 times i 0 plus equals l 1 times 3 l 1 equals 2 henrys therefore, this is 6. Therefore, i 0 plus equals 6 divided by l 1 plus l 2 which is 3 that is 2 amperes. Therefore, this current had jumped from 3 amperes to 2 amperes and this current had jumped from 0 to 2 amperes.

Now, this particular equation that we have here is the 1 that now fixes the new value of the current. And this is usually referred to and constant flux linkage theorem constant flux linkage principle. What it means is l 1 plus l 2 times i 0 plus is the flux linkage associated with this circuit. So, total inductance times the current passing through that and this is initial flux linkage in the circuit because l 2 does not carry any current. This is the flux linkage associated with l 1.

So, flux linkages in a circuit which do not have any net impulse e m f in the circuit cannot change suddenly. The result is that the old flux linkages must be equal to the new flux linkages and therefore, that fixes the new value of the current. This is another way of looking at it. In any case, what we want to demonstrate through this example is that there may be cases where, inductor currents can be discontinuous.

And such cases arise whenever as a result of a switching operation a new restriction of the inductor currents is brought into effect or brought into force where previously none existed. Therefore, whatever currents which they are having at t equals 0 minus will no longer satisfy Kirchhoff's current law in the new regime. Therefore, 1 or more of the currents have to jump and that can be dissolved using principles like this by trying to match the impulse voltages that arise are using the principle of constant flux linkages.

Remember that, the y we normally say inductor current is continuous in elementary treatment is because; we say, if the inductor current had jumped there must be an impulse voltage infinite voltage across the inductor. Since, all other voltages are finite this infinite voltage cannot exist therefore, inductor current must be continuous. But, in situation like this no doubt an impulse voltage arises across an inductor.

But that is matched by another impulse voltage across the second inductor and third inductor as the case may be and the whole Kirchhoff's current law can be satisfied only if such impulse voltages exist. Therefore, this is a particular case which 1 has to keep in mind whenever as a result of a switching operation a new constraint on inductor currents is brought about.

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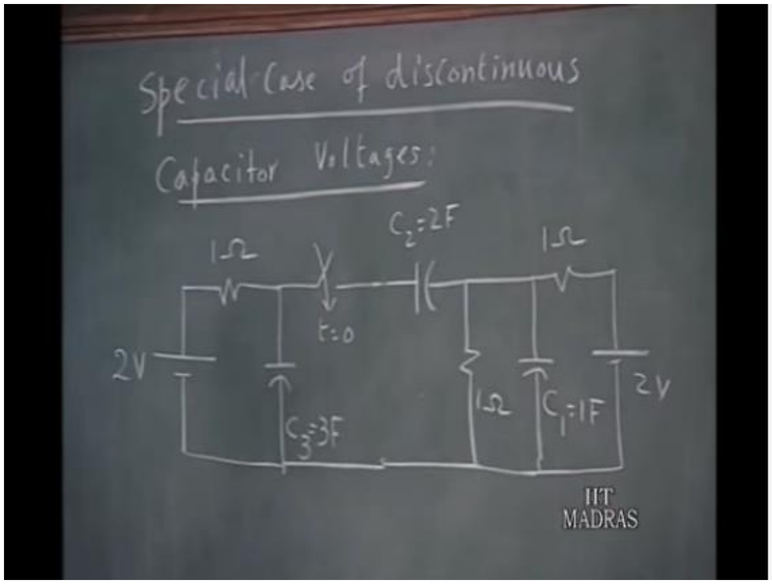

Just as inductor currents can be discontinuous in special cases and we have to calculate the t equals 0 plus values given the t equals 0 minus values using techniques, as we have seen in the last example. The similar situation arises in special cases where the capacitor voltages at t equals 0 minus need not be the same as the values at t equals 0 plus. Let us consider this example, let us imagine that switch is kept open for a long time till all the voltages get stabilized and let us assume that this c 2 is also initially not charged.

In which case, the voltage across this capacitor becomes 1 volt because this 2 volts get divided across the 2 1 ohm resistors. Therefore, v c 1 will be 1-volt whereas, this becomes 2 volts this is 2 volts this is 1 volt and this is 0 volts. But once you close the switch these 3 capacitors form a loop. Therefore, all the 3 voltages must add up to 0 and if this is 2 volts and this is 1 volt and this is 0 volts there is no way in which all the 3 voltages are going to add up to 0.

Therefore, something must give in and what happens is all the voltages of the capacitors change to satisfy Kirchhoff’s voltage law. And therefore, the voltages of the capacitors can be discontinuous. Let us see how we go about it. At t equals 0 minus we are interested only in the capacitor voltages because all the other voltages all the other elements carry finite currents. And because of the jump in capacitor voltages infinite currents or impulse currents must flow through the capacitors.

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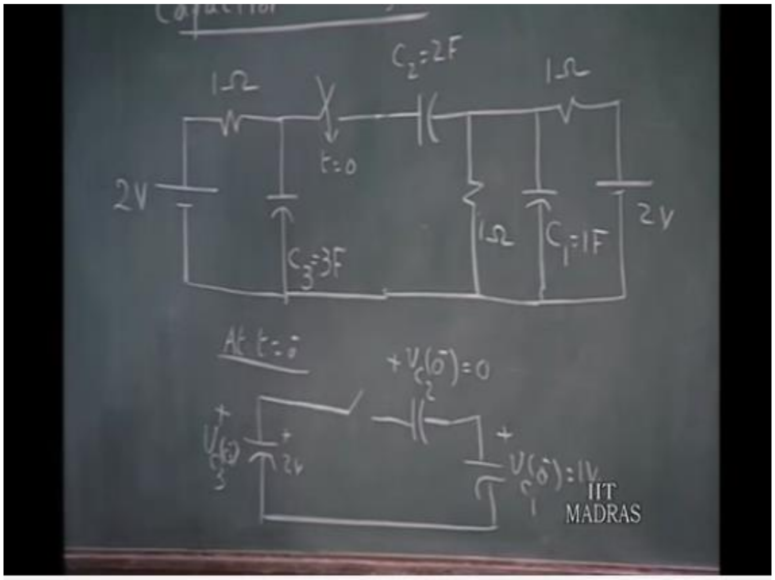

And therefore, we are taking stock of only the impulse currents in this transition. Therefore, all other element currents are finite we can disregard them. So, we have this c 3 i will call that v c 3 0 minus which is 2 volts and there is a switch of course, and this i will say v c 2 0 minus that is of course, 0. And then, you have this capacitor c 1 v c 1 0 minus that happens to be 1.

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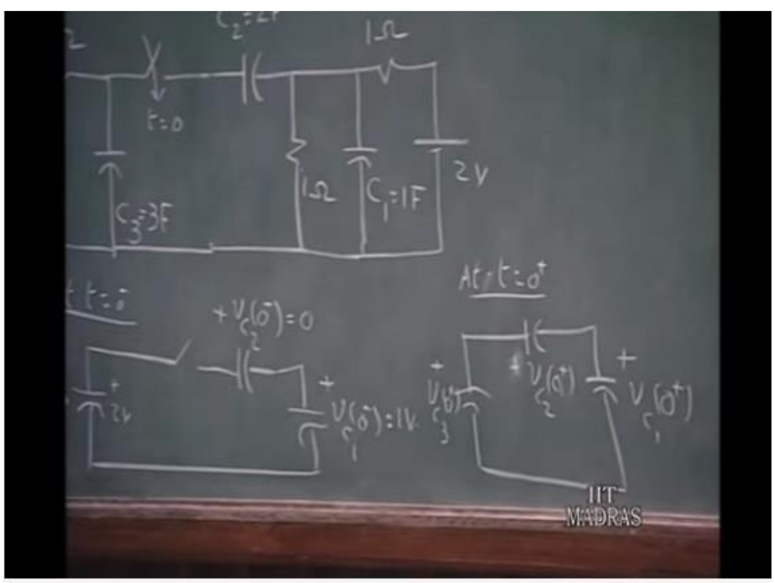

So, that is the situation at t equal 0 minus and because this switch is open there is no necessity for all the voltages to lead up to 0 because, the balance residual voltage is absorbed by the switch. Now, at t equals 0 plus you have again the switch been closed therefore, these 3 capacitors form a loop v c 1 0 plus. And let us say this is v c 2, this is the reference sign we have taken v c 2 0 plus and this is v c 3 0 plus.

So, we have a new regime at t equals 0 plus v c 3 0 plus v c 2 0 plus and v c 1 0 plus. Kirchhoff's voltage law tells us v c 3 0 plus equals v c 1 0 plus plus v c 2 0 plus right.

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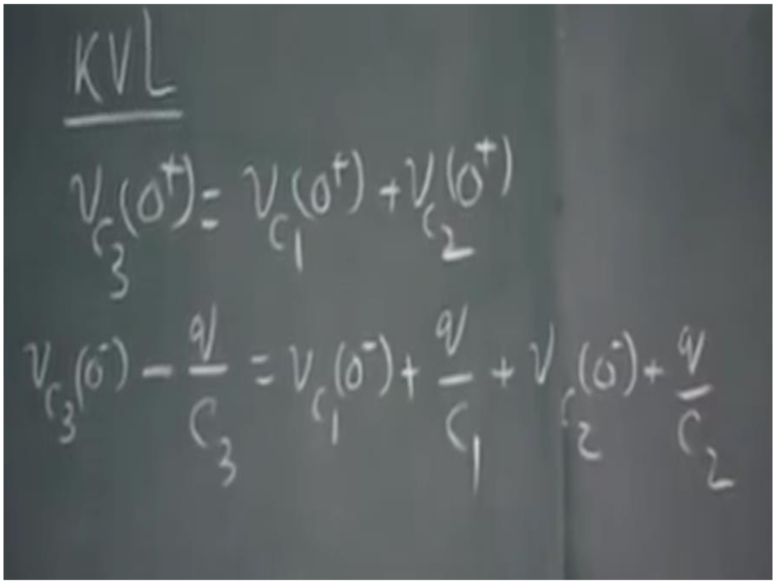

But what is v c 3 0 minus 2 volts? V c 3 0 plus may be not be 2 volts. So, there must be some initial instantaneous change of capacitor voltage. That means, in the process between t equals 0 minus to 0 plus and charge q must flow through these 3 capacitors. This q is the charge flow through the loop in the interval t equals 0 minus to t equals 0 plus. So, there must be a sudden increase or decrease in charge in the capacitors. That means impulse current must flow.

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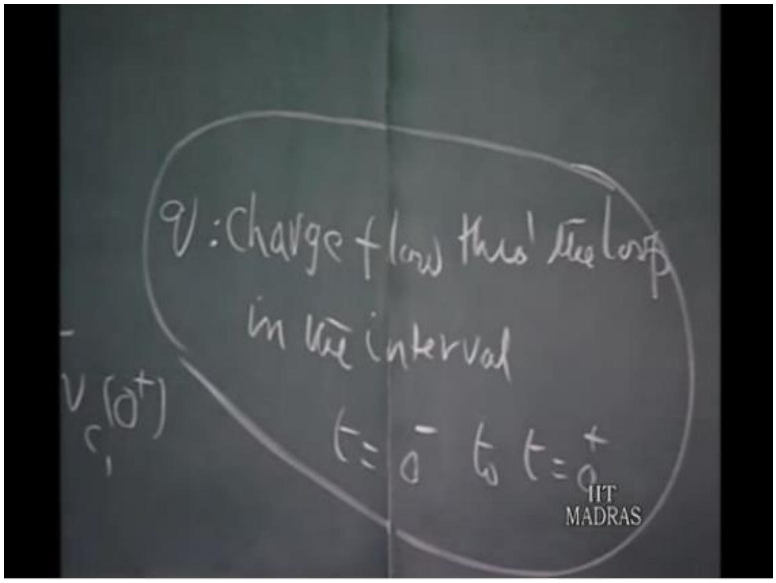

D q by d t will turn out to be impulse the current is impulse, but the area under the impulse is finite. Therefore, instantaneously an additional charge must flow and what is the value of this charge v c 3 0 plus equals v c 3 0 minus this is v c 3 0 minus, And there is a charge q coming out therefore, the resulting voltage drop is q by c three. V c 3 0 minus q by c 3 because, there is some charge when q comes out some of it is getting discharged and the amount of charge presents a voltage drop of q minus c 3.

So, v c 3 0 plus is v c 3 0 minus divided by minus q minus c 3 and v c 1 0 plus is equal to v c 1 0 minus plus q divided by c 1. Some initial charge is put on this capacitor and similarly v c 2 0 minus plus q by c 2 and this q is a common charge which flows through this loop. That means, there is an impulse current; which integrated over the interval from 0 minus to 0 plus represent a non-0 amount of charge.

And in this equation, we know v c 3 0 minus v c 1 0 minus and v c 2 0 minus, we can calculate q due to the numerical values. That turns out to be if you solve this you will have let me write this down here q times 1 over c 1 plus 1 over c 2 plus 1 over c 3 solving this equation you will get this as 1.

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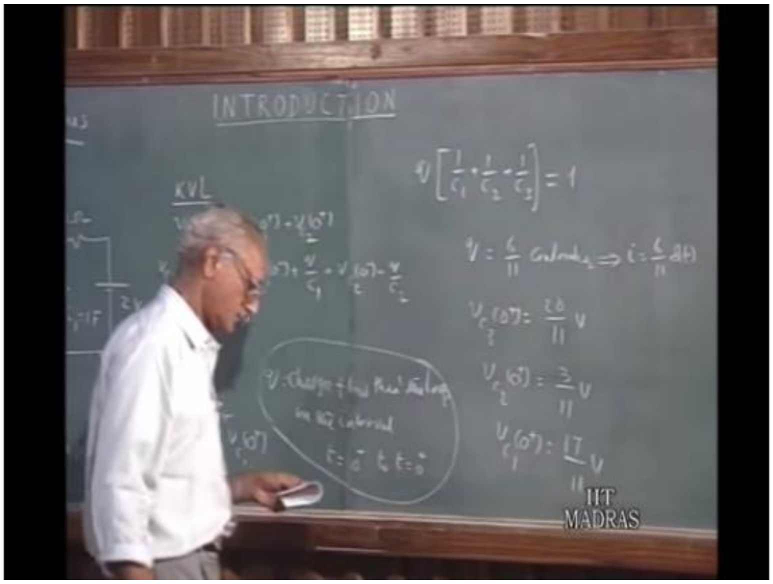

That is the balance the unbalanced voltages that we get here and from we get q equals 6 by eleven coulombs. And therefore, you have v c 3 0 plus we can calculate using this. It becomes 20 up on 11 volts and v c 2 0 plus equals 3 up on 11 volts and v c 1 0 plus will be 17 up on 11 volts. So, the capacitor voltage is indeed had jumped and this represents an impulse current of 6 up on 11 delta t.

This is the description of the current in the interval t equals 0 minus to t equals 0 plus all the other the components which are finite we have ignored.

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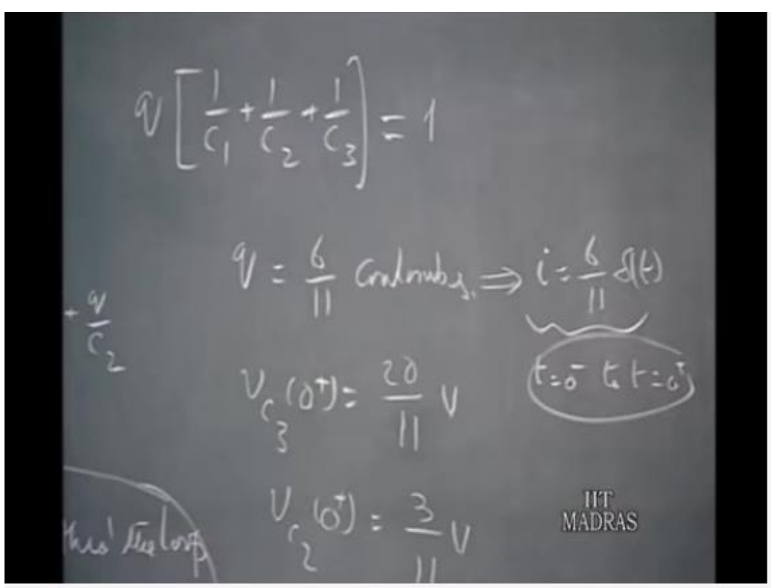

So, this example illustrates the situation where the capacitor voltages have to change and therefore, when you want to solve this for the transient analysis of this problem. If you know the capacitor voltages at t equals 0 minus and for the solution of the differential equations we need to know the initial conditions at t equals 0 plus, we have to calculate these values.

This point of course, 1 of the difficulties in the classical method of solution of differential equations from t equals 0 minus; you have to calculate the t equals 0 plus conditions. Not only for various response quantities, but even for reactive elements sometimes you may have to calculate these values using principle of that like this.

What we have really assumed here is that as far the capacitors are concerned the charge is concerned whatever is discharged here it goes to charge these 2. Therefore, this is just like the principle of conservation of flux linkages as applicable to inductors is what we discussed earlier, this is the principle of conservation of charge across the capacitors.

So, we will leave this discussion at this stage all my intention is to point out that in the calculation of initial conditions, you have problems in the classical differential equation approach. Not only to calculate the initial have conditions and the various initial values liked the various derivatives of the response quantities. But even for reactive elements themselves it may or may not be continuous depending up on the special situation that we have on hand.